

Research Article

Hanane Serhane*, Kouider Bendine, Farouk Benallel Boukhoulda, and Abdelkader Lousdad

Numerical analysis based on finite element method of active vibration control of a sandwich plate using piezoelectric patches

<https://doi.org/10.2478/mme-2020-0005>

Received Sep 18, 2019; accepted Jun 01, 2020

Abstract: An active method of vibration control of a smart sandwich plate (SSP) using discrete piezoelectric patches is investigated. In order to actively control the SSP vibration, the plate is equipped with three piezoelectric patches that act as actuators. Based on the classical plate theory, a finite element model with the contributions of piezoelectric sensor and actuator patches on the mass and stiffness of the sandwich plate was developed to derive the state space equation. LQR control algorithm is used in order to actively control the SSP vibration. The accuracy of the present model is tested in transient and harmonic loads. The applied piezoelectric actuator provides a damping effect on the SSP vibration. The amplitudes of vibrations and the damping time were significantly reduced when the control is ON.

Keywords: Sandwich smart piezoelectric plate; control; vibration; sandwich, modeling; finite element; LQR

1 Introduction

Sandwich structures are used in many engineering fields, such as aeronautics and astronautics, due to their superior mechanical characteristics such as high rigidity and bending strength provided by the skins as well as for their lightness and flexibility offered by the core. This flexibility can cause significant problems, such as structural instability, and allows undesirably large vibration amplitude. It is thus important to know the vibration characteristics for the structures. Recently, different investigations dealt with the vibration performances of the sandwich structures. [1] presented the dynamic response for free and forced vibration. They measured the sandwich as orthotropic and represented elastic constants that vary with the core configuration. In another study, [2] investigated a bending behavior of sandwich structures using foam core. [3] investigated the vibration response of clamped circular monolithic and sandwich plates under impact loading. The results show that the sandwich plates have higher impact strength than monolithic plates when the masses are similar.

In order to reduce the problems caused by vibrations and to improve the structures' efficiency and reliability, damping and control of the undesirable vibrations are required. Conventional methods such as passive damping are widely used to reduce the structure vibration and they consist of adding an additional viscoelastic material. This caused a significant increase in weight that makes it unusable in the aerospace applications [4, 5]. A new way to damp the structures' vibration called active vibration control was introduced by the research community to overtake the weakness of the passive vibration control. The active vibration control is based on three key points: actuators, sensors and control design. The structures are coupled with piezoelectric sensors and actuators to create self-control intelligent structures. The measures provided by the sensors are processed by an appropriate control system that sends a signal to the actuators, which is capable to change the structure behavior and adapting it to the required position.

***Corresponding Author: Hanane Serhane:** Mechanics of Structures and Solids Laboratory, Department of Mechanical Engineering, Faculty of Technology, Djillali Liabes University of Sidi Bel-Abbès, Algeria; Email: hserhane90@gmail.com

Kouider Bendine: Mechanics of Structures and Solids Laboratory, Department of Mechanical Engineering, Faculty of Technology, Djillali Liabes University of Sidi Bel-Abbès, Algeria; Email: Kouider84@live.com

Farouk Benallel Boukhoulda: Mechanics of Structures and Solids Laboratory, Department of Mechanical Engineering, Faculty of Technology, Djillali Liabes University of Sidi Bel-Abbès, Algeria; Email: boukhoulda_22000@yahoo.fr

Abdelkader Lousdad: Mechanics of Structures and Solids Laboratory, Department of Mechanical Engineering, Faculty of Technology, Djillali Liabes University of Sidi Bel-Abbès, Algeria; Email: a_lousdad@yahoo.com

Various researches have focused on piezoelectric sensor and actuator applications in smart structures (beams, plates and shells). The active control of vibration for composite beam distributed with piezoelectric sensors and actuators was investigated by several authors as [6] and [7]. In the study conducted by [8], a numerical modeling of composite plate with piezoelectric layers was proposed and the authors used the classical plate theory to study the shape control of the plate. [9] proposed a finite element model using eight-noded isoparametric element based on the first order shear deformation theory to investigate the stretching-bending coupling effect of the piezoelectric patches on the stability of smart composite plate. [10] developed a finite element model based on the third order shear theory of a composite plate with four piezoelectric patches. [11] developed finite element to mold a composite plate with piezoelectric patches for active vibration control based on negative velocity feedback theory. While, [12] dealt with H2 controller for the active damping. The controller was designed to study the control performance of an elastic plate equipped with three piezoelectric patches. In the work of [13], a comparison was made between PID control and LQR optimal control of piezoelectric bonded smart plates. The results of this study showed that the LQR control is better. The paper of [14] examined the active vibration control based on linear quadratic regulator (LQR) and negative velocity feedback for the conical shells.

Less contributions papers have been made on active vibration control in sandwich structures with the piezoelectric materials. The first interesting study to mention is that of [15]. They provided a finite element model of smart beam containing shear actuators by modeling the skins using classical Euler–Bernoulli beams and the core as a Timoshenko beam. In another study, [16] studied the piezoelectric material in smart sandwich structures for dynamic and static behavior using the method of Ritz, which is based on analytical solutions. [17] had proposed LQR algorithm on sandwich beam with extension and shear actuators to study the effect of extension and shear actuator to damp the vibration of structures. However, in the work of [18], an analytical solution was used to study a sandwich plate with a piezoelectric core based on Raleigh–Ritz and stationary potential energy methods. Sanjay *et al.* (2009) developed a finite element model for the analysis of three layers sandwich plate treated with Passive Constrained Layer Damping (PCLD) and Active Constrained Layer Damping (ACL D) to find the effect of constrained layer damping in controlling the vibration of sandwich structure. [19] investigated the active vibration control of a free-edge rectangular sandwich plate. They proposed a control algorithm

based on the (PPF) technique and in order to control the first four normal modes.

Concerning the present work, a contribution compared to the previous works is provided. An active vibration of sandwich plate with three piezoelectric patches bonded on the top and bottom faces core is considered. An important motivation of this work is to present a finite element model based on the plate classical theory using Ansys to design structure and algorithm controller (LQR) to suppress the vibration. In the first step, the natural frequencies are calculated and validated using Ansys and Matlab software. Then, a linear quadratic regulator algorithm (LQR) is developed to investigate the active vibration control of cantilever smart sandwich plate boundary conditions under two different excitations, transient and harmonic loads.

2 Mathematical Modeling

In the following subsections, the strategy of the developed method is described and implemented for active vibration control of smart sandwich plate (SSP) using discrete piezoelectric patches.

2.1 Behaviors law

Consider a sandwich plate with three layers discredited using a four-noded rectangular element, based on the classical plate theory [20, 21]. The element used include four node finite element model with three displacement degrees of freedom for each node, namely the displacement \bar{w} in the z direction, two rotations φ , ψ about (x, y) axis as well as one additional electrical degree of freedom (Φ) for piezoelectric patches Figure 1. The three layers of sandwich are restricted to linear elastic material behavior and assumed to be perfectly glued together, ensuring the continuity of the displacement field at the interfaces, in which the cross-section remains straight after the deformation during bending and the displacement is the same in the three layers. If we include the nonlinearity of the core in the model, it will face an enormous rising of the calculation time. Hence, we have proposed a linear model based on the fact that the section shape will maintain its initial form when the thickness is too small when we compare it to the length and the width; thus, the shear effect is too low to be considered. According to the reference [22], the difference between displacements through the thickness of the

core (viscoelastic material) and the skins (isotropic material) is very small.

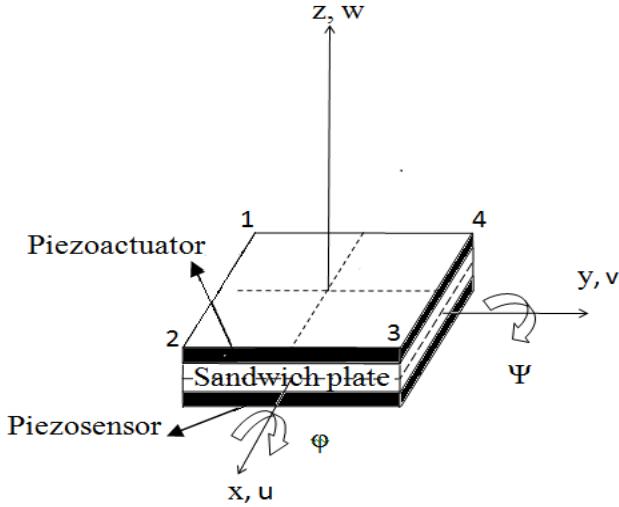


Figure 1: Coordinate system of sandwich plate finite element with integrated piezoelectric material.

Using Kirchhoff supposition, the relation between plane stress σ and the deformation can be written as follows:

$$\sigma = [D] \{ \epsilon \} \quad \text{with } \sigma = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_{xy} \end{bmatrix}^T \quad (1)$$

where $[D]$ is the rigidity matrix and it is expressed by:

$$[D] = \frac{E}{(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (2)$$

ν is Poisson's ratio and E the Young's modulus.

Each node of the element has a displacement \bar{w} in the z direction, a rotation φ about x axis and a rotation ψ about y -axis.

The transverse displacement field w can be expressed by:

$$w(x, y) = \{ \bar{p}(x, y) \}^T \{ \bar{a} \} \quad (3)$$

where the coefficients of the vectors $\{ \bar{a} \}$ and $\{ \bar{p} \}$ are represented by the following equations:

$$\bar{a} = [a_1 \ a_2 \ a_3 \ a_4 \ a_5 \ a_6 \ a_7 \ a_8 \ a_9 \ a_{10} \ a_{11} \ a_{12}]^T \quad (4)$$

$$\bar{p} = [1xyx^2 \ xy y^2 \ x^3 \ x^2yx y^2 \ y^3x^3 \ yxy^3]^T \quad (5)$$

The vector $\{ \beta_i \}$ is defined as the nodal displacements fields of an element and they are given by the following expression (Eq. (6)):

$$\beta_i = [\bar{w}_1 \ \varphi_1 \ \psi_1 \ \bar{w}_2 \ \varphi_2 \ \psi_2 \ \bar{w}_3 \ \varphi_3 \ \psi_3 \ \bar{w}_4 \ \varphi_4 \ \psi_4]^T \quad (6)$$

The global displacement can be discredited by the following expression:

$$\begin{cases} \bar{w} = \bar{p}^T \bar{a} \\ \varphi = \frac{dw}{dx} \\ \psi = \frac{dw}{dy} \end{cases} \quad (7)$$

Substituting (x, y) coordinate values for the four nodes of Eq. (3) and Eq. (6) in Eq. (7) yields to the following matrix expression:

$$\{ \beta_i \} = [PM] \{ \bar{a} \} \quad (8)$$

where $[PM]$ is a 12×12 matrix by replacing the coordinate values (x, y) for the four nodes:

$$[PM]_{12 \times 12} = \begin{bmatrix} \bar{P}^T(0, 0) \\ \frac{\partial \bar{P}^T(0,0)}{\partial x} \\ \frac{\partial \bar{P}^T(0,0)}{\partial y} \\ \bar{P}^T(x, 0) \\ \frac{\partial \bar{P}^T(x,0)}{\partial x} \\ \frac{\partial \bar{P}^T(y,0)}{\partial y} \\ \bar{P}^T(x, y) \\ \frac{\partial \bar{P}^T(x,y)}{\partial x} \\ \frac{\partial \bar{P}^T(x,y)}{\partial y} \\ \bar{P}^T(0, y) \\ \frac{\partial \bar{P}^T(0,y)}{\partial x} \\ \frac{\partial \bar{P}^T(0,y)}{\partial y} \end{bmatrix}_{12 \times 12} \quad (9)$$

Therefore, the coefficient vector $\{ \bar{a} \}$ can be computed from Eq. (8):

$$\{ \bar{a} \} = [PM]_{12 \times 12}^{-1} \{ \beta_i \} \quad (10)$$

The coefficients of the vector $\{ \bar{a} \}$ are the polynomial coefficients. However, since the nodal displacements are variable, so these will use the coefficients form to get the shape function.

Substituting Eq. (10) into Eq. (3) yields:

$$w = [N_w] \{ \beta_i \} \quad (11)$$

where $[N_w]$ is the shape function matrix in the z axis given by:

$$[N_w] = \{ \bar{p} \}^T [PM]^{-1} \quad (12)$$

The relation of deformation-displacement can be written as follow:

$$\epsilon = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -z \frac{\partial^2 \bar{p}^T}{\partial x^2} [PM]_{12 \times 12}^{-1} \\ -z \frac{\partial^2 \bar{p}^T}{\partial y^2} [PM]_{12 \times 12}^{-1} \\ -z \frac{\partial^2 \bar{p}^T}{\partial x \partial y} [PM]_{12 \times 12}^{-1} \end{Bmatrix} \{ \beta_i \} = [B] \times \{ \beta_i \} \quad (13)$$

By mathematical manipulating of Eq. (13), it is to get:

$$\{\epsilon\} = -z [L_K] [PM]^{-1} [\beta_i] \quad (14)$$

$$[L_K] = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 6x & 2y & 0 & 0 & 6xy & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 6xy \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 6y^2 \end{bmatrix} \quad (15)$$

The displacement field u , v and w is obtained by vector $\{\beta\}$ as follow:

$$\{\beta\} = \{w \ u \ v\}^T \quad (16)$$

By substituting Eq. (14) in Eq. (16), we obtain the equation mentioned below:

$$\{\beta\} = [H] [L_M]^T [\beta_i] \quad (17)$$

where

$$[L_M]^{T1} = \left\{ \begin{array}{l} \{\bar{P}\}^T \\ \frac{\partial \{\bar{P}\}^T}{\partial x} \\ \frac{\partial \{\bar{P}\}^T}{\partial y} \end{array} \right\} \quad (18)$$

and

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -z & 0 \\ 0 & 0 & -z \end{bmatrix} \quad (19)$$

2.2 Linear piezoelectric equation

Considering the electromechanical behavior of piezoelectric layer, the electrical potential varies linearly through the thickness and is constant in the course of the piezoelectric layer. Linear piezoelectric equations are used because the literature [22, 23] show that they have efficiently predicted well the piezoelectric material behavior.

The electromechanical equations are given by:

$$\begin{cases} \sigma_p = c^e \epsilon_p - e^T \bar{E}_p \\ D_p = e \epsilon_p + Xp \end{cases} \quad (20)$$

where σ_p , ϵ_p , D_p et \bar{E}_p are respectively the mechanical stress, the mechanical deformation, the electric displacement and the electric field. c^e , e , Xp are respectively the matrix of elasticity, the matrix of the piezoelectric coefficients and the electric matrix.

2.3 Dynamic system equation

The general equation governing the motion of the structure with piezoelectric actuators with bonded sensors to

the surface of the structure was established by the Hamilton principle.

$$\int_{t_1}^{t_2} \delta [(K - S + w_e - \ddot{w})] dt = 0K \quad (21)$$

is kinetic energy, S potential energy, w_e and \ddot{w} the work done by external electric and mechanical forces. t_1 , t_2 are the times of variation of the energy.

The kinetic energy and the potential energy of the structure are expressed by the following relations:

$$K = \frac{1}{2} \int_V \rho \{\dot{\beta}\}^T \{\dot{\beta}\} dv \quad (22)$$

$$S = \frac{1}{2} \int_V \{\epsilon\}^T \{\sigma\} dv \quad (23)$$

where $\{\dot{\beta}\}$ is the differentiation of $\{\beta\}$ and dV is established by:

$$dV = dV_p + dV_a + dV_s \quad (24)$$

The indices p , a and s represent the plate, the actuator and the sensor elements, respectively.

According to [24], the global matrix equations governing the present SSP can be written as:

$$[M] \{\ddot{\beta}\} + [C_{damp}] \{\dot{\beta}\} + [K_u - K_{u\varphi} K_{\varphi\varphi}^{-1} K_{\varphi u}] \{\beta\} = -[K_{\varphi u}] \{\Phi\} \quad (25)$$

where $[M]$, $[K_u - K_{u\varphi} K_{\varphi\varphi}^{-1} K_{\varphi u}]$, $[C_{damp}]$, $K_{\varphi u}$ and $[F_m]$ are mass matrix, stiffness, damping, elastic-electric coupling stiffness matrices and the applied mechanical force, respectively. $\{\beta\}$ and $\{\Phi\}$ denotes structural displacement and electric potential.

Assuming that the system response is governed by the eigenmodes, the displacement can be expressed as:

$$\{\beta\} = [\Omega] \{Y\} \quad (26)$$

where $\{Y\}$ are the modal coordinates and $[\Omega]$ is the modal matrix.

Introducing the variable $X = \{Y \ \dot{Y}\}^T$, the state space equation for the dynamic system equation can be written as:

$$\dot{X} = [A] \{X\} + [B] \{u\} \quad (27)$$

where $[A]$ is the system state matrix and $[B]$ Input control matrix which are given by:

$$\begin{cases} [A] = \begin{bmatrix} 0 & I \\ -[\hat{M}]^{-1} [\hat{K}] & -[\hat{M}]^{-1} [C_{damp}] \end{bmatrix} \\ [B] = \begin{bmatrix} 0 \\ [\hat{M}]^{-1} [K_{u\varphi}] \end{bmatrix} \end{cases} \quad (28)$$

where $[\hat{M}]$, $[\hat{K}]$, $[\hat{C}_{damp}]$,
 $[\hat{K}_{u\varphi}] = [\Omega]^T ([M] [K_u K_{u\varphi} K_{\varphi\varphi}^{-1} K_{\varphi u}], [C_{damp}]) [\Omega]$ and
 $[\hat{K}_{u\varphi}] = [\Omega]^T K_{u\varphi}$.

The output equations can be written as:

$$\{Y\} = [\hat{C}]\{X\} \quad (29)$$

$[\hat{C}]$ is the output matrix, which depends on the modal matrix $\{X\}$ and sensor piezoelectric stiffness matrix.

2.4 Linear quadratic algorithms

The proposed method is designed to minimize an “objective” function of a dynamic system proportional to the response to measure the performance of the control. The function used is given by:

$$J = \int_0^{\infty} (\{\bar{y}\}^T [Q] \{\bar{y}\} + \{\phi_a\}^T [R] \{\phi_a\}) dt \quad (30)$$

where $[Q]$: is the output weighting matrix, $[R]$: input weighting matrix.

It assumed that the most desirable state is $x = 0$; however, the initial condition is non-zero, so the matrix Q penalizes the state error in a mean-square sense. Similarly, the matrix R penalizes the control effort, that is, limits the control signals. $\{\phi_a\}^T [R] \{\phi_a\}$ is the scalar quantity of the output of the system under control and $\{\bar{y}\}^T [Q] \{\bar{y}\}$ is the scalar quantity of the system input.

The optimal feedback control force by $\{\phi_a\}$ is the application of classical LQR control method:

$$\{\phi_a\} = K \cdot \bar{y}(t) \quad (31)$$

The gain matrix $K = R^{-1} B^T P$ that minimizes J can be found by solving a matrix Riccati equation that given by:

$$[P] [A] + [A]^T [P] + [Q] - [P] [B] [R]^{-1} [B]^T [P] = 0 \quad (32)$$

3 Numerical modeling Ansys

3.1 Material of the sandwich plate

The sandwich plate used is rectangular. It is constituted by three layers. The top and the bottom layer are made of the same steel used by [21], while the core is made of polymer.

The properties of the materials constituting the sandwich plate are summarized in (Table 1). The geometrical properties of the structure are noted: L_x (for the length),

L_y (for the width) and h (for the thickness). The mechanical and geometrical properties of each layer are summarized in the Table 1 bellow.

Table 1: Mechanical and geometrical properties of the layers of the sandwich plate.

Property	Plate 1-3 (Skins)	Plate 2 (Core) [25]
E : Young’s modulus, (N.m ⁻²)	21e10	35e 06
ρ : Density, (Kg.m ⁻³)	7810	32
ν : Poisson’s coefficient	0.38	0.38
h : Thickness, (mm)	2	3
L_x : length, (mm)	500	500
L_y : width, (mm)	500	500

3.2 Piezoelectric material

The SSP considered is bounded by three piezoelectric patches. The piezoelectric material used is composed of lead zirconate titanate is an intermetallic inorganic compound with the chemical formula $Pb [ZrxTi1-x]O_3$ ($0 \leq x \leq 1$), called PZT. The mechanical, electrical and geometrical properties of piezoelectric patch used are summarized in the Table 2 bellow.

Table 2: Mechanical, electrical and geometrical properties of the piezoelectric layer.

Property	Piezoelectric (PZT)
E (Young’s modulus) (N.m ⁻²)	69e10 ⁹
ρ (Density) (Kg.m ⁻³)	7700
ν (Poisson’s ratio)	0.3
ϵ^e (Piezodielectric constant) (Fm-1)	1.6e10 ⁻⁸
E (Piezoelectric stress) (NV-1 m-1)	-12.5
C (Capacitance) (F)	6.3e10 ⁻⁷
l_x (mm)	50
l_y (mm)	50
e_p (mm)	0.5

3.3 Meshing and boundary conditions

Ansys computer code is used to perform the numerical modeling of the system structure. The finite element model of the SSP is created in Ansys software. Two kinds of element type are used. Three-dimensional continuum brick finite element SOLID45 (Figure 2.a) was used to mesh the three layers of the sandwich. SOLID5 (Figure 2.b) element is used for piezoelectric layer and has an additional Volt degree of freedom, which is the key of coupling the strain field and electrical field. In fact, SOLID5 has a three-dimensional magnetic, thermal, electric, piezoelectric and

structural field capability with limited coupling between the fields. The element has eight nodes with up to six degrees of freedom at each node. Scalar potential formulations (reduced RSP, difference DSP, or general GSP) are available for modeling magnetostatic fields in a static analysis. The plate was discretized into 10×10 finite elements. Analysis will be done for cantilever sandwich plate with boundary conditions presented in (Figure 2.c).

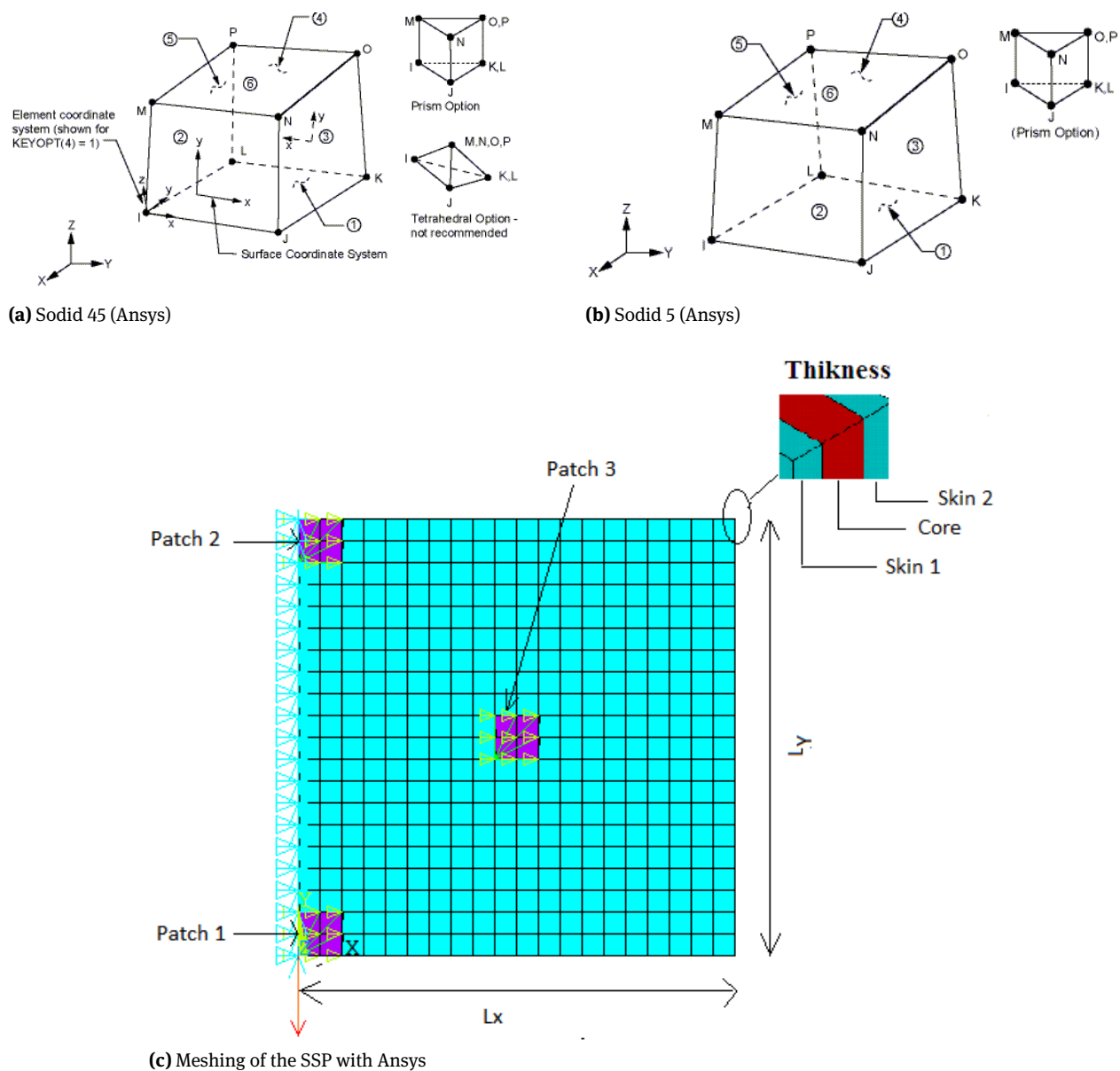


Figure 2: Modeling the sandwich plate with Ansys.

3.4 Algorithm of LQR control scheme

The flowchart of (Figure 3) below shows the Ansys algorithm proposed for the simulation of the active vibration control including LQR control schematic. The discrete linear time invariant state space system defined by A, B matrix and the LQR gain K have been computed based on the finite element method, MATLAB program and then implemented in Ansys.

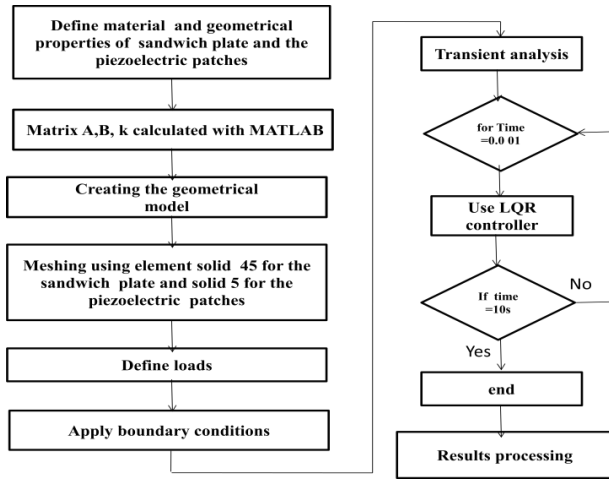


Figure 3: Control scheme used to validate the proposed LQR control algorithm of the sandwich plate

4 Results and discussion

4.1 Model validation

In order to validate the present finite element model, the first step is to develop and validate a finite element program under Matlab and Ansys to obtain the first six natural frequencies of an isotropic plate made of steel. The results found were compared with those of literature [21]. The error varies from 2 to 6% as shown in (Table 3). The error has been estimated based on the following formula:

$$Error = (Present - literature)/literature$$

The table shows that the obtained results are in good agreement, which prove the precision of the present methodology.

In the second step of validation, a Matlab code is developed to perform a modal analysis to illustrate the eigenfrequencies and eigenmodes for the sandwich plate studied. Table 4 and Figure 4 presents the results obtained of the

Table 3: Natural frequencies of skins.

Modes	Present FEM code (Matlab) (Hz)	Code Ansys (Hz)	Reference (Hz) [21]	Error (%)
1	7.37	6.93	7.19	3
2	17.32	17.99	17.90	5
3	44.15	42.55	43.68	2
4	55.33	54.36	55.64	2
5	63.81	61.88	64.57	4
6	109.03	108.58	109.22	6

five natural modes and forms respectively. The results obtained with the present developed code are in good agreement with those obtained by using Ansys. The error varies from 3 to 12%.

Table 4: Natural frequencies of the sandwich plate.

Modes	Code Ansys (Hz)	Present FEM code (Matlab) (Hz)	Error (%)
1	23.57	22.55	3
2	43.00	44.39	6
3	122.54	139.31	11,5
4	186.92	171.74	12

4.2 Dynamic analysis and control

To demonstrate the capability of the LQR control algorithm developed under Ansys Apdl, a sandwich plate with the same properties as in Table 1 has been used as prototype specimen. The required space state equation is defined based on a finite element Matlab program.

The value of the state matrices A and B as well as the LQR gain K calculated by the developed Matlab program are given below:

$$A = \begin{bmatrix} 0 & 1 \\ -0.1736e^5 & -0.7906 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -0.0009 & 0.0001 & -0.0003 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.066e^5 & -1.062e^5 \\ -0.019e^5 & 0.55e^5 \\ 0.31e^5 & -1.60e^5 \end{bmatrix}$$

The gain matrix $K = R^{-1}B^T P$ that minimizes the cost function J is found by solving a matrix Riccati (Eq. 32). The main stages of programming of the LQR control are:

- Storing A and B state space matrices in Ansys Apdl using Ansys code-specific commands:
 - * DIM: for the size of finite elements
 - * SET: for the elements defined
- Programming the state equation under Ansys Apdl and applying the control in a dynamic analysis.

4.2.1 First simulation: Impulsion load

In the first vibration control analysis, the proposed control algorithms are applied to a cantilever SSP bounded by three actuators. The smart structure is subject to an impulsion load of 1N for a sampling time equal to 0.01 second. The deflections of the plate with and without control are presented in (Figure 4). It is clearly visible from the figure that the macro developed under Ansys is successful in reducing the vibration in terms of amplitude and settling time. The figure highlight that the plate vibration is rapidly attenuated in less than 1.5 s when the control is ON and the damping coefficient is equal to 49%. When the control is OFF, the vibration disappears in about 8 s and the damping coefficient is equal to 35%.

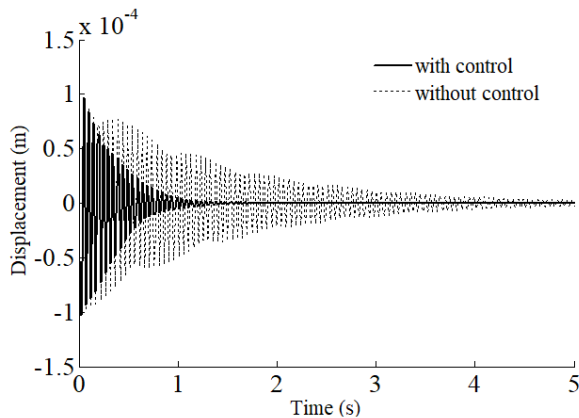


Figure 4: Transient vibration response of SSP with and without control

The damping coefficient of the structure is calculated with Eq. (33) presented below by the application of the logarithmic decrement.

$$\xi = \frac{1}{\sqrt{1 + \left(\frac{4\pi^2}{P^2}\right)}} \quad \text{with } P = \log \frac{y_1}{y_2} \quad (33)$$

where y_1 and y_2 are the amplitude of the first and second peak successively.

The actuators' voltages obtained by the control algorithm are shown in Figure 5 for actuator 1, actuator 2 and actuator 3. It must be noticed that a saturation condition has been added to the actuators 1 and 2 to prevent their failure.

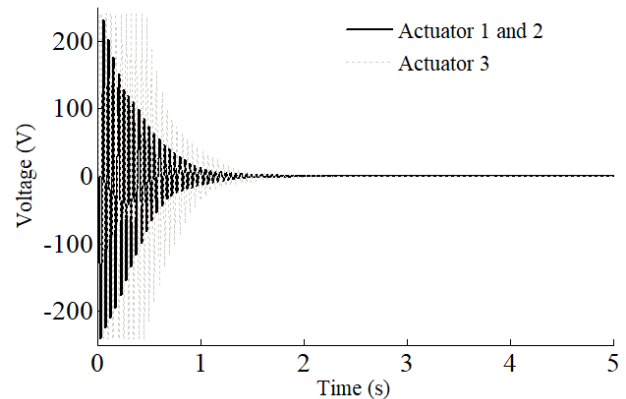


Figure 5: Control voltage applied on the actuators

4.2.2 Second simulation: Harmonic load

The numerical results presented in Figure 6 were obtained from Ansys Apdl code. We consider that the sandwich plate by deforming the plate by harmonic load $f = 10 \times \sin(w_1 t)$ at the center of the free edge. The vibrations are given with and without electromechanical coupling as observed in (Figure 6). The LQR method is effective to reduce

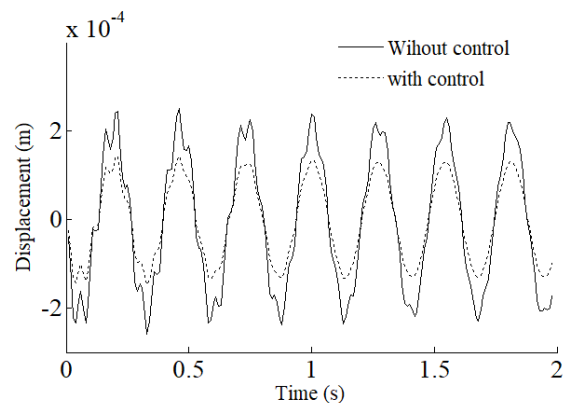


Figure 6: Harmonic vibratory response of SSP with and without control

the amplitude in harmonic load. The damping coefficient is 15% when the control system is ON.

Figure 7 shows the feedback voltage for each actuator. These figures reveal that the actuators 1 and 2 received the same voltage due to the symmetry of the plate. The control voltage is small and is easy to realize in practice.

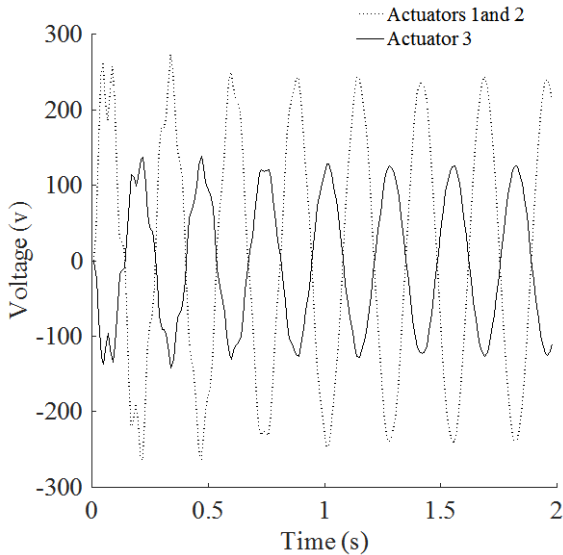


Figure 7: Control voltage applied on the actuators

5 Conclusion

In this paper the problem of damping flexural vibrations of a sandwich plate by using piezoelectric sensors and actuators has been addressed. To this end, LQR algorithm has been applied in the controller design through Ansys and Matlab. This technique, which has not been considered nor investigated in the cited literature, is developed in this study. A finite element model based on the classical plate theory is adopted and programmed in Matlab to achieve the space state equation. Moreover, the obtained equation was coupled with LQR controller and integrated into Ansys through a macro file for the case of two types of loads. The results show that the vibration of the plate is reduced with great damping coefficient when LQR controller is applied compared to the other without LQR controller and settling time for the case of transient analysis and active vibration control of a sandwich plate in harmonic load is analyzed. It is also noticed that the settling time is decreased from 8 s to 1.5 s when the control is ON and the vibration can be effectively suppressed.

The results obtained illustrate that the LQR control method is effective for rapidly attenuating the vibration amplitudes of the sandwich plates and that the control voltage is practically achievable.

References

- [1] Lok T.S., Cheng Q.H.: Free and forced vibration of simply supported, orthotropic sandwich panel. *Comput. Struct*, 79(3), 2001, 301–312.
- [2] Frostig Y., Baruch M.: Bending of sandwich beams with transversely flexible core. *AIAA J.*, 28 (3), 1990, 523–531.
- [3] McShane G. J., Radford D. D., Deshpande V. S. and Fleck, N. A.: The response of clamped sandwich plates with lattice cores subjected to shock loading. *Eur. J. Mech. ASolids*, 25 (2), 2006, 215–229.
- [4] Lam M. J., Inman D. J. and Saunders, W. R.: Vibration control through passive constrained layer damping and active control, *J. Intell. Mater. Syst. Struct.*, 8 (8), 1997, 663–677.
- [5] Soong T. T., Dargush G. F.: *Passive energy dissipation systems in structural engineering*. Wiley, 1997.
- [6] Han J.H., Lee I.: Analysis of composite plates with piezoelectric actuators for vibration control using layerwise displacement theory. *Compos. Part B Eng*, 29 (5), 1998, 621–632.
- [7] Sun B., Huang D.: Analytical vibration suppression analysis of composite beams with piezoelectric laminae. *Smart Mater. Struct.*, 9 (6), 2000, 751.
- [8] De Abreu G., Ribeiro J. F., and Steffen Jr, V.: Finite element modeling of a plate with localized piezoelectric sensors and actuators, *J. Braz. Soc. Mech. Sci. En*, 26, (2), 2004, 117–128.
- [9] Wang S. Y., Quek S. T. and Ang, K. K. Vibration control of smart piezoelectric composite plates, *Smart Mater. Struct.*, 10 (4), 2001, 637.
- [10] Yasin M. Y., Ahmad N. Alam, and M. N.: Finite element analysis of actively controlled smart plate with patched actuators and sensors, *Lat. Am. J. Solids Struct.*, 7 (3), 2010, 227–247.
- [11] Peng X. Q., Lam K. Y. and Liu, G. R.: Active vibration control of composite beams with piezoelectrics: a finite element model with third order theory, *J. Sound Vib.*, 209 (4), 1998, 635–650.
- [12] Caruso G., Galeani S. and Menini, L.: Active vibration control of an elastic plate using multiple piezoelectric sensors and actuators, *Simul. Model. Pract. Theory*, 11 (56), 2003, 403–419.
- [13] Zhang S., Schmidt R. and Qin, X.: Active vibration control of piezoelectric bonded smart structures using PID algorithm, *Chin. J. Aeronaut.*, 28 (1), 2015, 305–313.
- [14] Li F.M., Song Z. G. and Chen, Z. B.: Active vibration control of conical shells using piezoelectric materials, *J. Vib. Control*, 18 (14), 2012, 2234–2256.
- [15] Zhang X. D., Sun C. T.: Formulation of an adaptive sandwich beam. *Smart Mater. Struct.*, 5 (6), 1996, 814.
- [16] Koconis D. B., aszló L. P., Kollá ar. and Springer, G. S.: Shape control of composite plates and shells with embedded actuators. I. Voltages specified, *J. Compos. Mater.*, 28 (5), 1994, 415–458.
- [17] Trindade M. A., Benjeddou A. and Ohayon R.: Parametric analysis of the vibration control of sandwich beams through shear-based piezoelectric actuation, *J. Intell. Mate. Syst. Struct.*, 10 (5), 1999, 377–385.

- [18] Zhang X. D., Sun C. T.: Analysis of a sandwich plate containing a piezoelectric core. *Smart Mater. Struct.*, 8 (1), **1999**, 31.
- [19] Zippo A., Ferrari G., Amabili M., Barbieri M. and Pellicano F.: Active vibration control of a composite sandwich plate, *Compos. Struct.*, 128, **2015**, 100–114.
- [20] Bendine K., Wankhade R. L.: Vibration control of FGM piezoelectric plate based on LQR genetic search, *Open J. Civ. Eng.*, 6 (1), **2016**, 1–7.
- [21] Kumar K. R., Narayanan S., The optimal location of piezoelectric actuators and sensors for vibration control of plates, *Smart Mater. Struct.*, 16 (6), **2007**, 2680.
- [22] Moita J. S., Araujo S., A finite element model for the analysis of viscoelastic sandwich structures, *computer . Struct.*, 89, **2007**, 1874–1881.
- [23] Bendine K., Boukhoulda B. F., Nouari M., and Satla Z.: Structural modeling and active vibration control of smart FGM plate through Ansys, *Int. J. Comput. Methods*, 14 (4), **2017**, 1750042.
- [24] Bendine K., Wankhade R. L.: Optimal shape control of piezolaminated beams with different boundary condition and loading using genetic algorithm. *Int. J. Adv. Struct. Eng.*, 9(4), **2017**, 375–384.
- [25] Trindade, M. A., Contrôle hybride actif-passif des vibrations de structures par des matériaux piézoélectriques et viscoélastiques: poutres sandwich/multicouches intelligentes, PhD Thesis, Conservatoire national des arts et metiers-CNAM, **2000**.