

Research Article

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Self-Maintenance Orbits Using The perturbations Due to Air Drag and Due to Earth's Oblateness

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Abstract: The focus of this paper is the design of a self-maintenance orbit using two natural forces against each other. The effect of perturbations due to Earth's oblateness up to the third order on both the semi-major axis and eccentricity for a low Earth orbit satellite together with the perturbation due to air drag on the same orbital parameters were used, in order to create self-maintenance orbits. Numerical results were simulated for a low earth orbit satellite, which substantiates the applicability of the results.

Keywords: Orbital Mechanics, Orbit Maintenance, Air Drag, Earth's Oblateness

1 Introduction

The study of satellite motion and its lifetime has been one of the most important topics of research over the past few decades. Many analytical treatments deal with the problem of satellite motion in order to calculate the perturbation on this motion, especially the perturbations due to Earth's oblateness and air drag.

The problem of satellite motion under atmospheric drag was studied by King-Hele *et al.* (1958) [1], wherein they showed that the air drag acted on an Earth's satellite by reducing the period of revolution T . The motion of satellite in the terrestrial upper atmosphere was studied by Sehna (1980) [2]. The potential of the non-spherical Earth models was initiated by Kozai (1959) [3]. The Hamilton equations for the satellite motion under the Earth's oblateness and atmospheric drag were solved using canonical transformation by Khalil (2002) [4]. A semi-analytic theory for a low Earth orbit satellite, taking into account the perturbations due to Earth's oblateness and atmospheric drag

was considered by Bezdek (2004) [5]. Ammar *et al.* (2012) [6] studied the satellite motion under the Earth's oblateness up to the third order. They showed that the Earth's oblateness had a secular perturbation on the semi-major axis and on the eccentricity of the satellite orbit.

Yin ka Fan *et al.* (2013) [7] simulated the orbit decay of LEO satellites, taking into account the perturbations due to Earth's oblateness and atmospheric air drag. Akram Masoud *et al.* (2018) [8] investigated the problem of artificial frozen orbits around the Earth; they designed such orbits taking into account the main zonal harmonics up to J_4 and solar radiation pressure with the minimum control thrust required.

The objective of this paper is to determine the perturbations due to oblateness and air drag in order to not only take them into account in the satellite maintenance operations, but also to use them against each other to fade the resultant perturbations on the satellite and create a self-maintenance orbit.

2 Formulation of the problem

2.1 The air drag model

The effect of perturbation due to air drag on the semi-major axis and on the eccentricity of the satellite can be written as [9]:

$$\left(\frac{da}{dt}\right)_D = -n a^2 \frac{A}{m} \rho C_D \frac{(1 + 2e \cos f + e^2)^{\frac{3}{2}}}{(1 - e^2)^{\frac{3}{2}}} \quad (1)$$

$$\left(\frac{de}{dt}\right)_D = -n a \frac{A}{m} \rho C_D \left(\frac{e + \cos f}{\sqrt{1 - e^2}}\right) \sqrt{1 + e^2 + 2e \cos f} \quad (2)$$

where

$$\rho = \rho_0 e^{[-(\eta - \eta_0)/H]} \quad (3)$$

a – is the satellite semi-major axis,

e – the satellite eccentricity,

A – the satellite's average cross-sectional area,

m – the satellite mass,

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n – the mean motion,
 f – the true anomaly,
 C_D – the aerodynamic drag coefficient,
 η – the satellite altitude at any time,
 η_0 – the satellite altitude at the perigee,
 ρ – the density of air at altitude η ,
 ρ_0 – the density of air at the standard altitude,
 H – the scale height, and
 E – is the eccentric anomaly.

The secular perturbation on the semi-major axis and eccentricity due to air drag can be formulated from equations (1) and (2) up to the $O [e^7]$ as

$$(\Delta a)_D = -n a^2 \frac{A}{m} \rho C_D \left(1 + \frac{3}{4} e^2 + \frac{21}{64} e^4 + \frac{55}{256} e^6 \right) t \quad (4)$$

$$(\Delta e)_D = -n a \frac{A}{m} \rho C_D \left(\frac{1}{2} e - \frac{5}{16} e^3 - \frac{9}{128} e^5 \right) t \quad (5)$$

2.2 The perturbation due to oblateness

The perturbations on the semi-major axis and on the eccentricity due to oblateness are [6]:

$$\begin{aligned} (\Delta a)_{ob} = & \frac{J_2 D_4 R^6 n t}{a^5 (1 - e^2)^{21/2}} (\alpha_1 + \alpha_2 \cos(2i)) \quad (6) \\ & + \alpha_3 \cos(4i) + \alpha_4 \cos(6i) \\ & + \frac{J_2^3 R^6 n t}{a^5 (1 - e^2)^{21/2}} (\alpha_5 + \alpha_6 \cos(2i) + \alpha_7 \cos(4i) \\ & + \alpha_8 \cos(6i)) \\ & + \frac{J_2^3 R^6 n^2 t^2}{a^5 (1 - e^2)^{21/2}} \left[(\alpha_9 + \alpha_{10} \cos(2i) + \alpha_{11} \cos(4i) \right. \\ & + \alpha_{12} \cos(6i)) + (\alpha_{13} + \alpha_{14} \cos(2i) + \alpha_{15} \cos(4i) \\ & + \alpha_{16} \cos(6i)) \cos 2\omega + (\alpha_{17} + \alpha_{18} \cos(2i) \\ & \left. + \alpha_{19} \cos(4i) + \alpha_{20} \cos(6i)) \cos 4\omega \right] \end{aligned}$$

$$\begin{aligned} (\Delta e)_{ob} = & \frac{J_2 D_4 R^6 n t}{a^6 (1 - e^2)^{19/2}} (\beta_1 + \beta_2 \cos(2i)) \quad (7) \\ & + \beta_3 \cos(4i) + \beta_4 \cos(6i) + \frac{J_2^3 R^6 n t}{a^6 (1 - e^2)^{19/2}} (\beta_5 \\ & + \beta_6 \cos(2i) + \beta_7 \cos(4i) + \beta_8 \cos(6i)) \\ & - \frac{J_2 J_3 R^5 n^2 t^2}{a^5 (1 - e^2)^{19/2}} (\beta_9 \sin(i) + \beta_{10} \sin(3i)) \\ & + \beta_{11} \sin(5i) \sin(\omega) + \frac{J_2^3 R^6 n^2 t^2}{a^6 (1 - e^2)^{19/2}} \left[\beta_{16} \right. \\ & \left. + \beta_{17} \cos(2i) + \beta_{18} \cos(4i) + \beta_{19} \cos(6i) \right] \end{aligned}$$

$$\begin{aligned} & + (\beta_{20} + \beta_{21} \cos(2i) + \beta_{22} \cos(4i) \\ & + \beta_{23} \cos(6i)) \cos(2\omega) + (\beta_{24} + \beta_{25} \cos(2i) \\ & + \beta_{26} \cos(4i) + \beta_{27} \cos(6i)) \cos(4\omega) \left. \right] \\ & + \frac{J_2 D_4 R^6 n^2 t^2}{a^6 (1 - e^2)^{19/2}} (\beta_{12} + \beta_{13} \cos(2i) + \beta_{14} \cos(4i) \\ & + \beta_{15} \cos(6i)) \cos(2\omega) \end{aligned}$$

where the subscript (ob) stands for oblateness,
 μ is the Newtonian constant times mass of Earth,
 R the mean equatorial radius of the Earth,
 ω the argument of the perigee,
 J_k the zonal harmonics, given by
 $J_2 = 1.08263 \times 10^{-3}$, $J_3 = -2.5322 \times 10^{-6}$, $J_4 = -1.6110 \times 10^{-6}$,
 $D_4 = J_2^2 + J_4 = -0.4389122 \times 10^{-6}$, and α_i and β_j , $i = 1, \dots, 20$; $j = 1, \dots, 27$ are the functions of eccentricity e and are given in Appendix A.

3 Determining the orbits

We search for self-maintenance orbits on both the semi-major axis and on eccentricity or, in mathematical words, we search for some orbits in which the resultant values of the perturbation due to oblateness and due to air drag on both the semi-major axis and on eccentricity are equal to zero, that is,

$$\begin{aligned} \Delta a &= (\Delta a)_D + (\Delta a)_{ob} = 0 \quad (8) \\ \Delta e &= (\Delta e)_D + (\Delta e)_{ob} = 0 \end{aligned}$$

This can be done by solving the system of equation (8) simultaneously, where the inclination i varies from $0^\circ \rightarrow 180^\circ$ and the time t is in days.

4 Numerical calculations

We solve the system of equation (8) numerically using the physical data shown in Table 1.

Table 1: The satellite physical data used in the calculations

Parameter of satellite	Value
m	175 kg
A	2.22 m ²
C_D	2.3

The results are tabulated in Table 2.

Table 2: Results for the numerical solution of equation (8)

Altitude rang (km)	i (degrees)	a (km)	Maximum deviation in a (km)	e	Maximum deviation in e	Time for self-maintenance (days)
>500				None		
500–600	24	6906.58	–1.24398	0.00207809	-2.09296×10^{-7}	54
		6921.50	–1.26840		-2.12605×10^{-7}	55
		6936.18	–1.29284		-2.15910×10^{-7}	56
		6950.63	–1.31730		-2.19212×10^{-7}	57
		6964.87	–1.34179		-2.22509×10^{-7}	58
		6978.88	–1.36631		-2.25802×10^{-7}	59
		6992.69	–1.390850		-2.29091×10^{-7}	60
600–700	24	7027.55	–0.062746	0.00207804	-1.39578×10^{-8}	13
		7088.99	–0.067876		-1.46707×10^{-8}	14
<-700				None		

We observe the following from these results:

1. Within the altitudes >500 km, there are no self-maintenance orbits because the air drag values are large to the extent that they cannot be offset by the oblateness values.
2. Within the altitudes 500–600 km, there is a wide range of self-maintenance orbits for the beginning of air drag values decline because it can be offset by the perturbation due to oblateness. These orbits require a relatively long period of time (54–60 days) to be self-maintained, but the maximum deviations in orbital parameters remain in the safe range (about 0.02% of their original values).
3. Within the altitudes 600–700 km, there are a small range of self-maintenance orbits with a short self-maintenance time (13–14 days) due to the low effect of the air drag in this altitude. It is also noted that the maximum deviations in orbital parameters are very small (about 0.0009% of their original values).
4. Finally, within the altitudes <700 km, we do not find any self-maintenance orbits because the effect of air drag is almost fading compared to the values of Earth's oblateness.

5 Conclusion

The present paper investigated how we used the perturbations due to oblateness and due to air drag versus each other in order to use them to maintain the low Earth orbit satellites. We solved for the orbits where these two perturbations balance each other and calculated the total time

needed to do so. Also, we calculated the maximum deviation in the semi-major axis and in the eccentricity during this operation to make sure that the deviation is in safe range.

The results give self-maintenance orbits within a range of altitude 500–700 km in a safe range.

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Appendix A

$$\begin{aligned} \alpha_1 &= \frac{1755}{16384} e^2 (8 + 46e^2 + 11e^4), \\ \alpha_2 &= \frac{8955e^2}{32768} (8 + 46e^2 + 11e^4), \\ \alpha_3 &= \frac{2925e^2}{16384} (8 + 46e^2 + 11e^4), \\ \alpha_4 &= \frac{4725e^2}{32768} (8 + 46e^2 + 11e^4), \\ \alpha_5 &= \frac{9}{262144} (-512 + 512e - 525808e^2 + 450416e^3 + 695544e^4 + 1325848e^5 + 3541729e^6), \\ \alpha_6 &= -\frac{9}{524288} (-15872 + 15872e - 54544e^2 + 3798288e^3 + 19551624e^4 + 12273896e^5 + 23257663e^6), \\ \alpha_7 &= \frac{9}{262144} (-11776 + 11776e - 989328e^2 + 1282320e^3 + 3807816e^4 + 2972968e^5 + 6674807e^6), \\ \alpha_8 &= -\frac{9}{524288} (-8704 + 8704e - 837616e^2 + 1068016e^3 + 4417784e^4 + 2930584e^5 + 6832241e^6), \\ \alpha_9 &= -\frac{9}{524288} (90944 - 95424e - 283632e^2 - 283296e^3 - 697384e^4 + 300024e^5 + 1051709e^6), \\ \alpha_{10} &= \frac{9}{1048576} (338112 - 614208e - 3533584e^2 - 2112864e^3 - 1476568e^4 + 1808136e^5 + 6758243e^6), \\ \alpha_{11} &= -\frac{9}{524288} (232128 - 253248e - 1026704e^2 - 549984e^3 - 834968e^4 + 831816e^5 + 2419387e^6), \\ \alpha_{12} &= \frac{9}{1048576} (209728 - 181440e - 881136e^2 - 487584e^3 - 904104e^4 + 566136e^5 + 2365581e^6), \\ \alpha_{13} &= \frac{9}{131072} (6144 - 20736e - 57920e^2 - 44160e^3 - 46768e^4 + 66000e^5 + 119985e^6), \\ \alpha_{14} &= -\frac{9}{262144} (2688 - 9600e - 90880e^2 - 61824e^3 - 21976e^4 + 23496e^5 + 143303e^6), \\ \alpha_{15} &= \frac{9}{131072} (1536 - 5376e - 41664e^2 - 26496e^3 - 1072e^4 + 14544e^5 + 50311e^6), \\ \alpha_{16} &= -\frac{27}{262144} (4224 - 14208e - 36096e^2 - 26496e^3 - 24568e^4 + 45864e^5 + 65763e^6), \\ \alpha_{17} &= -\frac{81}{524288} e^2 (-152 - 160e - 360e^2 - 272e^3 + 285e^4); \\ \alpha_{18} &= \frac{27}{1048576} e^2 (-3256 - 3680e - 3080e^2 - 1456e^3 + 8185e^4), \\ \alpha_{19} &= -\frac{27}{524288} e^2 (-1784 - 2080e - 520e^2 + 304e^3 + 5009e^4), \\ \alpha_{20} &= \frac{81}{1048576} e^2 (-408 - 480e - 40e^2 + 144e^3 + 1181e^4), \\ \beta_1 &= \frac{1755}{32768} e (8 + 46e^2 + 11e^4), \\ \beta_2 &= \frac{8955}{65536} e (8 + 46e^2 + 11e^4), \\ \beta_3 &= \frac{2925}{32768} e (8 + 46e^2 + 11e^4), \\ \beta_4 &= \frac{4725}{65536} e (8 + 46e^2 + 11e^4), \\ \beta_5 &= \frac{9}{524288} e (-309296 + 226352e - 269192e^2 + 1259352e^3 + 3797113e^4 + 829302e^5), \\ \beta_6 &= -\frac{9}{1048576} e (789296 + 2682704e + 14989192e^2 + 12290600e^3 + 25356103e^4 + 7530674e^5), \\ \beta_7 &= \frac{9}{524288} e (-402512 + 793168e + 1369288e^2 + 3106024e^3 + 7978655e^4 + 1108282e^5), \end{aligned}$$

$$\begin{aligned}
\beta_8 &= -\frac{9}{1048576}e(-74800 + 757168e + 2173688e^2 + 3047000e^3 + 7852265e^4 + 1416366e^5), \\
\beta_9 &= \frac{117}{8192}(-16 + 88e^2 - 198e^4 + 231e^6), \\
\beta_{10} &= \frac{315}{16384}(-16 + 88e^2 - 198e^4 + 231e^6), \\
\beta_{11} &= \frac{225}{16384}(-16 + 88e^2 - 198e^4 + 231e^6), \\
\beta_{12} &= -\frac{855}{16384}e(8 - 36e^2 + 63e^4), \\
\beta_{13} &= -\frac{855}{32768}e(8 - 36e^2 + 63e^4), \\
\beta_{14} &= \frac{495}{16384}e(8 - 36e^2 + 63e^4), \\
\beta_{15} &= \frac{1575}{32768}e(8 - 36e^2 + 63e^4), \\
\beta_{16} &= -\frac{9}{1048576e}(26944 - 40128e + 22288e^2 - 304032e^3 - 1001128e^4 + 136440e^5 + 895805e^6 + 390132e^7), \\
\beta_{17} &= \frac{9}{2097152e}(40640 - 328512e - 1988368e^2 - 2340960e^3 - 3379288e^4 + 1218312e^5 + 6697091e^6 \\
&\quad + 2810124e^7), \\
\beta_{18} &= -\frac{9}{1048576e}(50368 - 124224e - 180624e^2 - 695136e^3 - 1889048e^4 + 654408e^5 + 2513467e^6 + 538092e^7), \\
\beta_{19} &= \frac{9}{2097152e}(15680 - 98496e - 122352e^2 - 591264e^3 - 1717032e^4 + 473976e^5 + 2303085e^6 + 539316e^7), \\
\beta_{20} &= \frac{9}{131072e}(1008 - 1008e - 20392e^2 - 39024e^3 - 22117e^4 + 31425e^5 + 28034e^6 + 31254e^7), \\
\beta_{21} &= -\frac{9}{262144e}(144 - 144e - 25944e^2 - 31824e^3 - 61035e^4 - 3657e^5 + 124276e^6 + 54870e^7), \\
\beta_{22} &= \frac{9}{131072e}(144 - 144e - 11288e^2 - 15120e^3 - 26067e^4 + 1527e^5 + 54198e^6 + 22794e^7), \\
\beta_{23} &= -\frac{9}{262144e}(2160 - 2160e - 37416e^2 - 76464e^3 - 35333e^4 + 69561e^5 + 40188e^6 + 53226e^7), \\
\beta_{24} &= -\frac{27}{1048576}e(-168 - 48e - 1880e^2 - 1848e^3 + 1387e^4 + 2436e^5), \\
\beta_{25} &= \frac{27}{2097152}e(-1368 - 528e - 8360e^2 - 10728e^3 + 11797e^4 + 18876e^5), \\
\beta_{26} &= -\frac{27}{1048576}e(-792 - 336e - 3304e^2 - 5256e^3 + 6941e^4 + 10716e^5), \\
\beta_{27} &= \frac{27}{2097152}e(-552 - 240e - 2008e^2 - 3480e^3 + 4859e^4 + 7428e^5),
\end{aligned}$$