Exercise 12

EXPERIMENTAL IDENTIFICATION PROCESS CONTROL

1. Aim of the exercise

Theoretical and experimental identification of oscillatory unit parameters.

2. Theoretical introduction

Figure 12.1 shows a block diagram of the oscillatory unit identification system which consists of an oscillatory unit and an operational amplifier.

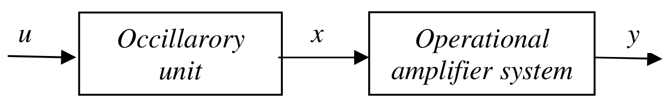

Fig. 12.1. Block diagram of the oscillatory unit identification system

2.1. Oscillatory unit parameters

An electrical scheme of the oscillatory unit is shown in Fig. 12.2. The resistor *R* and the inductor *L* are connected in series and the capacitor *C* is connected in parallel. The input voltage V_1 is the input signal, the output voltage V_2 is the output signal.

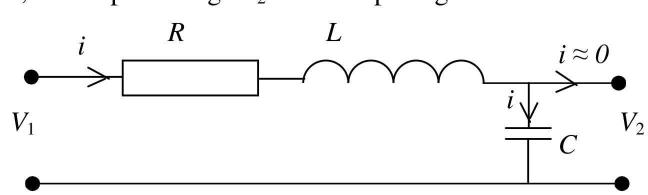

Fig. 12.2. Oscillatory unit scheme

The transfer function of the system shown in Fig. 12.2 can be derived on the basis of the Kirchhoff's law. Let us assume that the resistance of the system output is very high and the whole current *i* flows in the parallel branch. For the left closed loop, the input voltage drop is equal to the sum of voltage drops across the resistor *R*, the inductor *L* and the capacitor *C*.

$$\Delta V_1 = i\,R + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \tag{12.1}$$

On the basis of the above assumption, the change in the output voltage is equal to:

$$\Delta V_2 = \frac{1}{C}\int i\,dt \tag{12.2}$$

From Eq. (12.2), one can assign the current value:

$$i = C\frac{d\Delta V_2}{dt} \tag{12.3}$$

After substituting relation (12.3) to Eq. (12.1), we obtain:

$$\Delta V_1 = LC\frac{d^2 V_2}{dt^2} + RC\frac{d\Delta V_2}{dt} + \Delta V_2 \tag{12.4}$$

The oscillatory unit parameters are as follows:

$$T = \sqrt{LC}\,;\;\; \xi = \frac{R}{2}\sqrt{\frac{C}{L}}\,;\;\; \omega = \frac{1}{\sqrt{LC}}\,. \tag{12.5}$$

Then, the oscillatory unit equation has the following form:

$$\Delta V_1 = T^2 \frac{d^2 V_2}{dt^2} + 2\xi T \frac{d\Delta V_2}{dt} + \Delta V_2 \tag{12.6}$$

Finally, the transfer function of the system in the form of Laplace transforms is:

$$G(s) = \frac{1}{T^2 s^2 + 2\xi T s + 1}\,. \tag{12.7}$$

2.2. Operational amplifier (op-amp)

The op-amp is basically a differential amplifier, having a large voltage gain, very high input impedance and low output impedance. The op-amp has an "inverting" or (-) input and a "noninverting" or (+) input and a single output. The op-amp is usually powered by a dual polarity power supply in the range+/- 5 volts to +/- 15 volts.

2.2.1. Noninverting amplifier

The noninverting op-amp has the input signal connected to its noninverting input (Fig. 12.3), thus its input source recognizes infinite impedance. The input resistor R_G is grounded. There is no input offset voltage because $V_E = 0$, hence the negative input has to be at the same voltage as the positive input. The op-amp output drives current into R_F until the negative input is at the voltage V_{IN}. This action causes V_{IN} to appear across R_G.

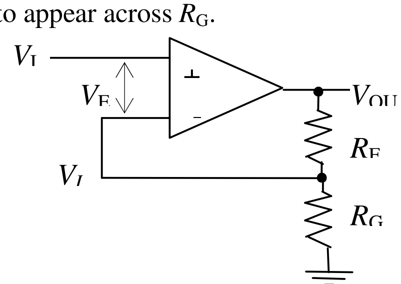

Fig. 12.3. Noninverting op-amp

The voltage divider rule is used to calculate the transfer function of the op-amp. On the basis of the electrical scheme in Fig. 12.3, one can write:

$$V_{IN} = V_{OUT} \frac{R_G}{R_G + R_F}\,, \tag{12.8}$$

thus

$$\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_F}{R_G} \tag{12.9}$$

The voltage gain is always more than 1. As the signal moves in either direction, the output will follow in phase to maintain it at the same voltage as the input signal.

2.2.2. Inverting amplifier

In this case, the inverting op-amp is connected by means of two resistors R_G and R_F, such that the input signal is applied in series with R_G, and the output is connected back to the inverting input through R_F (Fig. 12.4).

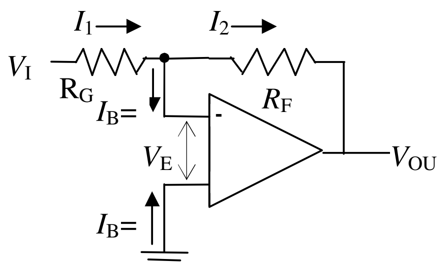

Fig. 12.4. Inverting op-amp

The noninverting input of the inverting op-amp circuit is grounded. To derive the input-output relationship, it is assumed that the input error voltage is zero. The current flow in the input leads is assumed to be zero, hence the current flowing through R_G equals the current flowing through R_F. Using the Kirchhoff's law, one can write:

$$I_1 = \frac{V_{IN}}{R_G} = -I_2 = -\frac{V_{OUT}}{R_F} \qquad (12.10)$$

The algebraic manipulation of (12.10) gives:

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_F}{R_G}$$

During the operation, as the input signal tends to be positive, the output will move negative and vice versa. It is worth noticing that in this case the gain is only a function of the feedback and gain resistors. The actual resistor values are determined by the impedance levels that the designer aims to establish.

3. Identification procedure

Similarly as the sinusoidal signal, white noise can also be used as the test signal in the identification process. Theoretically, the white noise signal contains a full spectrum of frequencies of the same amplitudes. Practically, the white noise signal is characterized by a constant value of the spectral concentration in the frequency bandwidth.

When the stochastic stationary input signal characterized by the spectral power density $P_i(\omega)$ acts in a linear system, then

$$P_0(\omega) = |G(j\omega)|^2 P_i(\omega), \qquad (12.11)$$

where: $|G(j\omega)|$ - sinusoidal transfer function amplitude of the system,

$P_0(\omega)$ - spectral power density of the output signal.

On the basis of Eq. (12.11), for the constant value $P_i(\omega) = K$, one can determine the sinusoidal transfer function amplitude of the system:

$$|G(j\omega)| = \sqrt{P_0(\omega)/K} \qquad (12.12)$$

Equation (12.12) determines the identification procedure when the oscillatory unit system is excited by the white noise input signal.

4. Course of the exercise

Figure 12.5 shows a block diagram of the oscillatory unit identification system with input and output blocks.

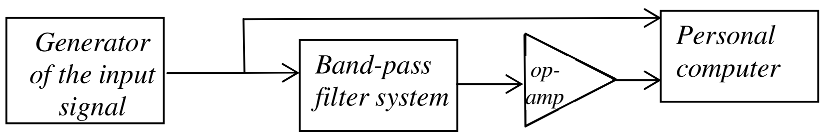

Fig. 12.5. Block diagram of the identification system

On the basis of the data of the oscillatory unit system and the dependences presented in Part 2, one should calculate theoretical values of oscillatory unit parameters, namely: the time constant (T), the dimensionless damping coefficient (ξ) and the natural frequency of the unit (ω).

The experimental identification process will be carried out in two different ways. At first, the oscillatory unit system is excited with a sinusoidal input signal. In this part of the investigations, the personal computer plays the role of a two-channel oscilloscope. For different frequency values, the input and output signals and their phase angles should be measured. On the basis of the experimental data, the frequency response and oscillatory unit parameters are to be determined.

In the second part, the oscillatory unit system is excited with the white-noise input signal. On the basis of the identification procedure (12.12) and by means of the FFT computer algorithm, the frequency response and oscillatory unit parameters are determined in another way.

5. Laboratory report should contain:

1. Aim of the exercise.
2. Calculation results of the oscillatory unit parameters.
3. Experimental results of the frequency response obtained in the classical way.
4. Frequency response in a graphic form.
5. Unit parameters determined in the experimental way.
6. Computer printout of the time response of the white-noise signal.
7. Amplitude frequency response (results of the investigations in a graphic form).
8. Phase angle frequency response of the system.
9. System parameters determined from the frequency response.
10. Comparison of results, conclusions and remarks.

References

1. Ogata K.: *Modern Control Engineering,* IV-th Edition, Prentice Hall 2002.
2. Wolovich W.A.: *Automatic Control Systems, Basic Analysis and Design,* Harcour Brace College Publishers, 1994.
3. Randal R.B.: *Application of B & K Equipment to Frequency Analysis,* Brüel & Kjǽr Publishing, 1987.