Exercise 14

HARMONIC ANALYSIS OF AIR-COMPRESSOR VIBRATIONS

1. Aim of the exercise

Analysis of a complex signal recorded from the machine and the determination of its elementary components.

2. Theoretical introduction

2.1. Measurement signals

In order to make proper measurements, different signals, met in typical kinds of vibrations, should be known. Two groups of signals can be mentioned, namely:

1. Deterministic signals – those whose values are predictable in time. Such signals contain:
   a) periodic vibrations:
   - harmonic,
   - complex (polyharmonic),

   b) aperiodic signals:
   - almost periodic,
   - transient (pulse, i.e., impacts starting and finishing with zero values)
   - chaotic performed by deterministic system (deterministic chaos).
2. Random, stochastic signals – they have unpredictable, random values at any time instant. Their properties are described with statistic characteristics, i.e., averaging parameters applied to amplitudes, frequencies and time. Here can be found:
   a) stationary signals with statistic characteristics (mean, mean square values) which are not functions of time,
   b) non-stationary signals.

As an example of deterministic signals, one can mention vibrations of a gearbox (Fig. 14.1a) or piston movements in an engine with two frequencies ω and 2ω (Fig. 14.1b). Part c) of this figure shows a time history and a power spectrum of the non-continuous signal, i.e., an impact. Typical examples of random vibrations are those resulting from fluid flows, noise, disturbances, and vibrations of a car body moving on a rough road, spatter of rain. They are characterized by random behaviour, have no characteristics of their frequency, and the spectrum seen as a function of frequency is almost evenly distributed.

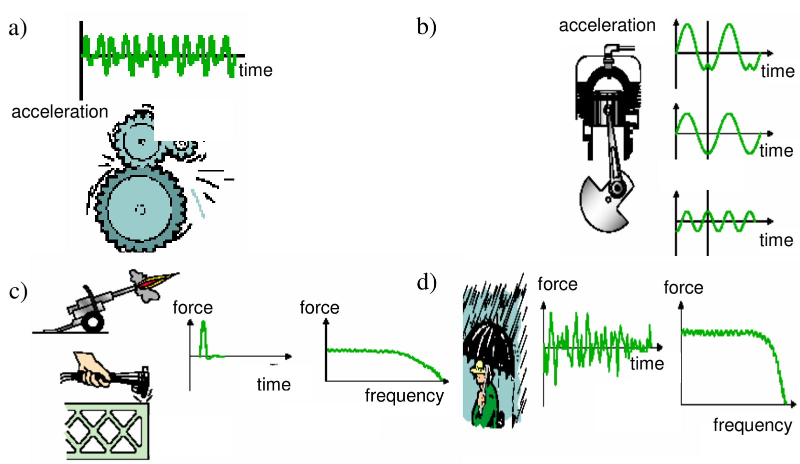

Fig. 14.1. Different signals a) and b) deterministic, c) pulse type, d) random

If measurements and analysis have to deal with a particular range of frequencies and there are no demands to be fulfilled, say, imposed by standards, then the general rule is to measure the value which has the most flat character as a function of frequency, Fig. 14.2. This allows us to cover the widest range of signal dynamics of the system under investigation. But in the case when the characteristics are unknown, there is a general suggestion to choose a vibration velocity signal as the basis for analysis. This is important, especially in the case when the characteristic curve is not flat. Then, all components of the level below the average one will have less influence on the final result and, in the case of measurements in the full range of frequencies, the lowest components cannot be detected at all.

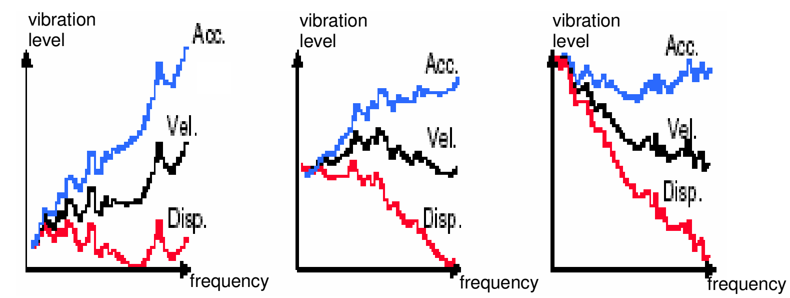

Fig. 14.2. Choice of parameters of vibrations measured with respect to the power spectrum: a) displacement, b) velocity, and c) acceleration

The demand for flat characteristics means that velocity is to be measured in most cases of machine vibration measurements. In some cases, however, this should be rather acceleration, but for the majority of machines, accelerations of high amplitude are found only at high frequencies. It is rather rare to have flat characteristics of the displacement signal because such high amplitudes take place only at very low frequencies in most real cases. There can be more reasons why we cannot use particular sensors – e.g., a mass of the sensor is too high, compared to the object under investigation, or its measuring range is not wide enough to use it in testing.

Analysing relations between displacement, velocity and acceleration (differentiation or integration), one can see that for a particular level of velocity of vibrations, displacement amplitudes decrease (since they are divided by the ω value) and acceleration amplitudes increase (multiplied by ω) with an increase in frequency – see Fig. 14.3.

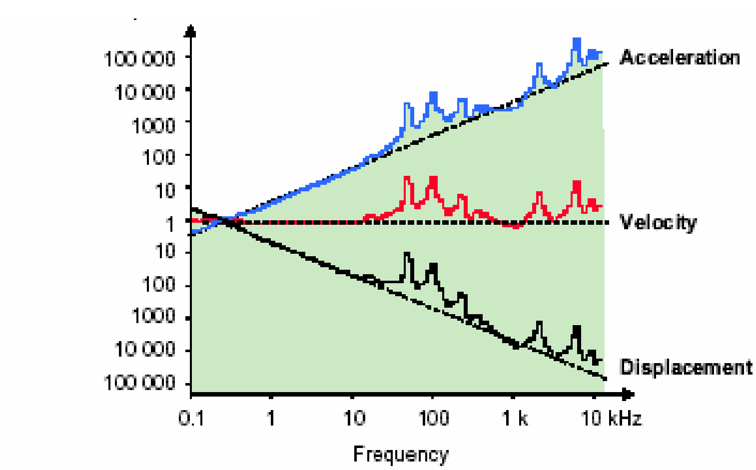

Fig. 14.3. Example of spectrum characteristics of the vibration signal shown as displacement, velocity and acceleration signals

In some cases of vibration measurements, observed time histories are enough to carry out the

analysis. They comprise data to determine the amplitude, frequency (as $1/T$) or phase shift of signals. But usually time histories present complex shapes, giving information about the overall level of vibrations only,
Fig. 14.4.

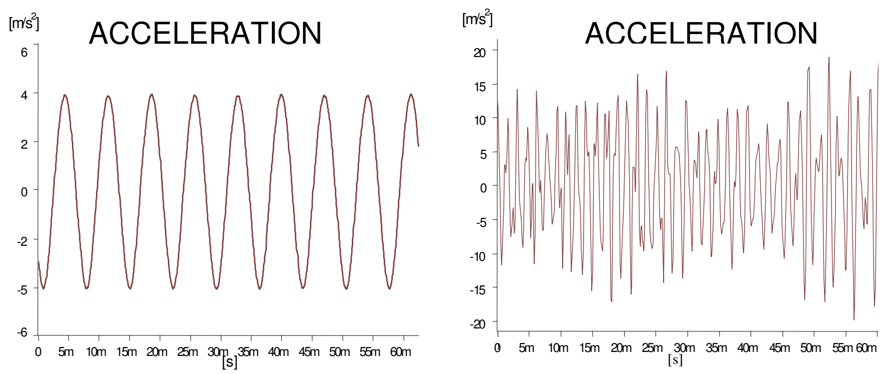

Fig. 14.4. Example of vibration time histories, time in [ms]

2.2. Spectral analysis of vibrations

In the case when the detailed information about all components of the signal is needed, one has to conduct a power (frequency) analysis of the recorded time history signal. Such an analysis can be done by means of analogue or digital methods. In the first case, power spectrum analysing devices are applied. These include a set of filters with different characteristics of the signals passed or tuned narrow-band filters. In the digital method, the Fast Fourier Transform (FFT) is applied.

A periodic function can be represented with the Fourier series by the following components: a constant part, a_0, and harmonic parts of the frequencies ω_1, $2\omega_1$, …, $n\omega_1$, where ω_1 is the basic frequency and the terms $n\omega_1$ are harmonic frequencies, whereas n is a natural number. The basic frequency is described as:

$$\omega_1 = \frac{2\pi}{T}, \qquad (14.1)$$

where T denotes the period of the function.

The complete equation describing all components of a periodic function $x(t)$ with the Fourier series terms has the following form:

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_1 t + b_n \sin n\omega_1 t \right). \qquad (14.2)$$

Let us consider a rectangular wave as an example – it can be represented by an infinite trigonometric series of odd harmonics (1, 3, 5, 7, …) with diminishing amplitudes, see Fig. 14.5. This is a representation in the *time domain*.

$$x(t) = \frac{4A}{\pi}(\sin\omega_1 t + \frac{1}{3}\sin 3\omega_1 t + \frac{1}{5}\sin 5\omega_1 t + \frac{1}{7}\sin 7\omega_1 t + \ldots) \qquad (14.3)$$

Periodic signals can be represented in a graphical form in the *frequency domain*. The horizontal axis presents the frequency values f [Hz] (or $\omega = 2\pi f$ measured in [rad/s]), whereas the vertical one shows the amplitudes measured in the same way as in time histories or using some relative values. The height of a component is proportional to the values of amplitude of the particular harmonic term, Fig. 14.6. Such a graph is a power spectrum graph or an amplitude graph. Spectra of periodic signals have a discrete form of separated bars, while non-periodic signals, e.g., pulse or stochastic, have continuous forms, see Figs.

14.6, 14.11, 14.12 and 14.13.

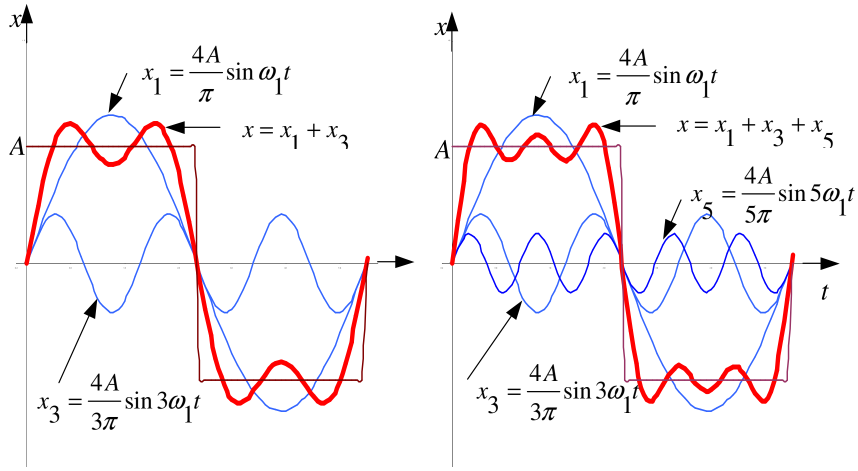

Fig. 14.5. Approximation of the rectangular wave with a limited number of harmonics

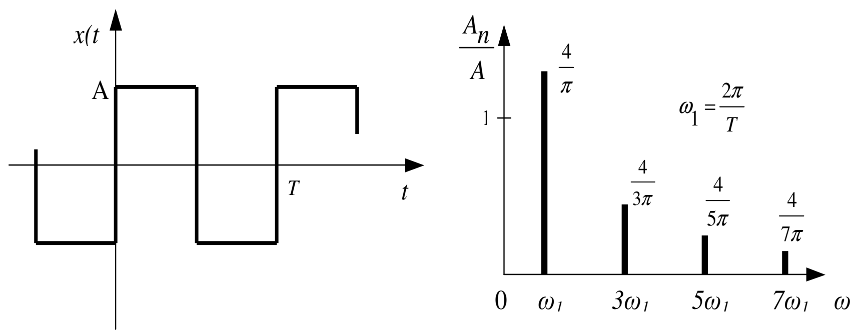

Fig. 14.6. Power spectrum of the rectangular signal

2.3. Filters: band passing

The spectral analysis is used in many different areas of mechanical applications, especially in machine diagnostics and vibration measurements. In analogue devices, electrical filters with band passing characteristics are usually employed. Such filters pass only those components of the analysed signal, whose frequencies are within the filter characteristics.

Figure 14.7 presents both ideal and real band pass filters. In the ideal case, there is zero damping inside the passing band of frequencies and infinitely high elsewhere. Therefore, the filter characteristic curve has a rectangular shape. Usually filter damping is expressed in decibels (dB) as:

$$N\,[dB] = 10\log\left(\frac{U_{out}^{\;2}}{U_{in}^{\;2}}\right) = 20\log\left(\frac{U_{out}}{U_{in}}\right), \tag{14.4}$$

where U_{in} and U_{out} are input and output filter signals, respectively. Real filters characteristics are said to be correct if they are flat in the mid section of the passing band and edge slopes are steep. Band pass filters are characterised by their mid-frequency f_0 and the band width $B = f_2 - f_1$, which is the difference of its limit frequencies, upper and lower, at which damping of the signal diminishes by –3dB, the signal power halves: $\frac{U_{out}^2}{U_{in}^2} = \frac{1}{2}$, and the amplification factor changes from $k = 1$ to $k = 1/\sqrt{2}$ in comparison with the average level in the passing band, see Fig. 14.7.

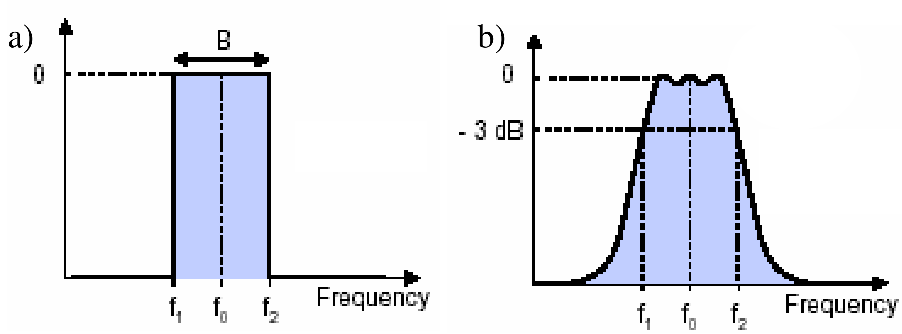

Fig. 14.7.
Characteristics of a) ideal and b) real filters

In the frequency analysis of vibration signals, two types of filters are used:

- with a constant absolute bandwidth, e.g., 3 Hz or 100 Hz, etc.
- with a constant relative bandwidth measured with respect to the mid frequency f_0 in percents, e.g., 3%, 10% or 30%, see Fig. 14.8. They are also called constant relative bandwidth filters.

The characteristic frequencies of these filters are expressed by the following relationship:

$$f_0 = \sqrt{f_1 \cdot f_2} \tag{14.5}$$

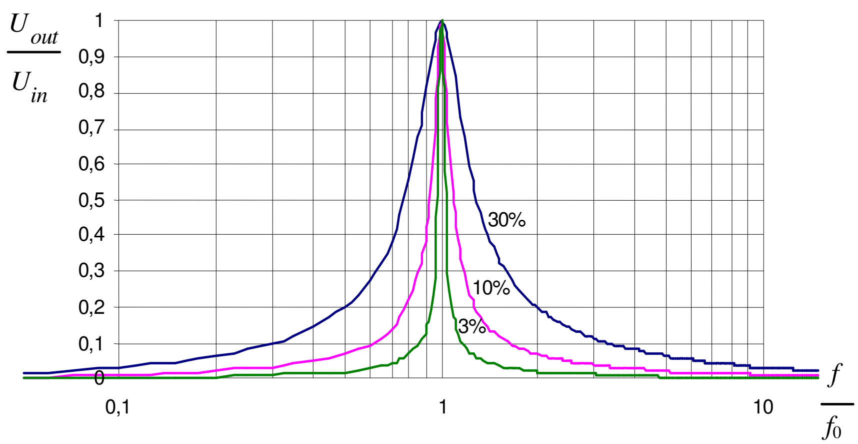

Fig. 14.8. Characteristics of the narrow-band filter with widths of 3%, 10%, and 30%

An octave means the width when the upper frequency doubles the lower one, whereas the third is the width when the upper frequency is $\sqrt[3]{2} \approx 1{,}26$ times higher than the lower one. If the bandwidth is equal to one octave, then we have an octave filter, B = 70%, if it is 1/3 octave – we have a third (interval) filter. The term octave comes from the fact that it covers eight basic sounds on the musical scale. The mid frequencies make the geometrical series and the normalized values are rounded, e.g., 1.0 Hz; 1.25 Hz, 1.6 Hz, 2.0 Hz, 2.5 Hz, 3.15 Hz; 4.0 Hz, …, etc.

The width of the passing part is proportional to the mid frequencies, therefore it is variable. There are filters of the width equal to 1/12 and 1/24 of the octave. The narrower the bandwidth is, the more detailed information can be obtained from the analysed signal, but the time of such an analysis is longer as well. The filter characteristics can be presented both in linear (with a constant absolute bandwidth) and logarithmic (with a constant relative bandwidth) scales, as seen in Fig. 14.9. Different scaling makes it easier to understand filters.

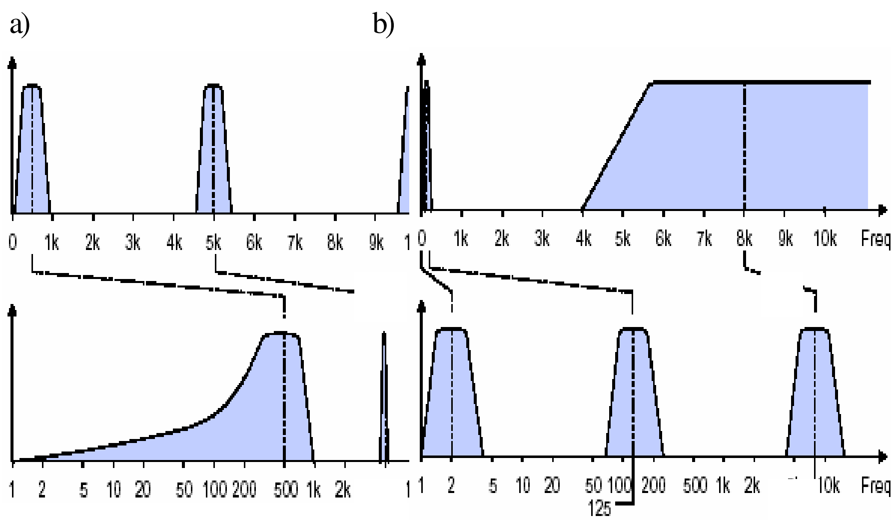

Fig. 14.9. Examples of characteristics of filters in both, linear and logarithmic scaling, a) filter with the constant bandwidth of 400 Hz, b) filter with the constant relative bandwidth of 1/1 octave, i.e., about 70% of the mid-frequency value

2.4. Fast Fourier Transform in the vibration analysis

Let the following forces interact with the investigated object:

$$P_1 \sin(\omega_1 t + \varphi_1), \; P_2 \sin(\omega_2 t + \varphi_2), \dots, P_k \sin(\omega_k t + \varphi_k), \qquad (14.6)$$

which results in a complex shape of the vibrations observed. To determine sources of frequencies in signals and their influence on vibrations of objects, the complete signal *x(t)* incoming from the measurement sensor (Fig. 14.10) should be divided into harmonic terms as follows:

$$x(t) = \sum_{i=1}^{k} A_i \sin(\omega_i t + \beta_i) \qquad (14.7)$$

A typical record from the data acquisition system is a set of discrete values x_i. The data are recorded every Δt time instants. Such a process is referred to as sampling, Fig. 14.10.

The time span of the recorded data set is called the recording time and it is usually limited due to some practical aspects. The sampling time is calculated as:

$$\Delta t = l / 2B, \qquad (14.8)$$

where *B* is the width of the frequency band.

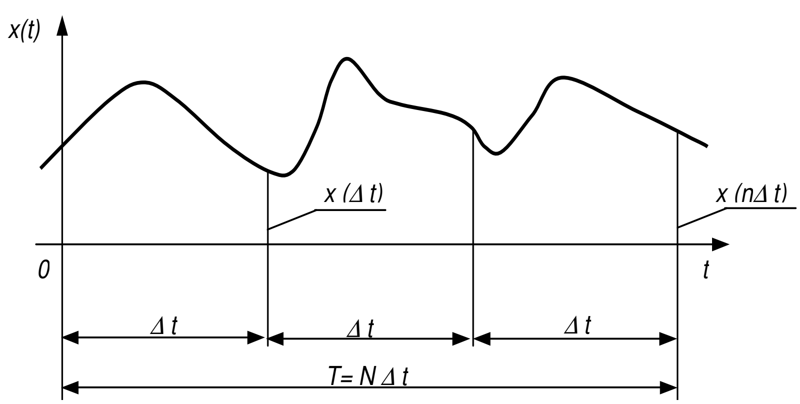

Fig. 14.10. Illustration of the sampling process

The sampling time is determined with respect to the upper limit of the frequency range, $f \in (0; B)$, which is to be assumed according to the needs of the analysis. A number of samples N also is to be set, usually a power of 2 is the best. The data are stored in the computer memory and processed according to the procedure described below. First, the Fourier transform of the data is calculated as:

$$X_k \sum_{i=0}^{N-1} x_i e^{-j\frac{2\pi ik}{N}}; \quad k=0, 1, ..., N-1 \tag{14.9}$$

The fast type of the transform is applied, which results in shortening of the computing time. Let:

$$W = e^{-j\frac{2\pi}{N}} \tag{14.10}$$

$$X_k = \sum_{i=0}^{N-1} x_i \ W^{ik}; \qquad k = 0, 1, ..., N-1 \tag{14.11}$$

Equation (14.11) can be rewritten as:

$$X_{(c+dA)} = \sum_{b=0}^{B-1} \sum_{a=0}^{A-1} x_{(b+aB)} W^{(b+aB)(c+dA)} \tag{14.12}$$

where:

$i = b + a B$ - index of time samples;

$k = c + d A$ - index of frequency samples;

$a, c = 0, 1, ... A$-1; $\quad b, d = 0, 1, ... B$-1.

The exponents of the term W from the previous equation can be rewritten as:

$$W^{(b+aB)(c+dA)} = W^{bc} W^{bdA} W^{acB} W^{adAB} = W^{bc} W^{bdA} W^{acB} \tag{14.13}$$

This results from the fact that a and d are integer numbers, and the exponent of the complex term W, which is a multiple of N, is equal to one. A part of the complex number in the exponential term can be excluded outside, and thus:

$$X^{(c+dA)} = \sum_{b=0}^{B-1} W^{bdA} \sum_{a=0}^{A-1} x_{(b+aB)} W^{acB} W^{bc}, \tag{14.14}$$

Such an exclusion is equivalent to the limitation of the number of multiplications applied, which means that instead of:

$$(AB)(AB) = N^2, \tag{14.15}$$

we have:

$$AB(A+B) = N(A+B). \tag{14.16}$$

Practically, using this procedure with the number of samples N that is a power of 2 is of special meaning. Then, the exponential terms take the values of +1 and –1, which results in avoiding the multiplication on complex numbers and additionally makes the procedure less time-consuming. Next, the power spectrum of the signal is determined as:

$$G_k = \frac{2\Delta t}{N} |X_k|^2; \qquad k = 0, 1, ..., \frac{N+1}{2}. \tag{14.17}$$

The frequency range obtained is divided in such a way that there is a distance

$$\Delta f = \frac{1}{N\Delta t} \tag{14.18}$$

between each consecutive values of discrete frequencies.

As a result, the measured signal is decomposed into components, which have their level of power of the signal. This allows us to determine which of them gives highest values in the signal, and thus enable us to look for the source of it.
Figs. 14.11, 14.12 and 14.13 show time histories and power spectra of real signals after applying the FFT procedure.

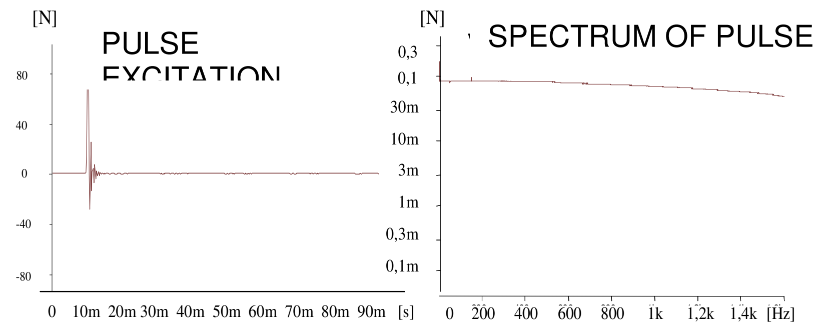

Fig. 14.11. Time history and amplitude spectrum of the pulse signal

For the pulse signal, see Fig. 14.11, the power spectrum has a continuous character. The theoretical pulse $\delta(t)$ (Dirac function) has signals of all frequencies, from $-\infty$ to $+\infty$, with the same amplitude of 1. At $t = 0$, all spectrum components are in phase. Such a concentration causes that a pulse can arise. A short pulse caused, for example, by hitting with a test hammer (hammer with a mounted force sensor, which is of the acceleration type), has components of the equal amplitude in a wide range of frequencies. Thus, the system is excited with a full frequency range and it allows us to obtain the so-called dynamic characteristics of stiffness or dynamic receptance by means of the FFT.

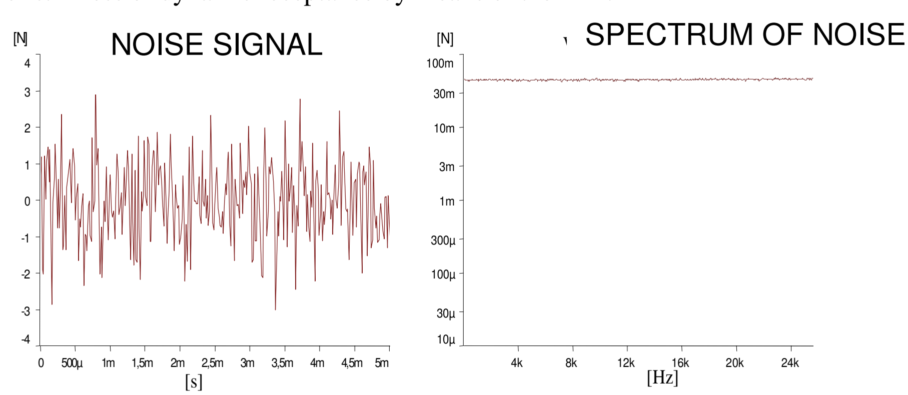

Fig. 14.12. Time history and power spectrum of the stochastic signal

The signal whose values are random at any instant is called stochastic. It is of high practical importance when it is totally disordered, with all frequencies in a chosen range. Its energy is uniformly distributed in the full frequency range, see Fig. 14.12. By analogy to optical science, it is called white noise. If such a signal is applied to the exciter input, then the exciting force has also all frequencies, which allows us to obtain the dynamic characteristics of the object under investigation.

An example of real vibrations recorded and its FFT frequency analysis results is shown in Fig. 14.13.

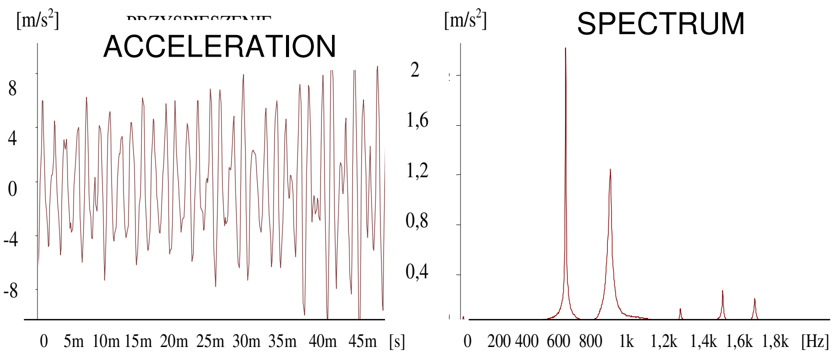

Fig. 14.13. Time history and acceleration spectrum of the real signal

3. Test stand

An air compressor is subject to testing. Its vibrations measured during normal operation show very complex behaviour. It is a result of the fact that the effects of forces with different frequencies, which are natural to operation of the machine, are superimposed. To determine where the exciting forces come from, a frequency analysis of vibrations is needed. The values of frequencies of these components that are equal to the frequencies of the exciting forces can be thus calculated. The analysis of the signal recorded from the measurement sensor is conducted in two ways, namely:

1. Direct analysis of the air compressor vibration with an SM 32 narrow-band analogue analyser.
2. Computer analysis of the data collected from a piezoelectric vibration sensor by the Ambex LC-010-1612 data acquisition system, then the Fast Fourier Transform (FFT) procedure is used.

1.1. Analysis of vibrations with an analogue narrow-band analyser

The behaviour of the object, i.e. the air compressor (Fig. 14.14), is analysed with a SM 231 vibration measurement device with an SM 32 narrow-band analyser and a KD 20 piezoelectric acceleration sensor.

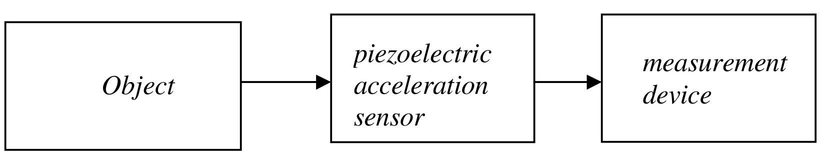

Fig. 14.14. Scheme of the test stand

The SM 231 vibration measurement device (Fig. 14.15a) is connected to piezoelectric acceleration sensors. Inside the device, two SM 10 integrating amplifiers (1), SM 40 measurement section (2), SM 50 oscilloscope section (3), SM 61 power supply section (4), and SM 32 narrow-band analyser (5) (see Fig. 14.15b) are included.

a) b)

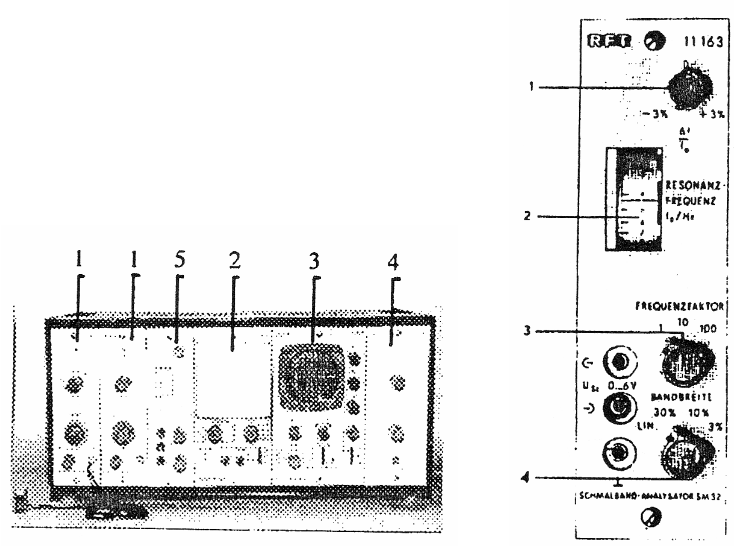

Fig. 14.15. a) SM 231 device, b) SM32 analyser

The power supply unit includes a switch of amplifier channels (denoted by 1, 2, 3 or 1, 2 when the internal filter is applied). The SM 50 oscilloscope screen allows us to observe the amplified signals incoming from the vibration measurement during the acceleration measurement, then integrated for the velocity value and integrated again for the displacement signal. A switch of the measured value is located in the SM 10 integrating amplifier – marked with letters *a, v*, ξ. The indexes at those letters present lower frequencies of the measured values. When the filter is not applied (the position marked by LIN in the SM 40 section), the whole spectrum of the investigated signal can be seen, whereas after applying the filter (the switch in the position INT), only the output signal from the analyser or the filter is investigated. In the SM 32 narrow-band analyser, Fig. 14.15 b), the following elements are presented: 1 – controller, range 3% … 0 … +3%, 2 – resistor with the knob setting frequency f_0, 3 – FREQUENCY MULTIPLIER switch (multiplier x1, x10, x100), 4 – BANDWIDTH switch (LIN, 30%, 10%, 3%) which is a selective amplifier with the amplification ratio of 1 for the chosen mid-frequency f_0. Other frequencies included in the analysed signal are suppressed – the more suppressed, the further they are located from the mid-frequency value f_0 (Fig. 14.16).

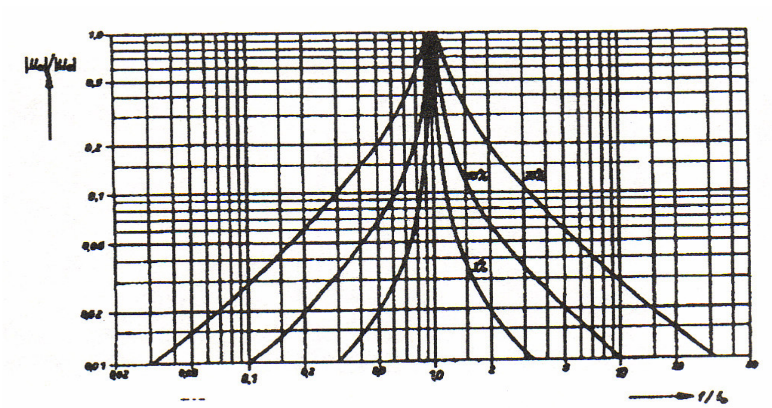

Fig.14.16. Filter characteristics

The SM 32 analyser enables the signal analysis in the frequency range 0.2 Hz to 20 kHz in three sub-ranges (multiplier x1, x10, x100). The BANDWIDTH switch has 4 positions: LIN (the complete bandwidth is passed through), then 30%, 10%, 3%.

The above-mentioned values determine the relative bandwidth of the frequencies passed. The mid-frequency f_0 is chosen by setting the resistor scaled in Hz. The bandwidth is the difference between the upper and lower passed frequencies, at which the damping level is equal to 3 dB. The resistor marked by ΔV on the SM 10 integrating amplifier allows us to change the amplifying ratio. Therefore, sensors, which have usually different sensitivities, can be used together.

4. Course of the exercise

Part 1

1. Place a vibration sensor on the air compressor.
2. Using the SM 61 channel switch, choose the proper measurement channel (1 or 2).
3. Set the measurement value switch on the SM 10 amplifier to the position a_2 (acceleration, lower measured frequency 2 Hz).
4. Set the range switch to its maximum value.
5. Set the BANDWIDTH switch to the position LIN (full range of frequencies).
6. Turn on the SM 231 device, start the air compressor.
7. By changing the position of the SM 10 amplifier switch, set the gauge on SM 40 over 1/3 of its range. Record the position.
8. To carry out the analysis, set the BANDWITH switch of the analyser to 30% and change the mid-frequency f_0 with its resistor until the gauge shows a distinctive value. To determine exactly the frequency, change the bandwidth to 3%.
9. Record the levels of the power spectrum components of the investigated vibrations and the frequencies at which they appear.

Part 2

1. Place a piezoelectric vibration sensor on the air compressor.
2. Switch on the PC and the SM 231 vibration measurement device.
3. Run the program called SYGNAL.
4. Start the compressor.
5. Complete the test with the program SYGNAL.

Part 3

1. Print out a test report.
2. Complete the report with the analyser measurement results.
3. Compare both the results.
4. Estimate the source of the excitations found – where they come from.

References

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