Exercise 2

EXPERIMENTAL ESTIMATION OF THE MOMENT OF INERTIA OF A MACHINE PART BY MEANS OF THE PENDULUM METHOD

1. Aim of the exercise

Determine experimentally the moment of inertia of a connecting rod with respect to the axis parallel to its vertical axis and passing through the mass centre (Part I). Determine the moment of inertia of a crankshaft by means of the string support method and check the shear modulus of the string material (Part II).

2. Theoretical introduction – Part I

The connecting rod supported in the point A other than B (Fig. 2.1) can be considered as a physical pendulum and its motion can be described as:

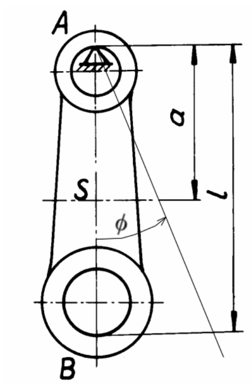


Fig. 2.1. Connecting rod

$$B_A \frac{d^2\varphi}{dt^2} = -mga\phi, \qquad (2.1)$$

where:

B_A – moment of inertia with respect to the point of support A,
m – mass of the connecting rod,
a – distance of the mass centre from the support axis
ϕ – angle of rotation.

The solution of Eq. (2.1) is as follows:

$$\phi = \Phi \sin\left(\frac{2\pi}{T_A} t + \beta\right), \qquad (2.2)$$

where:

$$T_A = \frac{2\pi}{\sqrt{\frac{mga}{B_A}}} \qquad (2.3)$$

is the period of free vibrations of the rod supported in the point A. When we apply the Steiner theorem, the moment of inertia B_A can be determined using the B_S value (moment of inertia of the rod through the centre of its mass and parallel to the axis through the point A) as:

$$B_A = B_S + ma^2 \tag{2.4}$$

From Eqs. (2.3) and (2.4), we get:

$$T_A = \frac{2\pi}{\sqrt{\dfrac{mga}{B_S + ma^2}}} \tag{2.5}$$

After the experimental determination of the values T_A and m, the last equation still contains two unknown values: B_S and a. We can calculate both of them determining the period of free vibration when the rod is supported upside down in the point B. Analogically:

$$T_B = \frac{2\pi}{\sqrt{\dfrac{mg(l-a)}{B_B}}} \tag{2.6}$$

$$B_B = B_S + m(l-a)^2 \tag{2.7}$$

$$T_B = \frac{2\pi}{\sqrt{\dfrac{mg(l-a)}{B_S + m(l-a)^2}}} \tag{2.8}$$

System of equations (2.3) and (2.8) allows one to determine the values of B_A and a as:

$$a = \frac{glT_B{}^2 - 4\pi^2 l^2}{g(T_A^2 + T_B^2) - 8\pi^2 l^2}, \tag{2.9}$$

$$B_S = \frac{mgaT_A^2 - 4\pi^2 ma^2}{4\pi^2}. \tag{2.10}$$

2.1. Error analysis – Part I

In Eqs. (2.9) and (2.10), we use the measured values T_A, T_B, m, and l, which can be encumbered with some errors. We assume the errors in the mass and displacement measurements as: $\Delta m = 0.005$ kg and $\Delta l = 0.001$ m, respectively. Assuming the measurement of 50 periods of free vibration and the time measurement accuracy of 0.5 s, we determine the error in estimation of the period as:

$$\Delta T_A = \Delta T_B = \frac{0.5}{50} = 0.01\,[\mathrm{s}] \tag{2.11}$$

The error in calculation of the moment of inertia is as follows:

$$\Delta B_S = \left|\frac{\partial B_S}{\partial T_A}\Delta T_A\right| + \left|\frac{\partial B_S}{\partial T_B}\Delta T_B\right| + \left|\frac{\partial B_S}{\partial m}\Delta m\right| + \left|\frac{\partial B_S}{\partial l}\Delta l\right|. \tag{2.12}$$

The partial derivatives take the form:

$$\frac{\partial a}{\partial t}=\frac{-2gT_A M}{N^2}, \qquad \frac{\partial a}{\partial T_B}=\frac{-2glT_B N-2glT_B M}{N^2},$$

$$\frac{\partial a}{\partial l}=\frac{(gT_B{}^2-8\pi^2 l)N+8\pi^2 M}{N^2},$$

$$\frac{\partial B_S}{\partial T_A}=\frac{1}{4\pi^2}\left(mgT_A^2\frac{\partial a}{\partial T_A}+2mgaT_A-8\pi^2 ma\frac{\partial a}{\partial T_A}\right),$$

$$\frac{\partial B_S}{\partial T_B}=\frac{1}{4\pi^2}\left(mgT_A^2\frac{\partial a}{\partial T_B}-8\pi^2 ma\frac{\partial a}{\partial T_B}\right), \tag{2.13}$$

$$\frac{\partial B_S}{\partial l}=\frac{1}{4\pi^2}\left(mgT_A^2\frac{\partial a}{\partial l}-8\pi^2 ma\frac{\partial a}{\partial l}\right), \quad \frac{\partial B_S}{\partial m}=\frac{1}{4\pi^2}\left(gaT_A^2-4\pi^2 a^2\right),$$

$$M=glT_B^2-4\pi^2 l^2, \qquad N=g\left(T_A^2-T_B^2\right)-8\pi^2 l.$$

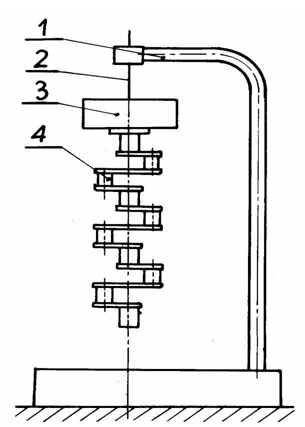
The relative measurement error is equal to:

$$\frac{\Delta B_S}{B_S}\times 100\%. \tag{2.14}$$

3. Theoretical introduction – Part II

The crankshaft is supported on a string (Fig. 2.2) and oscillates torsionally. We cannot support it in two ways to make it oscillate along two different axes. Then, we use a supporting string that creates the system a form of torsion physical pendulum. Its motion is described by the following equation:

$$B_c\frac{d^2\varphi}{dt^2}=-\frac{GI_0}{l}\varphi, \tag{2.15}$$



where:
$B_c = B + B_o$, B_o – mount moment of inertia,
B – moment of inertia of the element under investigation,
ϕ – angle of the string torsion,
G – shear modulus (rigidity of the string),
M – reaction moment of the string torsion,
$I_0 = \pi d^4/32$ – polar moment of inertia of the string,
d – string diameter,
l – string length.

Fig. 2.2. Measurement device (Part II)

The solution to Eq. (2.15) is following:

$$\phi=\Phi\sin\left(\frac{2\pi}{T_C}t+\beta\right), \tag{2.16}$$

where:

$$T_C=\frac{2\pi}{\sqrt{\frac{GI_0}{B_C l}}} \tag{2.17}$$

is the period of free vibration of the system. Now, B_C can be determined as:

$$B_C = \frac{T_C^2 \; GI_0}{4\pi^2 l} \tag{2.18}$$

If we have the T_C, G, d, and l measured. Then, the moment of inertia of the shaft B_K is expressed as:

$$B_K = B_C - B_0 \tag{2.19}$$

on assumption we know the value of B_0. We can determine it by calculating the moment of inertia of the disc of the known mass and radius. The value of G is obtained by the determination of a period of vibration of the system containing a string and a mount (flywheel).

$$T = \frac{2\pi}{\sqrt{\frac{GI_0}{B_0 l}}}, \tag{2.20}$$

$$G = \frac{4\pi^2 B_0 l}{T_0 I_0}. \tag{2.21}$$

Using the relation B_c=B+B_o and Eq. (2.21), one gets:

$$G = \frac{4\pi^2 B_0 l}{T_0^2 I_0}. \tag{2.22}$$

Then, the moment of inertia of the shaft is equal to:

$$B = B_o \left(T^2 / T_0^2 - 1\right). \tag{2.23}$$

Note the fact that there are NO parameters of the string properties in the above equation!

3.1. Error analysis – Part II

In Eq. (2.22), we use the measured values T_C, T_0 (we assume that the error in estimation of a value of the moment of inertia of the flywheel is negligible), which can be encumbered with some errors. Assuming the measurement of 20 periods of free vibration and the time measurement accuracy of 0.5 s, we determine the error in estimation of the period as:

$$\Delta T_C = \Delta T_0 = \frac{0.5}{20} = 0.025\,[\mathrm{s}]. \tag{2.24}$$

The error in calculation of the moment of inertia is:

$$\Delta B_K = \left|\frac{\partial B_K}{\partial T_C}\Delta T_C\right| + \left|\frac{\partial B_K}{\partial T_0}\Delta T_0\right|. \tag{2.25}$$

The partial derivatives are as follows:

$$\frac{\partial B_K}{\partial T_C} = \frac{2B_0 T_C}{T_0^2}, \qquad \frac{\partial B_K}{\partial T_0} = \frac{-2B_0 T_C^2}{T_0^3}. \tag{2.26}$$

The relative measurement error equals:

$$\frac{\Delta B_K}{B_K} \times 100\%. \tag{2.27}$$

4. Measurement devices

The measurement device used in the experimental determination of the moment of inertia of the connecting rod in Part I is shown in Fig. 2.3. Due to the fact that the connecting rod has a complex shape, we attempt to determine its moment of inertia experimentally. The measurement device consists of prism (1) and vertical supports (2), at which the prism is mounted. Connecting rod (3) is then supported on the prism.

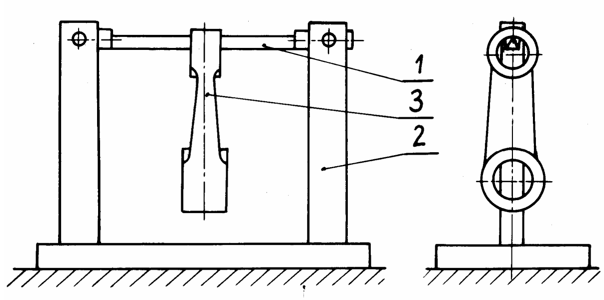


Fig. 2.3. Measurement device (Part I)

The measurement device for the experimental determination of the moment of inertia of the crankshaft in Part II is shown in Fig. 2.2. The device consists of base (1), string (2), mount (3) and analysed element (4), which is fixed in the mount (flywheel) in such a way that its axis and the string axis are in line.

5. Course of the exercise

Part I

- Place the rod to allow oscillations in the point A. Take care of keeping the axis of holes parallel to the prism edge.
- Assume: m – mass of the connecting rod equal to 1.850 kg, and l – distance between the points of support A and B to be 0.270 m.
- Allow the rod to oscillate with amplitude lower than 10°, and then measure the time of 50 oscillations three times. These results should be recorded in the report sheet and the mass centre and the moment of inertia are to be determined with the above-mentioned equations.
- Repeat the procedure supporting the rod in the point B.
- Estimate the error of the procedure.

Part II

- Assume: l – length of the supporting string equal to 0.590 m, and d – string diameter 0.005 m.
- Fix the investigated element in the mount in such a way that its axis is in line with the string axis.
- Introduce torsion oscillations with angular amplitude lower than 10°, then measure the time of 20 periods of oscillations (3 times) and determine its average value and the period T_C.
- Dismount the device from the flywheel, then measure the time of 20 periods' oscillations of the flywheel alone (3 times) and determine its average value and the period T_0.
- Calculate the value of B_K and G, and then estimate the error of the calculations.

6. Laboratory report should contain:

1) Aim of the exercise.
2) Results of the experimental investigations.
3) Calculation results.
4) Results of the error analysis.
5) Conclusions and remarks.

References

1. Rao S.S.: *Mechanical Vibrations*, Prentice Hall, NY, 1995.
2. Tse F.S., Morse I.E., Hinkle R.T.: *Mechanical Vibrations - Theory and Applications,* Allyn and Bacon Inc., 1978.
3. Kapitaniak T.: *Wstęp do teorii drgań*. Wydawnictwo Politechniki Łódzkiej, Łódź 1992.
4. Parszewski Z.: *Drgania i dynamika maszyn*. PWN, Warszawa 1982.