Exercise 4

INVESTIGATION OF THE ONE-DEGREE-OF-FREEDOM SYSTEM

1. Aim of the exercise

Identification of parameters of the equation describing a one-degree-of- freedom (1 DOF) mathematical model of the real vibrating system.

2. Test stand

The test stand consists of a plate undergoing oscillations forced by springs connected to an eccentric device mounted on an electric motor shaft (Fig. 4.1). Viscous damping comes from an oil damper and the system of supporting springs makes the system stiffness. In Fig. 4.1 there are vibration mass 2, which can rotate about the axis of support bearings 3, set of springs 4, oil reservoir 14, and oil damper 9 forming the support of mass 2. On base 1 there is a forcing subsystem, which consists of electric motor 6 with constant speed control, eccentric-arm device 7 and spring 8.

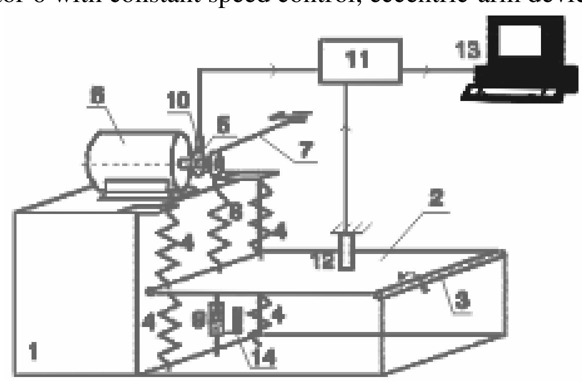

Fig. 4.1. Scheme of the test stand

The approximate frequency of forcing is measured with tooth disc 5 placed on the shaft of motor 6, photoelectric transducer 10 and analogue-measuring device 11 of the ZPA19 type. The measurement of the mass vibration is made with transformer 12 of the OT23 type, measuring device 11 and PC microcomputer 13 equipped with the AMBEX LC-010-1612 data acquisition system collecting data from displacement transducer 12. These data are sent then directly to the PC memory for further processing.

3. Theoretical introduction

For further discussion, the real system is modelled with a simplified one, having 1 DOF under harmonic excitation. Instead of the rotational motion, which takes place on the stand, we approximate its motion as the linear one. This is done by reducing rotations in the particular point (displacement sensor position) to the linear (vertical) ones, on assumption that the rotation angles are small. Then, simple trigonometric relations can be applied. A model of the equivalent system is shown in Fig. 4.2.

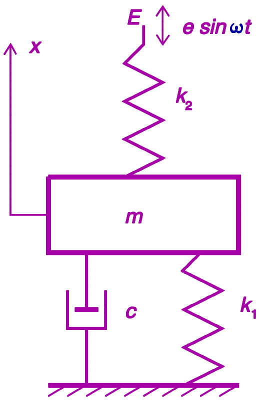

Fig. 4.2. Model of the experimental – equivalent system

Notations in Fig. 4.2:

m - reduced mass,
k_1 - reduced stiffness of spring set 4,
k_2 - reduced stiffness of spring 8,
c - reduced damping coefficient,
e - vibration amplitude of the point E (reduced value of the eccentricity),
ω - excitation force radial frequency equal to the motor rotational speed,
x - reduced displacement of the mass *m*.

All the values are reduced with respect to the point of the measurement (the contact point of measurement device 12 with the vibrating mass - Fig. 4.1).

The equation of motion of the mass *m* is as follows:

$$m\ddot{x} = (k_1 + k_2)x + c\dot{x} + k_2 e \sin \omega t \quad (4.1)$$

Equation (4.1) can be arranged as:

$$\ddot{x} + 2h\dot{x} + \omega_N^{\ 2} x = q \sin \omega t\ , \quad (4.2)$$

where:

$$2h = \frac{c}{m}, \quad \omega_N^{\ 2} = \frac{k_1 + k_2}{m}, \quad q = \frac{ek_2}{m}, \quad (4.3)$$

The particular solution to the equation of motion presents the state of the system when natural vibration disappears:

$$x = A\sin(\omega + \varphi)\,, \quad (4.4)$$

where A is the amplitude of forced vibration, ω is the forcing frequency of the system, and φ represents a phase shift between the forcing and the resulting displacement of the mass *m*:

$$A = \frac{q}{\sqrt{\left(\omega_N^{\ 2} - \omega^2\right)^2 + 4h^2\omega^2}}, \quad \varphi = \arctan\frac{-2h\omega}{\omega_N^{\ 2} - \omega^2}. \quad (4.5)$$

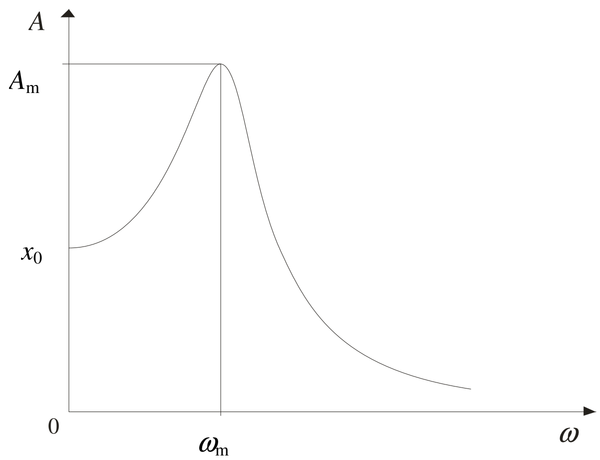

Fig. 4.3. Resonance graph

The amplitude A of the system vibration described with the function shown in the first equation of Eqs. (4.5) can be presented in a typical form as shown in Fig. 4.3, and is usually called a resonance graph.

During the experiment, one can obtain a similar graph recording the appropriate pairs of data (ω, A). The parameters q, ω_N, h are to be found by comparison of both graphs, the theoretical and the experimental one, assuming the curves are to be as close each other as possible. Mathematically, one can set the conditions possible to formulate the equations describing their values. Due to the fact that we need to determine the values of three parameters q, ω_N, h in Eqs. (4.5), three different conditions for the adjacency of both graphs should be posed. It is useful to choose the following conditions:

1) both curves have to pass the point (0, x_0) (see Fig. 2.4),
2) both curves have to come through the point (ω_m, A_m), the peak of the resonance graph,
3) the theoretical graph is to be tangent to the experimental one in the point (ω_m, A_m).

If we replace ω and A with 0 and x_0, respectively, in the first equation of Eqs. (4.5), the above conditions can be shown mathematically as:

$$x_0 = \frac{q}{\omega_N^2}, \tag{4.6}$$

then, the first of the above-mentioned conditions will be satisfied. Similarly, after replacing ω and A with ω_m and A_m respectively,

$$A_m = \frac{|q|}{\sqrt{(\omega_N^2 - \omega_m{}^2)^2 + 4h^2\omega_m{}^2}} \tag{4.7}$$

the second condition will be fulfilled.

The third condition means that the derivative $\partial A/\partial\omega$ disappears in the point (ω_m, A_m). Deriving the first equation of Eqs. (4.5), we get:

$$\frac{\partial A}{\partial \omega} = \frac{q\left[2\left(\omega_N^2 - \omega^2\right)(2\omega) + 8h^2\omega\right]}{2\sqrt{\left[(\omega_N^2 - \omega^2)^2 + 4h^2\omega^2\right]^3}} = 0, \tag{4.8}$$

which results in:

$$\omega_N^2 + \omega_m^{\ 2} + 2h^2 = 0. \tag{4.9}$$

Solving the system of equations (4.6), (4.7) and (4.9) with respect to the parameters q, ω_N, h, we obtain:

$$q = \frac{\omega_m^{\ 2} x_0}{\sqrt{1-\left(\frac{x_0}{A_m}\right)^2}}, \quad \omega_N = \frac{q}{x_0} = \frac{\omega_m^{\ 2}}{\sqrt{1-\left(\frac{x_0}{A_m}\right)^2}},$$
$$2h = \sqrt{2(\omega_N^2 - \omega_m^{\ 2})} = \omega_m \sqrt{\frac{1}{2\sqrt{1-\left(\frac{x_0}{A_m}\right)^2}} - 1}. \tag{4.10}$$

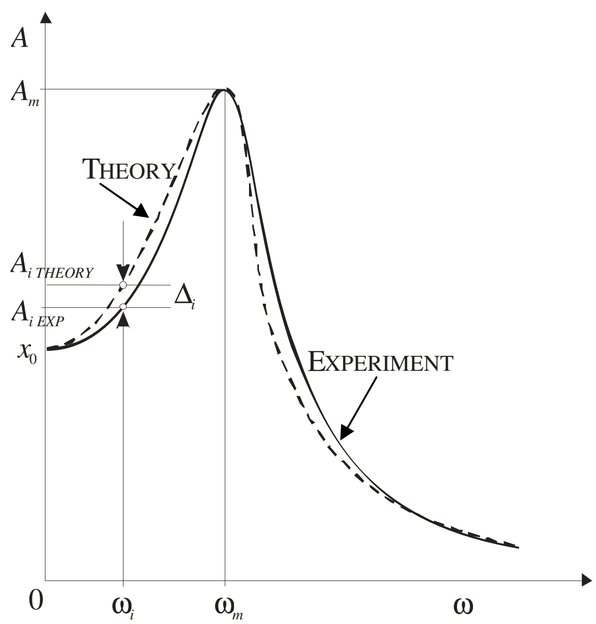

Fig. 4.4. Theoretical and experimental resonance graphs

From the above equations one can see that the necessary condition is to determine the coordinates of the graph peak (ω_m, A_m) and the displacement x_0 under static conditions.
We find here the situation shown in Fig. 4.4 – both the theoretical and experimental curves pass through the points (0, x_0) and (ω_m, A_m), and, additionally, they have a common tangent value in the point (ω_m, A_m). If the investigated system is linear, then both curves in Fig. 4.4 should be identical, but it is rather impossible that the experimental data are ideal. To evaluate how the discussed linear model approximates the real system, we employ the following formula:

$$\Delta = \sqrt{\frac{\sum(\Delta_i)^2}{\sum(A'_i)^2}} \tag{4.11}$$

Some remarks on the processing of the data.

1) The data from the measuring device have the form of an electric signal. Voltage values are measured, but before collecting the data, there is a need to scale the device. Applying the known displacement to the probe, its output value is to be measured and the characteristic curve as a function $U = f(x)$ is obtained. The resulting curve should form a broken line, which should be approximated with a line and its slope gives the recalculating coefficient of voltage to displacement values. Using different distance plates, we set up different known displacements and make 5 measurements of appropriate voltage at least.
2) There is a need to identify the excitation frequency in each point of the experimental resonance curve. Since the reading of the frequency on the ZPA-19 device is not accurate, this has been done applying the Fast Fourier Transform to the measured displacement signal. Its behaviour follows the excitation form and its frequency is equal to the excitation. The highest peak in the calculated spectrum is assumed to be such frequency. The initial values set with the device – 10 are to be used as approximate.
3) The program used in the experiment determines the theoretical resonance graph using the above described assumptions in two characteristic points $(0, x_0)$ and (ω_m, A_m). For the first one in x_0, a parabola is constructed on the basis of the first two data points of the recorded set with the condition for the tangent in x_0 to be horizontal (slope = 0). For the determination of the peak of the resonance curve, another parabola is used. Using coordinates of 3 experimental points found to be closest to the highest value, a parabola is mathematically constructed and its peak is assumed to be the experimental resonance curve peak, hopefully approximating better the value (see Fig. 4.5). Due to such a procedure, the peak coordinates are closer to the real ones – the control setup not always allows us to set the excitation frequency, which in fact gives the maximum displacement.
4) The amplitude of vibration of the mass is taken as a half of the detected peak values from probe 12 and multiplied by the coefficient determined in point 1.

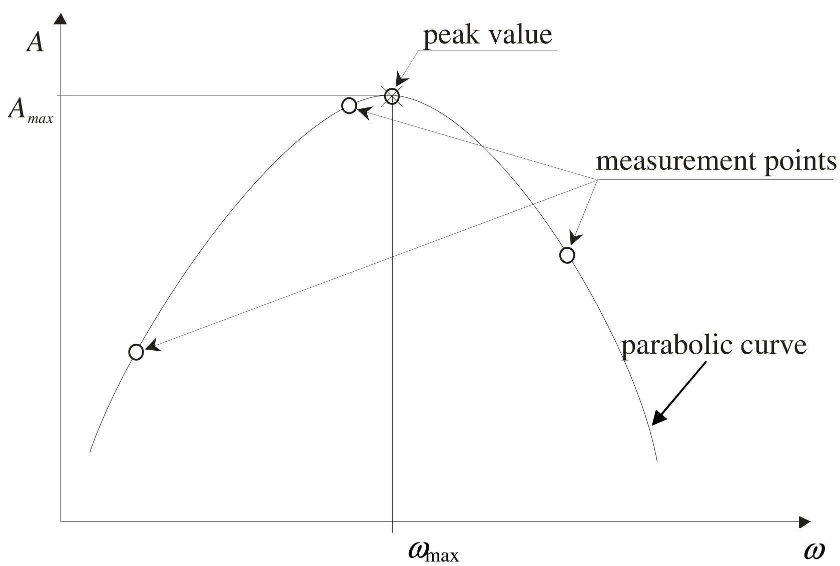

Fig. 4.5. Improved method for finding the peak value of the resonance graph

5) In some range, an oil reservoir position allows us to change the amount of damping in the system. The lowest position of the oil reservoir drains almost the whole volume of oil off the piston area, lowering thus the coefficient of viscous damping extremely. Such a position can result in dangerous behaviour of the system under forcing. The highest position (vertical) causes that

viscous damping is maximal, limiting also movements of the main mass. Other positions should allow mean values of damping.

4. Course of the exercise

Initial part

1) Start the PC computer and then the control device 11 (its knobs should be set to the minimum, i.e., to the left).
2) Run the program called CW1-3.BAT, choose either the basic or expanded version.

Scaling the displacement probe

1) Turning the shaft of the motor by hand (motor turned off!), set the eccentric device in mechanism 7, its marker should be in the highest position. It is assumed to be the zero position of the displacement probe. Take care not to shift this position during the whole process.
2) Run the program option called *Scaling*.
3) Run the measurement for the zero value of the displacement probe.
4) Put the thinnest plate under probe 12, enter its value, run the measurement.
5) Repeat point 8 for other plates.
6) Remove the last used plate and choose the option: E (*End of scaling*). The coefficient used in the calculation should be displayed on the screen.

Experimental resonance graph

1) Oil reservoir of damper 14 is to be set in the vertical position (Fig. 2.1).
2) Turn the motor drive on, using the knobs on the ZPA19 control device.
3) Set an approximate value of the excitation frequency
4) Suggested values of the excitation frequencies: 15, 20, 25, 30, 34, 36, 38, 40, 42, 45, 50, 70, 90 rad/s. Detailed measurements should be done around the resonance value. The measurements for the lowest values should be made especially carefully (determining the value of x_0). These values should be set from the range: 5-15 rad/s.
5) Setting the values close to the resonance, watch carefully indicators of the ZPA19 device. The values should be set a little lower and a little higher, but giving a visible difference on the scale.
6) Choose the option *Resonance graph* to start measuring the resonance graph.
7) Each measurement is started by pressing the key P (*Measurement*) on the computer keyboard. After collecting the data in the upper left corner, two traces of measured signals are visible. The upper part is used for the measurement of the amplitude of the signal. The lower one is used by the FFT procedure to calculate the excitation frequency.
8) If everything is satisfactory, the key Y (*Yes*) should be used. Otherwise, one should press the key N (*No*) and repeat the measurement by pressing P again.
9) Repeat points 15 and 16 for other excitation frequencies.
10) On completing this part of measurements, turn off the motor using the knobs.
11) Finish the measurements choosing the option K (*End*) with the confirmation by T (*Yes*).
12) Then, to analyse the real resonance graph, *Analysis* is to be performed.
13) To determine coordinates of the peak value (compare point 3), select 3 points closest to the maximum value with the cursor and mark them with the Z (*Mark*) key. Press the K key to finish this part. Pressing any other key means the beginning of the analysis.
14) The values of the parameters q, α, h and the theoretical graph are obtained automatically.

Expanded case: Characteristics at different damping levels

1) To obtain the dependence of resonance peak values on damping, change damping to a slightly lower value by decreasing the angle of the damper cylinder reservoir with respect to the base level.

2) After such a change, the collection of data points for the next resonance curve is to be performed, repeating procedure described in points 12 - 22. There should be at least 3 to 5 measurements at different damping values, and then the final analysis is needed to obtain the dependence.
3) Using the option A (*Analysis*), the graphical form of results is presented. It shows the dependence of the natural damped frequency of the system on the damping parameter level.
4) Print out the Report, and then draw your conclusions.

References

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3. Kapitaniak T.: *Wstęp do teorii drgań*. Wydawnictwo Politechniki Łódzkiej, Łódź 1992.