Exercise 3
Identification of parameters of the vibrating system with one degree of freedom

Goal

To determine the value of the damping coefficient, the stiffness coefficient and the amplitude of the vibration excitation with one degree of freedom. These are to be understood as parameters of vibrating system.

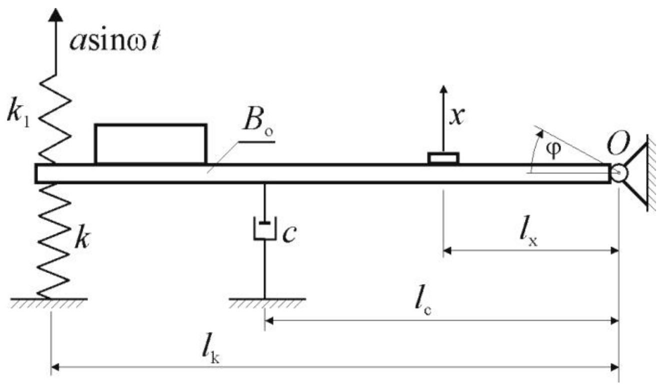

Fig. 1 Scheme of the rig

Drawing at Fig. 1 presents physical model of the vibrating system which possesses the following elements:

- stiff beam connected to the support rotating node O
- set of coil springs with stiffness coefficient k,
- oil damper with damping coefficient c,
- driving spring with stiffness coefficient k_1.

A spring with a stiffness factor k_1 is connected to the eccentric pin on the driving motor shaft. Rotation of the shaft creates a kinematic forcing of the upper end of this springs, approximately described as $a\sin\,\omega t$. Due to the force, the beam swings out of the balance position by an angle φ. The deflection of the beam is measured by a linear displacement sensor, defining the displacement x of the point of the beam, l_x away from the axis of rotation.

The equation of the linear motion of the physical model of the investigated system is as follows:

$$B_o\ddot{\phi}+\mathrm{c}l_c^2\dot{\phi}+(\mathrm{k}+\mathrm{k}_1)\phi=\mathrm{k}_1 l_k a\sin\omega t \quad (1)$$

where B_o – is mass moment of inertia relative to its rotation axis.

Dividing Eq. (1) by B_o and multiplying by l_x we obtain:

$$l_x\ddot{\phi}+\frac{\mathrm{c}l_c^2}{B_o}l_x\dot{\phi}+\frac{\mathrm{k}+\mathrm{k}_1}{B_o}l_x\phi=\frac{k_1 l_k \mathrm{a}l_x}{B_0}\sin\omega t \quad (2)$$

Introducing:

$$l_x\ddot{\phi}=\ddot{x},\quad \frac{\mathrm{c}l_c^2}{B_o}=2\mathrm{h},\quad l_x\dot{\phi}=\dot{x},\quad \frac{\mathrm{k}+\mathrm{k}_1}{B_o}=\alpha^2,\quad l_x\phi=\mathrm{x},\quad \frac{k_1 l_k \mathrm{a}l_x}{B_0}=\mathrm{q}, \quad (3)$$

we can rewrite Eq. (1) as:

$$\ddot{x} + 2h\dot{x} + \alpha^2 x = q\sin\omega t \quad (4)$$

where:

$2h$ - represents damping,

α - natural frequency of the system,

q - kinematic excitation amplitude.

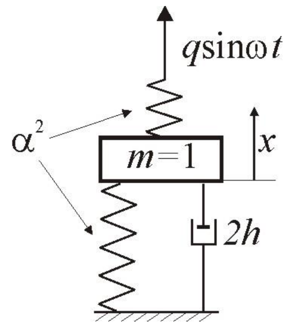

Fig. 2 Physical model of the rig

Equation (4) needs to be solved *(remind yourself – how to determine such solution?)* Its specific solution is function (5) which describes oscillatory motion of the model:

$$x = A\sin(\omega t + \beta), \quad (5)$$

where A is amplitude of the excited oscillations and β – defines phase angle shift between actual value of the kinematic excitation and the model oscillations.

The value of A is defined as:

$$A = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}}. \quad (6)$$

As ω is varying in mathematical sense from 0 to infinity values of A can be understood as a function $A(\omega)$. Its general form takes graphical representation shown in the drawing below.

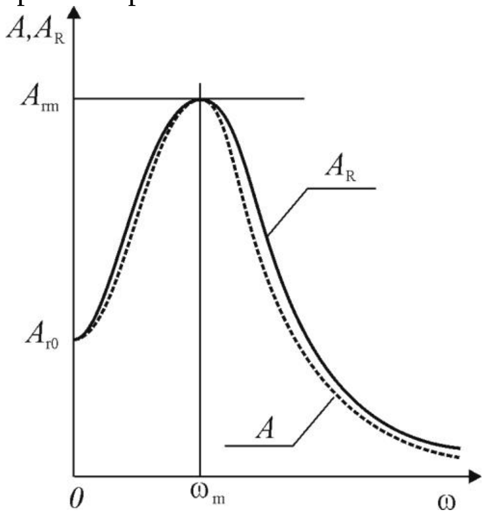

Fig. 3 General form of the function $A(\omega)$ – broken line overprinted on typical values collected from experimental measurements – continuous line

As Eq. (6) contains yet unknown values of 〈, *h*, *q*, these values are to be treated as parameters which we are looking for in the exercise. The way reaching their values can be as following:

- Determine experimental measurements graph as in Fig. 3 by measuring amplitudes ot the beam amplitudes at different excitations at the rig (continuous line).
- Amplitude at very small (practically close to zero) excitation ω is marked as A_{R0}. Maximal value of the beam motion appears at resonance frequency ω_m marked as A_{rm}.
- Theoretical resonance graph shown with broken line in Fig. 3 should be a result of calculations using formula (6) with some assumptions.

We assume both graphs have to fulfill three *(why?)* conditions:

1. For excitation frequency ω close to zero both amplitudes *A* and A_R are to be the same:

$$A(0) = \frac{q}{\sqrt{(\alpha^2-0^2)^2+4h^2 0^2}} = \frac{q}{\alpha^2} = \mathrm{A_{R0}}. \qquad (7)$$

2. At excitation equal to the natural (resonance) frequency ω_m, both amplitudes *A* and A_R are to be the same:

$$A(\omega_m) = \frac{q}{\sqrt{\left(\alpha^2-\omega_m^2\right)^2+4h^2\omega_m^2}} = \mathrm{A_{Rm}}. \qquad (8)$$

3. At excitation equal to the natural (resonance) frequency ω_m, both amplitudes *A* and A_R reach their maximum values:

$$\frac{\partial A}{\partial \omega}_{(\omega=\omega_m)} = \frac{q\left[-4\omega\left(\alpha^2-\omega^2\right)+8h^2\omega\right]}{2\sqrt{\left(\left(\alpha^2-\omega_m^2\right)^2+4h^2\omega^2\right)^3}} = 0 \qquad (9)$$

When Eqs. (7), (8), (9) are understood as set of algebraic equations they can be converted to the following results:

$$\mathrm{q}=\frac{\omega_m^2 A_{R0}}{\xi}, \quad \alpha^2 = \frac{\omega_m^2}{\xi}, \quad 2\mathrm{h}=\omega_m\sqrt{\frac{2-2\xi}{\xi}}$$
$$\text{where } \xi=\sqrt{1-\frac{A_{R0}^2}{A_{\mathrm{Rm}}^2}} \qquad (10)$$

which allows to calculate numerical values of the unknown parameters 〈, *h* and *q*.
Next, we can identify real rig parameters' values as:

$$\mathrm{c}=\frac{2\mathrm{hB_0}}{l_c^2}, \quad k_1\mathrm{a}=\frac{\mathrm{qB_0}}{l_k l_x}, \quad k = \frac{\mathrm{B_0}\alpha^2}{l_k^2} - k_1. \qquad (11)$$

Earlier, some physical values were measured or determined from basic engineering formulas:
$B_0 = 1.38$ kgm^2, a=0.003 m, $l_c = 0.54$ m, $l_k = 0.54$ m, $l_x = 0.24$ m. (12)

Course of the exercise:

1. Measure the A_R amplitude vibrations for different angular velocity values ω; number of measurements should be about 15. Determine the value of ω_m, for which the amplitude of vibrations reaches the maximum value of A_{Rm}. Determine the amplitude value A_{R0} at close to zero excitation frequency. Copy your results into the table as below.
2. Calculate the parameter values *α, h, q* using the formulas (10).

3. Calculate the amplitude values A of the theoretical resonance plot using the formula (6) for those ω values for which the A_R was measured.
4. Draw a real and theoretical resonance graphs.
5. Calculate the values of the real system parameters k, c, k_1 using formulas (11) and the values of the parameters given in (12).

Date: ………………….. Name and Family name: …………………….……..

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Exercise 3 Report

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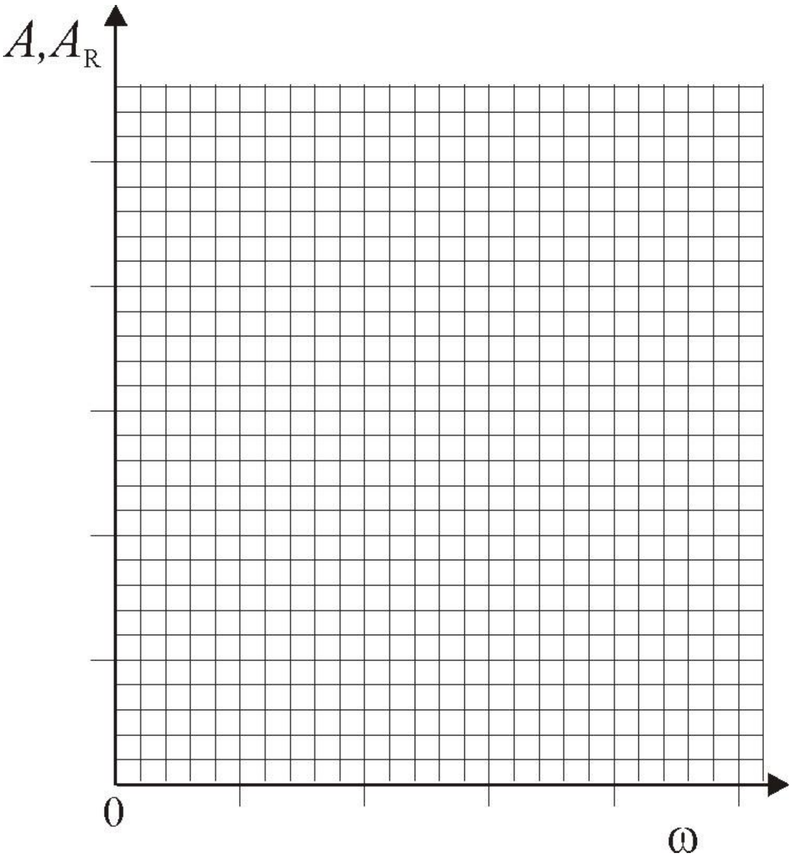
Measurements

ω	A_R	A



ω_m= A_{Rm}= A_{R0}=

Calculation of the physical model parameters:

$$\xi = \sqrt{1 - \frac{A_{R0}^2}{A_{Rm}^2}} = \qquad q = \frac{\omega_m^2 A_{R0}}{\xi} = \qquad \alpha^2 = \frac{\omega_m^2}{\xi} = \qquad 2h = \omega_m \sqrt{\frac{2 - 2\xi}{\xi}} =$$

Formula for calculation of the theoretical model amplitude:

$$A = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}} =$$

Calculation of the real model parameters:

$$c = \frac{2hB_0}{l_c^2} = \qquad k_1 = \frac{qB_0}{a l_k l_x} = \qquad k = \frac{B_0 \alpha^2}{l_k^2} - k_1 =$$