Exercise 3
Identification of parameters of the vibrating system with one degree of freedom

Goal

To determine the value of the damping coefficient, the stiffness coefficient and the amplitude of the vibration excitation in system with one degree of freedom. These are to be understood as parameters of the vibrating system.

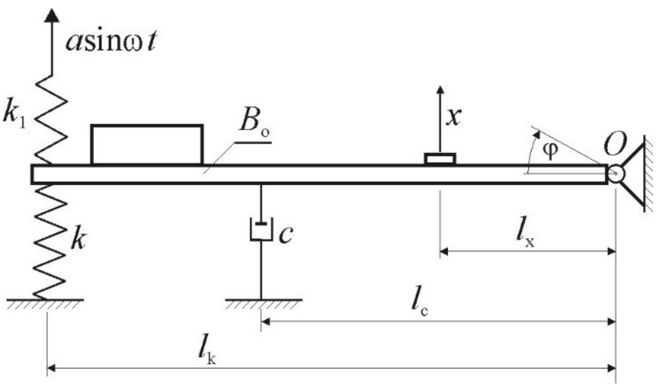

Fig. 1 Scheme of the rig

Drawing at Fig. 1 presents physical model of the vibrating system which possesses the following elements:

- stiff beam beam connected to the support rotating node O,
- set of supporting coil springs with stiffness coefficient k [N/m],
- oil damper with damping coefficient c [kg/s],
- driving spring with stiffness coefficient k_1 [N/m].

A spring with a stiffness factor k_1 is connected to the eccentric pin on the driving motor shaft. Rotation of the shaft creates a kinematic forcing of the upper end of this springs, approximately described as $a\sin(\omega t)$. Due to the force, the beam swings out of the balance position by an angle φ. The deflection of the beam is measured by a linear displacement sensor, defining the displacement x of the chosen point of the beam, being l_x away from the axis of rotation.

$$B_o\ddot{\phi}+c\,l_c^2\dot{\phi}+(k+k_1)\phi=k_1 l_k a\sin\omega t \qquad (1)$$

where: B_o – is mass moment of inertia relative to its rotation axis, then l_c, l_k – distances [m], a – eccentricity i.e., the excitation amplitude [m].

Dividing Eq, (1) by B_o and multiplying by l_x we get:

$$l_x\ddot{\phi}+\frac{c\,l_c^2}{B_o}l_x\dot{\phi}+\frac{k+k_1}{B_o}l_x\phi=\frac{k_1 l_k a\,l_x}{B_0}\sin\omega t \qquad (2)$$

Introducing:

$$l_x\ddot{\phi}=\ddot{x},\quad \frac{c\,l_c^2}{B_o}=2h,\quad l_x\dot{\phi}=\dot{x},\quad \frac{k+k_1}{B_o}=\alpha^2,\quad l_x\phi=x,\quad \frac{k_1 l_k a\,l_x}{B_0}=q, \qquad (3)$$

we rewrite Eq. (1) as:

$$\ddot{x}+2h\dot{x}+\alpha^2 x=q\sin\omega t \tag{4}$$

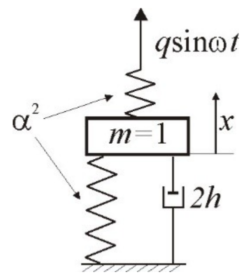

Fig. 2

where:

$2h$ - damping [1/s],

α^2 - natural frequency of the system [$1/s^2$],

q - kinematic excitation amplitude [m/s^2],

The specific solution of (4) is function (5) which describes oscillatory motion of the model:

$$x(t)=A\sin(\omega t+\beta), \tag{5}$$

where A is amplitude of the excited oscillations [m] and β – defines phase angle shift between actual value of the kinematic excitation and the model oscillations [rad].
Amplitude A is defined as dependence on varying excitation frequency ω:

$$A(\omega)=\frac{q}{\sqrt{(\alpha^2-\omega^2)^2+4h^2\omega^2}}. \tag{6}$$

As ω is varying in mathematical sense from 0 to infinity, values the amplitude A can take should be understood as a function $A(\omega)$, where ω is the function independent variable. Its general form takes graphical representation shown in Fig. 3.

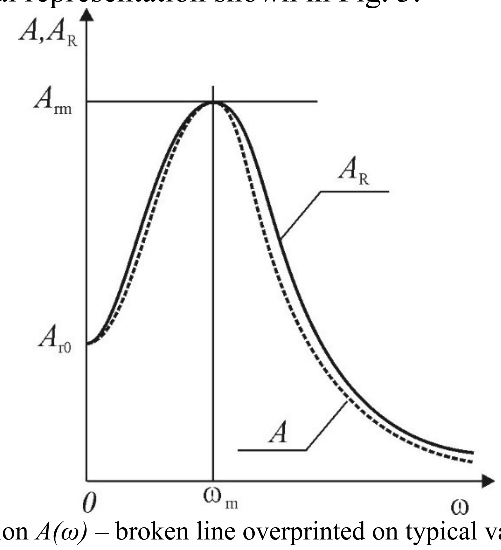

Fig. 3 General form of the function $A(\omega)$ – broken line overprinted on typical values collected from experimental measurements – continuous line

As Eq. (6) contains yet unknown values of α, h, q, these values are to be treated as parameters which we are looking for in the exercise. The way reaching their values can be as following:

- Determine experimental measurements graph as in Fig. 3 by measuring amplitudes of the beam at different excitation values at the rig (the continuous line)
- Amplitude at very small (practically close to zero) excitation ω is marked as A_{R0}. Maximal value of the beam motion appears at resonance frequency ω_m marked as A_{rm}.

Theoretical resonance graph shown with broken line in Fig. 3 should be a result of calculations using formula (6) with some assumptions.

We assume both graphs have to fulfill three conditions:

1. For excitation frequency ω close to zero both amplitudes A and A_{R0} are to be the same:

$$A(0)=\frac{q}{\sqrt{\left(\alpha^2-0^2\right)^2+4h^2 0^2}}=\frac{q}{\alpha^2}=A_{R0}. \qquad (7)$$

2. At excitation frequency equal to the natural (resonance) frequency ω_m, both amplitudes A and A_{rm} are to be the same:

$$A(\omega_m)=\frac{q}{\sqrt{\left(\alpha^2-\omega_m^2\right)^2+4h^2\omega_m^2}}=A_{Rm}. \qquad (8)$$

3. At excitation equal to the natural (resonance) frequency ω_m, both amplitudes A and A_{Rm} reach their maximum values (and the following condition is to be fulfilled – *why?*)::

$$\frac{\partial A}{\partial \omega}=\frac{q\left[-4\omega_m\left(\alpha^2-\omega_m^2\right)+8h^2\omega_m\right]}{2\sqrt{\left(\left(\alpha^2-\omega_m^2\right)^2+4h^2\omega_m^2\right)^3}}=0, \quad for \quad \omega=\omega_m \qquad (9)$$

When Eqs. (7), (8), (9) are understood as set of algebraic equations they can be converted to the following results:

$$q=\frac{\omega_m^2 A_{R0}}{\xi}, \quad \alpha^2=\frac{\omega_m^2}{\xi}, \quad 2h=\omega_m\sqrt{\frac{1}{2\xi}-1}, \text{ where } \xi=\sqrt{1-\frac{A_{R0}^2}{A_{Rm}^2}} \qquad (10)$$

which allows to calculate numerical values of the unknown parameters α, h, q.

Next, we can identify real rig parameters' values as:

$$c=\frac{2hB_0}{l_c^2}, \quad k_1=\frac{qB_0}{a l_k l_x}, \quad k=B_0\alpha^2-k_1. \qquad (11)$$

Earlier, some physical values were measured or determined from basic engineering formulas:

$$B_0=1.38 \text{ kg m}^2, \quad l_c=0.54 \text{ m}, \quad l_k=0.54 \text{ m}, \quad l_x=0.24 \text{ m}, a=0.003 \text{ m}. \qquad (12)$$

Course of experiment:

1. Measure the AR amplitude vibrations for different angular velocity values ω; number of
2. measurements should be about 15. Carefully determine the value of ω_m, for which the amplitude of vibrations reaches the maximum value of A_{Rm}. Determine the amplitude value A_{R0} at close to zero excitation frequency. Copy your results into the table as below.
3. Calculate the parameter values α, h, q, using formula (10).
4. Calculate the amplitude values A of the theoretical resonance plot using the formula (6) for those ω values at which the A_R was measured.
5. Draw both, experimental and theoretical resonance graphs.
6. Calculate the values of the real system parameters k, c, k_1 using (11) and the values
7. of the parameters given in (12).

Note: take care of units used.

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Exercise 3 Report

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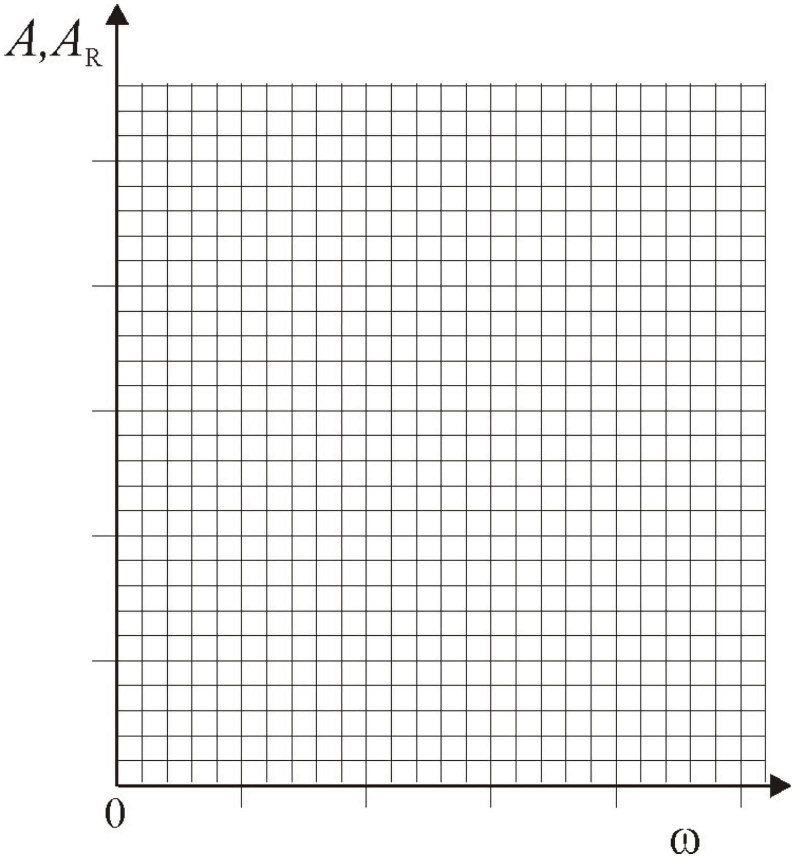
Measurements

ω	A_R	A



ω_m= …… A_{Rm}= …… A_{R0}= ……

Calculation of the physical model parameters:

$$\xi=\sqrt{1-\frac{A_{R0}^2}{A_{Rm}^2}}=.....[\ \ ] \qquad q=\frac{\omega_m^2 A_{R0}}{\xi}=.....[\ \ ] \qquad \alpha^2=\frac{\omega_m^2}{\xi}=.....[\ \ ] \qquad 2h=\omega_m\sqrt{\frac{2-2\xi}{\xi}}=.....[\ \ ]$$

Formula for calculation of the theoretical model amplitude:

$$A=\frac{q}{\sqrt{\left(\alpha^2-\omega^2\right)^2+4h^2\omega^2}}=..........[\ \ \ ]$$

$B_0=1.38\ \text{kg}\,\text{m}^2,\ \ l_c=0.54\ \text{m},\ \ l_k=0.54\ \text{m},\ \ l_x=0.24\ \text{m}, a=0.003$ m.

Calculation of the real model parameters:

$$c=\frac{2hB_0}{l_c^2}=.......\ [\ \ ] \qquad k_1=\frac{qB_0}{al_k l_x}=.......[\ \ ] \qquad k=\frac{B_0 a^2}{l_k^2}-k_1=........[\ \ ]$$