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Emergence of extreme outbreak events in population model

S. Dinesh Vijay[1,a], S. Leo Kingston[2,b], Suresh Kumarasamy[3,c], and Tomasz Kapitaniak[2,d]

[1] Center for Nonlinear and Complex Networks, SRM TRP Engineering College, Tiruchirappalli, Tamil Nadu 621105, India
[2] Division of Dynamics, Lodz University of Technology, Stefanowskiego 1/15, Lodz 90-924, Lodz, Poland
[3] Center for Artificial Intelligence, Easwari Engineering College, Chennai, Tamil Nadu 600089, India

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Abstract This study demonstrates the formation of distinct extreme outbreak dynamics in population models. We have investigated various intricate dynamics inclusive of the outbreak events in a single and two different coupling configurations. The system manifests abrupt deviations from the bounded region in an aperiodic interval owing to blowout bifurcation when the instability occurs in the invariant subspace. In addition, we have analyzed different transitions of the extreme outbreak dynamics using bifurcation plots and phase diagrams. The appearance of critical outbreak dynamics reveals a phenomenon known as dragon-kings, where the extreme values in the tail of the distribution depart from the expected pattern described by a power law. We performed comprehensive statistical measures of the dragon-kings test and cumulative probability distribution function to verify the emergence of dragon-kings dynamics in the system.

1 Introduction

Understanding diverse categories of population dynamics is a great deal of importance to unveil how discrete species interact with their environment. In particular, the numerous classes of population models are used in many fields, such as ecology [1, 2], population biology [3, 4], economics [5, 6], and medicine [7, 8] to decode discrete complex dynamics. In recent decades, researchers have proposed various population models, each of them focusing on exploring various aspects of intrinsic dynamics. For example, the exponential growth model elucidates the entanglement of population growth rate and size [9, 10]. On the other hand, the stochastic models deal with random variations in birth and death rates of different species [11, 12]. Similarly, the logistic model focuses on factors like intrinsic growth rate and environmental carrying capacity [13, 14]. The important role of age and survival rates while considering the age structure-based population system is examined in detail using distinct nonlinear dynamics perspectives in Refs. [15, 16]. Consequently, a deeper insight into population dynamics can be more beneficial to interpret divergent patterns of growth, stabilization, fluctuations, and future trends in population size and distribution [17]. In this paper, we focus on the logistic map-based population model, to explore atypical outbreak dynamics that pop up competition between the species.

Along this line, in recent decades, studies on acquiring deeper insight into numerous clusters of anomalous destructive catastrophic events investigated in different areas ranging from hydrodynamic and optics [18–20], geophysics [21], economics [22], power grid [23], different classes of complex systems [24–28] and to name a few. The dynamical origin and their variations are the substantive issues to accomplish better intuition on discrete extreme events. From this perspective, many underlying mechanisms which cause the formation of rare and extreme large-amplitude events have been demonstrated via different processes, namely the discrete type of intermittencies [29, 30], interior-crisis [31], stick–slip dynamics [32], blowout bifurcation [33], attractor hopping dynamics [34] and others [35].

[a] e-mail: sdvgreatphysics@gmail.com
[b] e-mails: kingston.cnld@gmail.com; leo.sahaya-tharsis@p.lodz.pl (corresponding author)
[c] e-mail: sureshscience@gmail.com
[d] e-mail: tomasz.kapitaniak@p.lodz.pl

Besides, a few recent reports elucidate the formations of EEs in different population models [36–38]. A parametrically driven coupled population model manifests extremely large biomass values for an appropriate coupling strength that signifies a sporadic deviation from its mean values [36]. The three-level tropical system of vegetation prey-predator populations elicit explosive burst-like dynamics in the predator density discussed in Ref. [37]. Various destructive activities of the population, either on their own carrying capacity or their coexisting species' carrying capacity, give rise to finite-time extinction or extreme blooms in their population dynamics [38].

In this work, we have explicitly focused on an extreme outbreak event in a Franke-Yakubu (FY) population model [39]. This model proves intermittent-rarity dynamics for a specific set of system parameter values that follow a universal scaling law behavior. Conversely, we have obtained remarkable dynamics in the FY model for a different set of system parameters so-called extreme outbreak dynamics. Noteworthy, we have identified a different scaling law behavior in the considered model that reveals Dragon-Kings (DK) dynamics. In principle, the formation of unforeseen rare events can be predictable (or) unpredictable, and exemplify low (or) high probabilities of occurrence based on the specific circumstances. In literature, two well-known approaches are used to deal with rare events, such as black swans and dragon-kings [21, 40] behavior. The black swan dynamics introduced by Nicholas Taleb [40] and classified as high-consequence events that are mostly unpredictable. Also, the low probability of appearance in numerous situations highlights the difficulty of its anticipation [41]. On the other hand, the DK formation, introduced by Sornette [21], focuses on distinct extreme dynamics that are both large in magnitude and relatively frequent [42]. These events exhibit sudden spikes or hump-like structures in probability at the tail of their distribution, manifesting typical behavior from the regular events. Hence, the advent of rare events in the tail region are outliers, that are deviating from the power law. The DK are particularly important while emphasizing the early warning signals approach, which provides some indication of the potential predictability of extreme events [41, 42].

Furthermore, in the realm of extreme events literature, an abnormal amplitude of temporal dynamics is affirmed by the different threshold measures. However, the formation of extreme events is effectively demonstrated, using the probability distribution function. Certainly, the frequent events hold a higher amount of probabilities, whereas the rare events possess lower probabilities. When the probability distribution function follows a power law fit, it indicates decreasing probabilities with the size of the events, thereby confirming the occurrence of extreme events. On the other hand, the dragon-kings shows a higher probability in the tail, which is caused by the consecutive typical events, and surpassing the power law fit in the tail of the distribution. Hence, the dragon-kings test is employed as a unique measure to elucidate discrete extreme large-amplitude dynamics in complex systems [43–45]. In our considered model, the appearance of a DK suggests a greater likelihood of species extinction or abnormal population growth. Consequently, the species can experience abnormal growth rates within a short time frame, leading to its potential extinction.

Moreover, blowout bifurcation plays a significant role in the formation of various intrinsic dynamics in nonlinear dynamical system [39, 46–48]. In general, a system can persist in a chaotic state in an invariant subspace for a specific range of system parameters. However, varying one of the parameters leads to gain or loss in the stability of chaotic attractors and can originate a critical dynamics termed as blowout bifurcation [46, 47]. Besides, understanding blowout bifurcation (or) outbreak dynamics in biological, ecological, and disease-spreading models is prime important to unravel a few unusual complex dynamics. Various synchronization behaviors and destabilization of its cooperative dynamics have been illustrated using blowout bifurcation in a coupled β-cell model [49]. The effect of environmental volatility has created a potential impact that leads to abrupt population breakout and their interrelation with blowout dynamics discussed in a Host-Parasitoid model [50]. Also, it has been established that the capability of the susceptible-infected-recovered (SIR) model to elucidate different stages of outbreak dynamics of the COVID-19 pandemic as well as their possible control strategies [51, 52]. In recent years, the formation of extreme events owing to blowout bifurcation has been explored in time-delay FitzHugh-Nagumo neuron model [33], coupled logistic maps with quasi-periodic forcing and thermo-fluid systems [44]. In this present work, we demonstrate the formation of critical outbreak dynamics and their transition in a specific population model with different configurations. In the context of population dynamics, the blowout bifurcation refers to sudden collapses or rapid growth of populations within a species. In our study, we have investigated the extreme outbreak dynamics resulting from both inter and intraspecific competition within the Franke-Yakubu model. This research focuses mainly on the reproductive behavior of species in the ecosystem and how extreme outbreak dynamics can lead to species extinction or abnormal growth.

The remainder of this article is organized as follows: Section 2 explains the details of the model taken into account for this study as well as the emergence of different dynamics in a broader set of parameter ranges of the system. Section 3 delves into the formation of extreme outbreak dynamics and their relevant mechanism using dynamical and statistical measures. Section 4 outlines the formation of critical outbreak dynamics in a diffusive and conjugately coupled population models. In the final section, we have presented the overall summary of our results.

2 Franke-Yakubu model

In order to illustrate the discrete intricate dynamics including a critical outbreak events in a population model, we have used Franke-Yakubu's (FY) model [39] which is denoted as the following discrete-time equations

$$\begin{aligned} x_{n+1} &= x_n \exp(r - s(x_n + y_n)) \\ y_{n+1} &= \frac{c_1 y_n}{c_2 + x_n + y_n}. \end{aligned} \tag{1}$$

Here, the variables x and y exemplify the contest between the species that can also intercepted using intra and intraspecific competition. The system parameters r and c_1 represent intrinsic growth rate, s and c_2 are the constants that are propositional to the carrying capacity of the environment, respectively. When we set the system parameters value $r = 2.916$, $c_1 = 20.25$, $s = 0.1$, and for a specific range of c_2, the system exhibits discrete complex dynamics. The formation of intermittency dynamics and riddle basin of attraction has been reported in the FY model for a set of system parameter values [39]. Nevertheless, in our illustration, we have chosen a slightly different set of parameter values, and demonstrate the existence of unusual dynamics of extreme outbreak events.

The system manifests a period-doubling route to chaos for a range of c_2 that is portrayed in a bifurcation diagram of Fig. 1a. Its equivalent Lyapunov exponent is showcased in Fig. 1b revealing the transition to chaos via successive period doubling sequence for a range of system parameter c_2. Besides, to exemplifying the formation of different dynamics in a larger range of parameter regions, we have presented a two-parameter phase diagram by varying $c_2 \in (0.5, 1.5)$ and $r \in (2.9, 3.0)$ as shown in Fig. 2. The dark gray region in Fig. 2 proves a different periodic dynamics, and the yellow region signifies chaotic motion. We discriminate the appearance of various dynamics in the system based on the Lyapunov exponent. Further, for the rigorous analysis of the chaotic dynamics (the

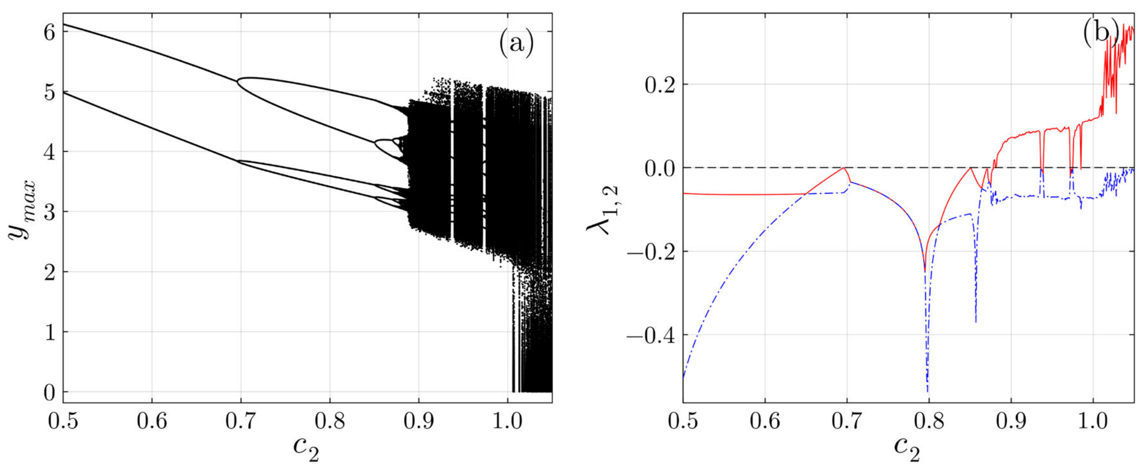
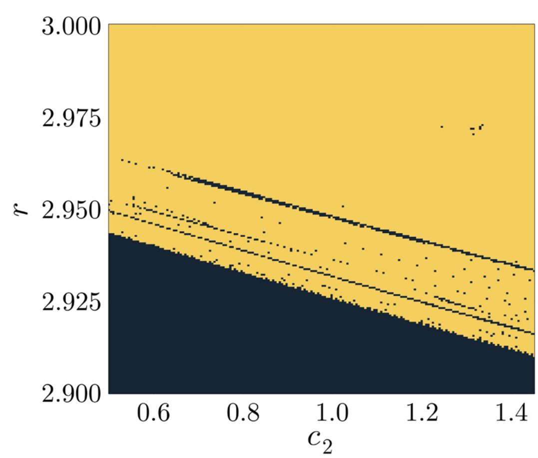


Fig. 1 **a** One parameter bifurcation diagram and **b** its corresponding Lyapunov exponent manifest period-doubling routes to chaos in a population model for a specific range of c_2

Fig. 2 Phase diagram for a wide range of c_2 and r proves the existence of periodic (dark gray region) and chaotic dynamics (yellow region) in the system, respectively



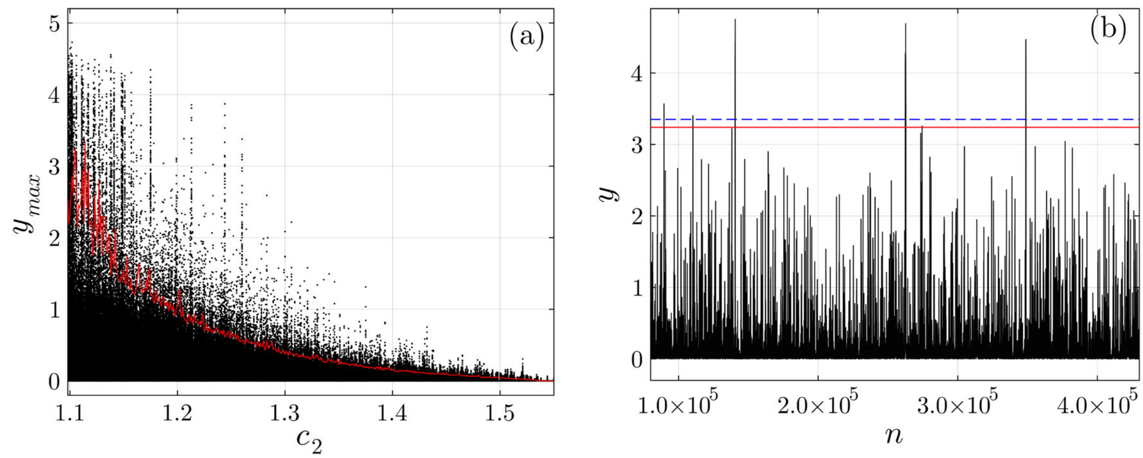


Fig. 3 **a** Bifurcation diagram of y_{max} for a range of $c_2 \in (1.1, 1.55)$ exemplifying the existence of larger outbreak dynamics in the population model. A significant height threshold is plotted as a red line to confirm the appearance of extreme events in the system. **b** Discrete temporal evolution of $y(n)$ proves extreme outbreak dynamics for $c_2 = 1.1$. The solid red line denotes a significant height threshold value, and the dashed blue line manifests 99th percentile value to ascertain the existence of extreme large-amplitude dynamics in the system

yellow region in Fig. 2), the system exemplifies an unexpected events known as extreme outbreak dynamics for a specific range of parameter values. In the forthcoming section, we will illustrate details on the formation of extreme outbreak events and their transitions using relevant measures.

3 Formation of extreme outbreak dynamics

In this section, we explored the origin of extreme outbreak dynamics in the FY model. For that, we embark on presenting a one-parameter bifurcation by plotting the local maxima of the y variable (y_{max}) for a specific range of the system parameter. Here, we set the parameter values of $r = 2.916$, $c_1 = 20.25$, $s = 0.1$, and for the gradual increase of $c_2 \in (1.1, 1.5)$, the FY model exhibits unforeseen large expansions from the bounded region which is pictured in Fig. 3a. Indeed, we have observed considerable large amplitude dynamics closer to 1.1 of the bifurcation parameter values, and subsequently, those large expansions gradually decay in height for the higher values of c_2. To discern the existence of extreme outbreak dynamics in the population model, we have estimated a significant height threshold that is overlapped as a solid red line in the bifurcation diagram of Fig. 3a. The significant height threshold is defined as $H_S = \langle y_n \rangle + N\sigma$, where y_n denotes an immense number of iteration (n) of y variable and σ signifies its corresponding standard deviation. Besides, N represents an arbitrary constant value that is chosen four to eight times to ascertain the appearance of discrete extreme events in a wide range of complex dynamical systems [29, 53]. However, here, we have taken $N = 12$ to reveal the advent of extreme outbreak dynamics in the FY model which has a remarkable impact then the other extreme events. Such large impact extreme dynamics entitled as superextreme events in the complex systems [54, 55].

In addition, to understand the aspect of these unusual dynamics, a temporal evolution for long iterations of extreme outbreak events for $c_2 = 1.1$ is illustrated in Fig. 3b. It can be obtained from Fig. 3b, the occasional rare bursting-like dynamics appearing from the bounded chaotic region. We have plotted the significant height threshold as a solid red in the time series plot of Fig. 3b to confirm the emergence of extreme outbreak dynamics. It is clear from the temporal plot a few extreme events appear in the system with an infrequent time of iterations. Moreover, some disagreements that, taking into account on significant height threshold as a generic measure to quantify the formation of extreme dynamics. In order to uncover such obscurity, we have calculated a 99th percentile measure [53, 56] for the extreme outbreak dynamics data set and the resultant value plotted as dashed blue line in Fig. 3b. Certainly, the significant height threshold and 99th percentile measure closely match with each other supporting the advent of atypical extreme dynamics in the FY model.

Moreover, we have analyzed the probability distribution function (PDF) for the sporadic appearance of extreme large-amplitude dynamics for $c_2 = 1.1$, portrayed in Fig. 4a. Intriguingly, the PDF elucidates a DK distribution in the log-log scale. Indeed, the hump-like structure appears in the tail region, owing to the emergence of rare events in the system. We have fitted the distribution with a power law (dashed line), which shows outliers deviate from the straight line showcased in Fig. 4a. The p-values of the DK test is estimated for the extreme outbreak events, and the result is presented in Fig. 4b. Indeed, only a few events in the whole data set prove distinct distribution

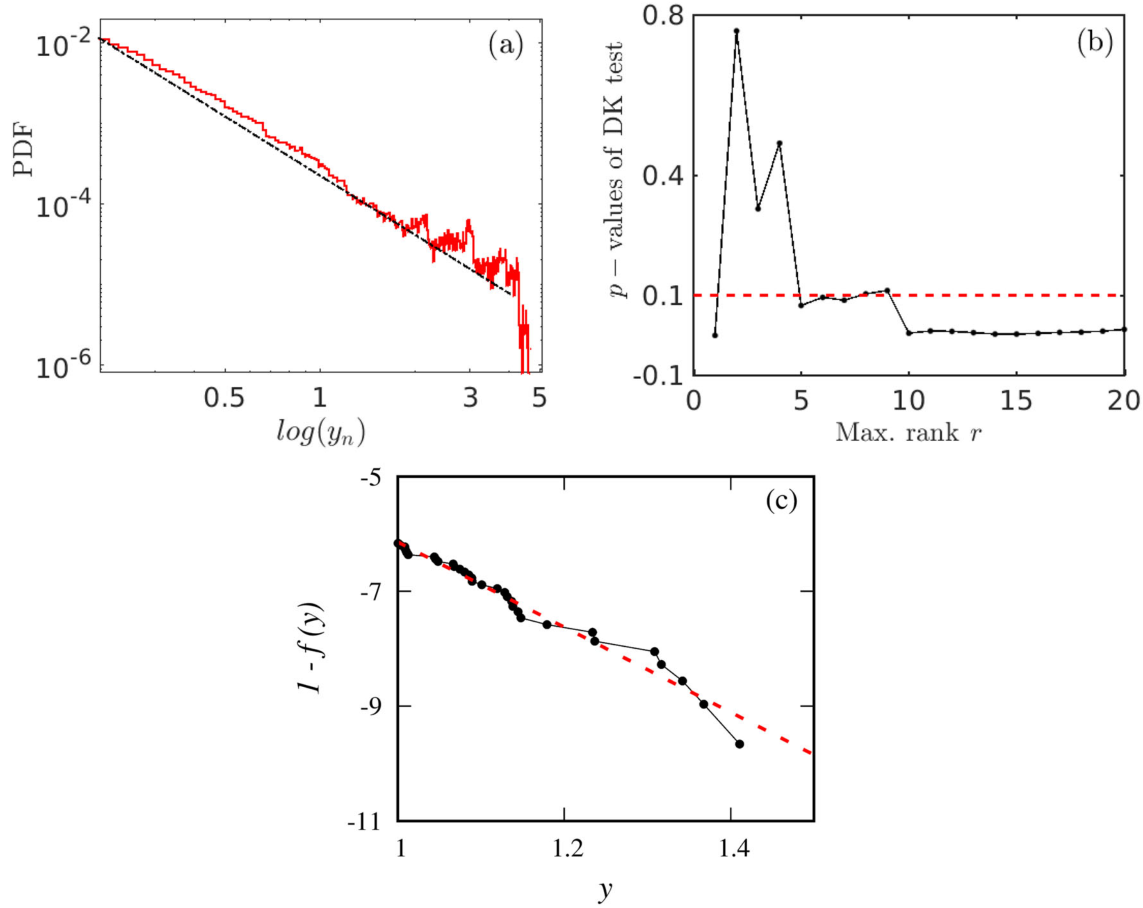


Fig. 4 **a** Probability distribution function for extreme outbreak dynamics elucidates a dragon-king shape distribution. Rare events in the tail region deviated from the normal population (dashed line) illustrate a hump-like structure. **b** DK test performed as a function of the maximal rank r (Max. rank r) by varying p-values signify the existence of extreme events in the system. **c** The complementary cumulative distribution function is fitted with a straight line fit (dashed red line)

from the power law denote the false null hypothesis. On the other hand, the majority of the events that are taken for analysis aligned with power law emanating the positive hypothesis, more details of p-values of the DK test given in Refs. [57, 58]. The appearance of p-values lesser than 0.1 signifies the occurrence of DK reported in a few complex dynamical systems [44, 57, 58]. In our case, the considered FY model manifests p-values less than 0.1 for a range of maximal rank of r confirming the existence of DK in the system.

In general, any events that deviate either from the power law or exponential fit distribution are classified as outliers or DK events. Besides, the cumulative distribution function (CDF) is estimated by integrating PDFs of all events. Subsequently, inverting the CDF gives rise to a complementary cumulative distribution function (CCDF), say, $1 - f(y)$. Here y denotes the peak values of the critical dynamics of the system. The obtained CCDF plot portrayed in Fig. 4c, where a few events deviated from the straight line fit (dashed red line) affirming the advent of DK behavior in the system.

In order to manifest the occurrence of extreme outbreak dynamics in a broader parameter region, we have presented a two-parameter phase diagram in Fig. 5. We have performed a comprehensive analysis in the chaotic region of Fig. 2 disclosing the emergence of extreme outbreak dynamics in the system, i.e., embedded in the chaotic region. The emergence of chaotic (light yellow) and extreme events (dark gray) is distinguished using the significant height threshold measure. Consequently, the FY model demonstrates critical outbreak dynamics in only an appropriate range of parameter values, as compared with a predominant appearance of chaotic dynamics.

Now, we pay attention to the emerging mechanism of extreme outbreak dynamics in the population model. The FY model manifests extreme outbreak dynamics as a consequence of blowout bifurcation. When the system transits from periodic to chaotic dynamics it remains in the same state for a certain range of control parameter values, say, $c_2 \in (0.9, 1.02)$ elucidating that the system is in the invariant subspace. Upon increasing the control parameter value, the system demonstrates infrequent unexpected larger deviations from the bounded state owing

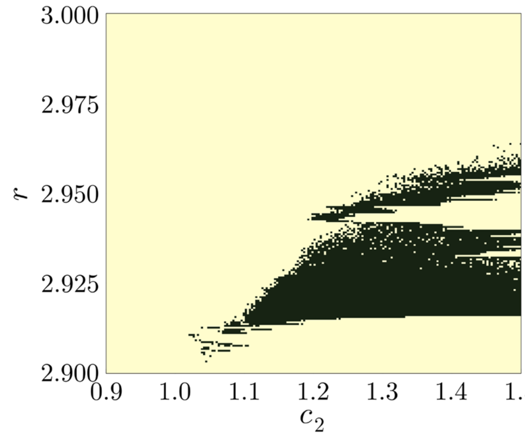


Fig. 5 The phase diagram for the single population model proves the existence of extreme (dark gray) and non-extreme events (light yellow) in the two-parameter space

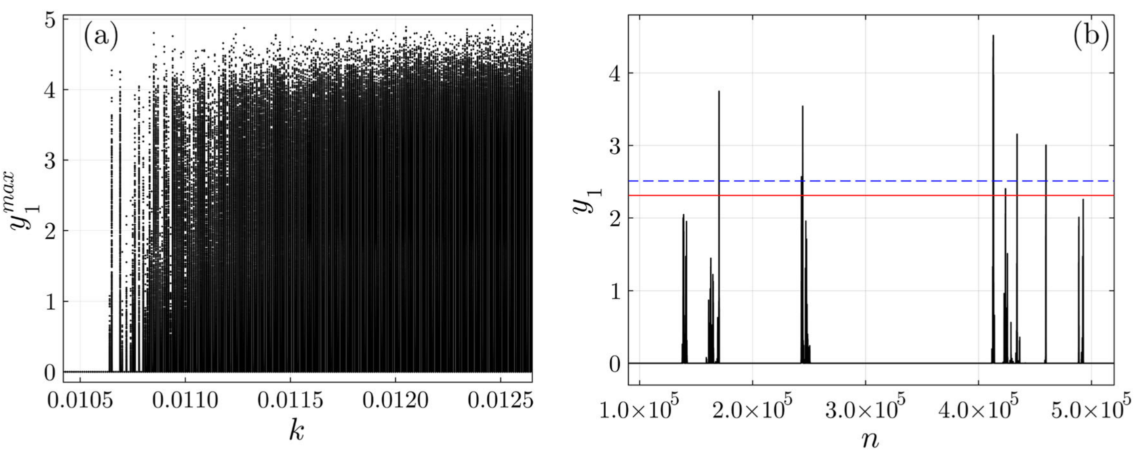


Fig. 6 **a** The bifurcation diagram of the diffusive coupled population model proves an abrupt increase in the y_1^{max} for the critical coupling parameter value $k = 0.0108$. **b** Discrete temporal evolution of $y_1(n)$ represents on-off intermittent dynamics that exceed a qualifier threshold. The solid red and dashed blue lines in **b** denote the significant height threshold and 99th percentile value, respectively

to loss of stability in the invariant region in the system. Hence, abrupt large amplitude dynamics occurred in the population model.

4 Extreme outbreak dynamics: coupled system

Next, we have extended our illustration into a different configurations of coupled population models, such as diffusive and conjugate coupling scheme. The coupled FY model exemplifying unexpected outbreak dynamics for the specific range of system parameter regions are elaborate in the following subsection.

4.1 Diffusive coupling scheme

We commence with illuminating formation of specific outbreak dynamics in the diffusive coupled FY model, that has significance relevance to grasp diversified population dynamics. For example, the predator–prey model with diffusive coupling scheme used to exemplify various synchronous and asynchronous dynamics in ecological system [59]. The coupled population model described as follow,

$$x_{n+1}^i = x_n^i \exp(r - s(x_n^i + y_n^i)) + k(x_n^j - x_n^i),$$
$$y_{n+1}^i = \frac{c_1 y_n^i}{c_2 + x_n^i + y_n^i} \qquad i,\, j \in 1,\, 2; i \neq j. \tag{2}$$

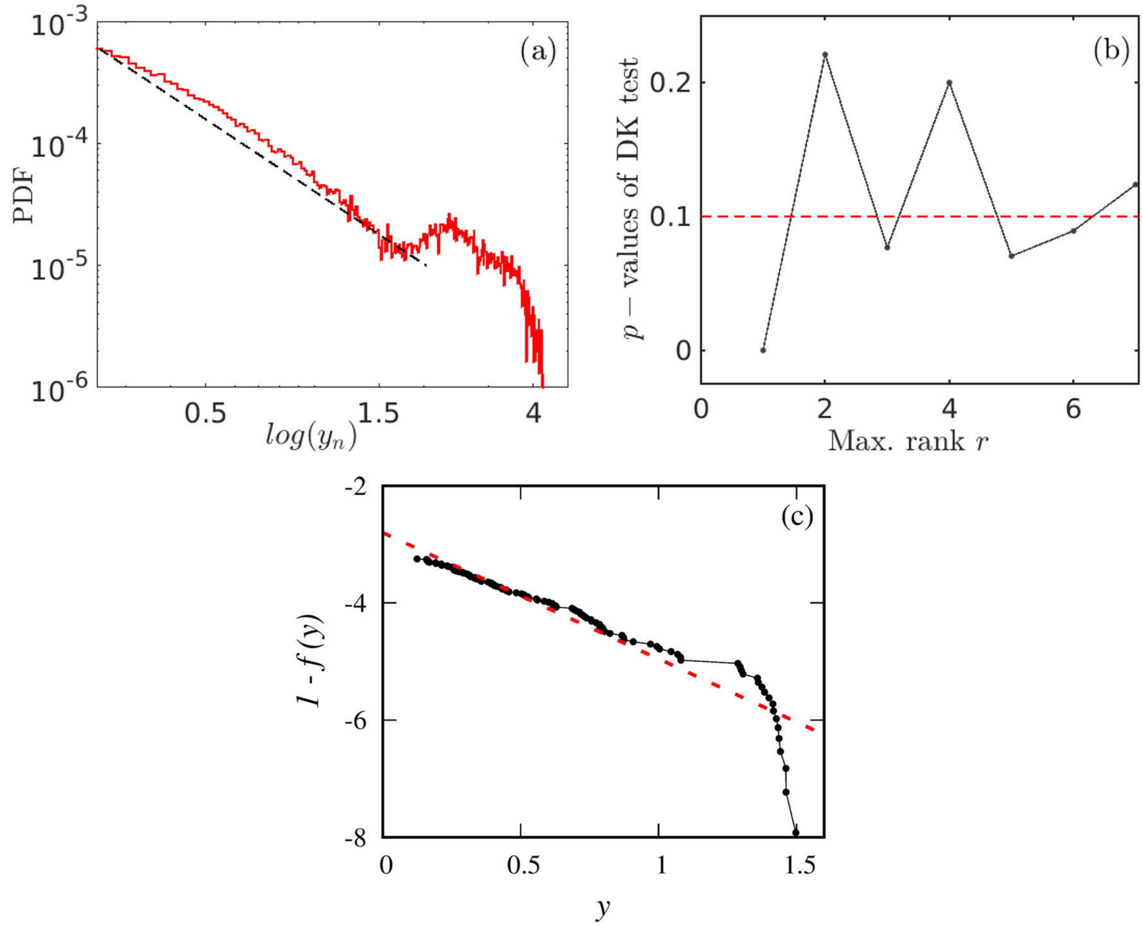


Fig. 7 **a** Extreme outbreak dynamics for the coupling parameter values $k = 0.0108$ proves a dragon-kings distribution. Normal events fitted with the power law (gray dashed line) and the rare events in the tail region deviated from the power law. **b** DK test for p-values less than 0.1 elucidates the appearance of extreme outbreak dynamics in the coupled system. **c** In the complementary cumulative distribution function outliers in the tail are deviated from the straight line fit (dashed red line)

Here, we fixed the parameter values of the model Eq. (2) as $r = 2.9$, $s = 0.1$, $c_1 = 20.25$, $c_2 = 1.0$, and for a fine turning of coupling strength k, the system elucidate transition from periodic to sudden large expansion.

We have presented a one-parameter bifurcation diagram for the coupled population model in Fig. 6a. The bifurcation plot portrays the influence of the coupling parameter in the advent of critical outbreak dynamics at $k = 0.0108$, that is, within the intra-species populations, the origin of unforeseen large expansion identified in the system. Here, the sudden surge originated via on-off intermittency, which is a specific dynamical behavior that appears in many complex dynamical systems as the consequence of blowout bifurcation [46]. In the coupled FY model, such on-off intermittent dynamics elicit extremely large amplitude events and create an abnormal rare event in the system for a range of parameter values. Additionally, to identify the outbreak dynamics within the coupled interaction of species, we have plotted the discrete temporal evolution of the system dynamics for $k = 0.0108$, as illustrated in Fig. 6b. We have computed the significant height threshold and 99th percentile value for the extensive length of extreme outbreak dynamics data and superimposed with the temporal dynamics as solid red and dashed blue lines, respectively. Both measures confirm the existence of critical outbreak dynamics in the system since the extremely large amplitude events cross the threshold values.

The probability distribution function is depicted in Fig. 7a, which is estimated from the larger numbers of peaks of iterations. In the PDF plot, recurrent events up to a maximum amplitude of 1.5 following a power law, that is superimposed by a dashed gray line. Notably, in Fig. 7a, the events which are not aligned with the power law, manifest a dragon-kings dynamics. We have determined the p-values of the DK test and CCDF for the on-off intermittency-induced extreme outbreak events as shown in Fig. 7b, c. The DK test proves a lesser than 0.1 for the specific rank values, and the CCDF exemplifies the deviation of anomalous events that are diverted from the straight line fit (dashed red line) to ascertain the existence of DK event in the coupled population model.

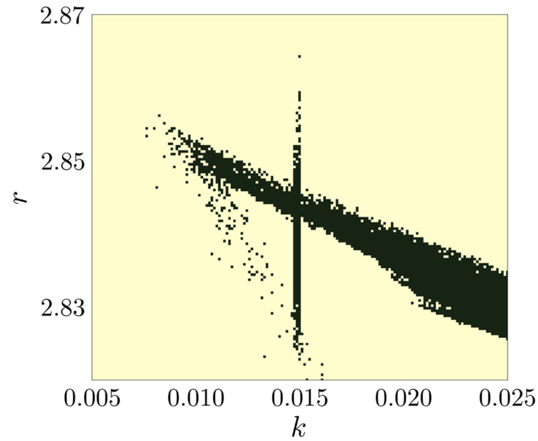


Fig. 8 Two parameter phase diagram for the coupled population model illustrates the emergence of extreme outbreak events (dark gray) and non-extreme events (light yellow) in a specific parameter region

Moreover, the existence of extreme and non-extreme event regions are depicted in a two-parameter phase diagram across a wide range in the k versus r plane is illustrated in Fig. 8. Here, the dark gray color signifies extreme outbreak dynamics within the intra-species populations. On the other hand, the light yellow region portrays the appearance of non-extreme events in the system. As compared with the single population model, the diffusively coupled system proves extreme outbreak events only a small range of parameter space.

4.2 Conjugate coupling scheme

In this subsection, to unravel the dissimilar (or) nonlinear interaction between the two population models, we have demonstrated the inception of various complex dynamics and extreme events in a conjugately coupled FY model. The model equations read as follows,

$$\begin{aligned} x^i_{n+1} &= x^i_n \exp(r - s(x^i_n + y^i_n)) + k(y^j_n - x^i_n), \\ y^i_{n+1} &= \frac{c_1 y^i_n}{c_2 + x^i_n + y^i_n} \qquad i,\, j \in 1,\, 2; i \neq j. \end{aligned} \tag{3}$$

Here, we set the parameter values of the model Eq. (3) as $r = 2.85$, $s = 0.1$, $c_1 = 19.5$, $k = 0.0075$, and for a gradual increase of carrying capacity parameter c_2, the system shows period-doubling route to chaos and extreme outbreak dynamics for a range of system parameter. Note that, the conjugately coupled population model has valid physical interpretation [60]. For example, while studying coupled predator–prey model using conjugate coupling which is also understood as cross-predator process [60]. Each predator has its prey in their environment. However, in a particular situation, the overconsumption of prey from one population to another population environment can lead to critical collapse (or) formation of outbreak dynamics in the ecosystem. Based on the aforementioned approach, we have elucidated the advent of outbreak dynamics owing to the interaction of two population models using a conjugate coupling scheme.

Two bifurcation diagrams plotted for the conjugately coupled FY population model for a long range of $c_2 \in (0.2,\, 1.8)$ which are portrayed in Fig. 9a, b. For the carrying capacity values lesser than 0.81, the system proves the period-doubling route to chaos as clearly attests in Fig. 9a. Particularly, for this specific parameter range, the system endures in the invariant subspace and signifies distinct periodic and chaotic dynamics. Upon increasing the c_2 values, the system experiences instability in its invariant region. As a consequence, we have seen occasional rare bursting from the bounded chaotic region due to the blowout bifurcation. The advent of infrequent outbreak dynamics for a range of c_2 is demonstrated in Fig. 9b. On the other hand, for larger values of c_2, the system turns into an extinction state. To comprehend the exact parameter region for the existence of extreme events, we have drawn the significant height threshold (red line) in the bifurcation diagram of Fig. 9b. Note that, to correlate the formation outbreaks dynamics in single and conjugately coupled FY models, we have presented the transitions of the conjugately coupled system by varying the carrying capacity parameter instead of coupling strength. The obtained results illustrate that the FY model proves outbreak dynamics irrespective of the number of populations involved during their interactions. Further, evidence for the emergence of extreme dynamics in the conjugately coupled system, we have pictured the evolution of y_1(n) in Fig. 9c. The rare appearance of outbreak dynamics from the chaotic domain is apparently shown in Fig. 9c.

Moreover, to establish the existence of dragon-kings dynamics in the conjugately coupled configuration of the FY model, we have presented the probability distribution in Fig. 10a. The regular events deviated from the power law (dashed line) signify a hump-like structure in the tail region (see Fig. 10a). Once again, we have performed the

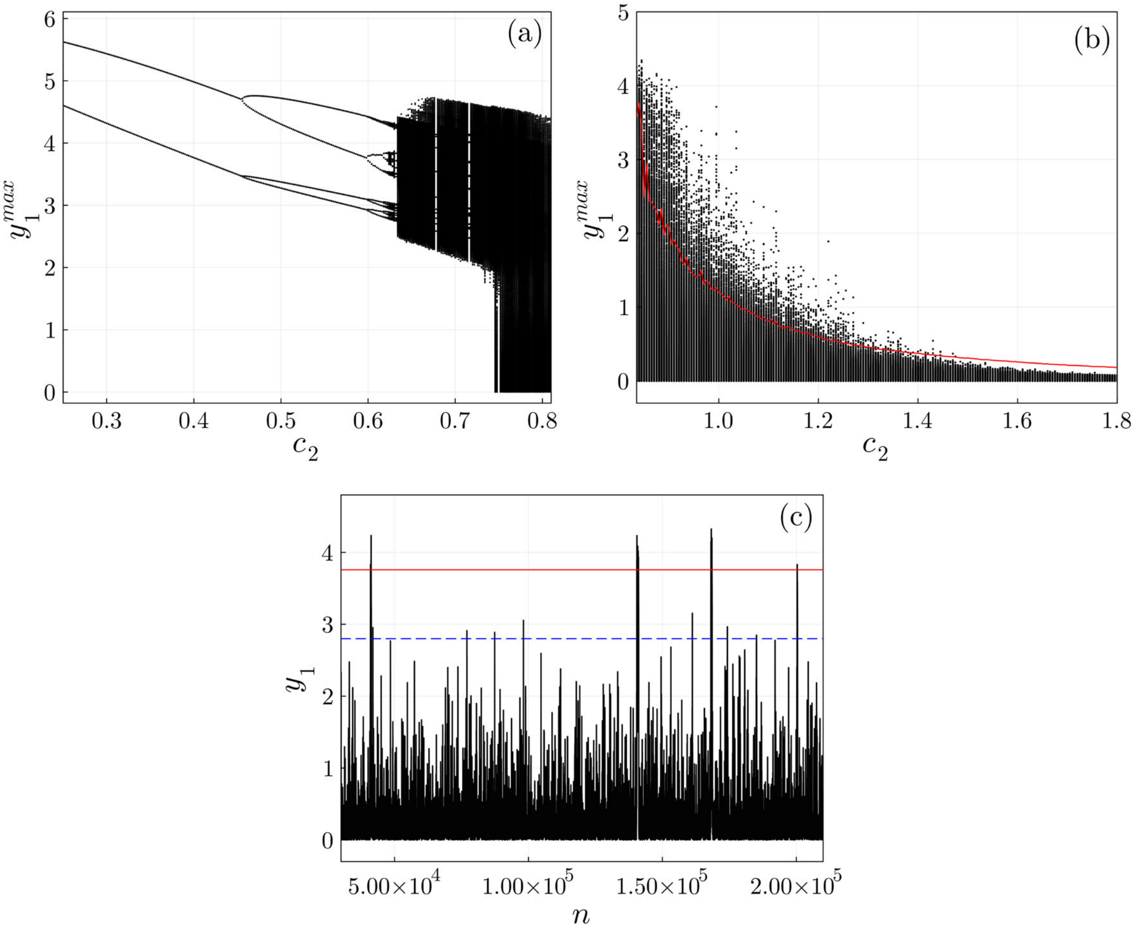


Fig. 9 **a**, **b** Bifurcation diagrams for conjugately coupled FY model: successive period doubling routes to chaos proved in **a** and for $c_2 > 0.81$ the advent of outbreak dynamics presented in **b** for a range of c_2, then the system transit to the extinction state. **c** Discrete temporal dynamics of conjugately coupled FY model manifests extreme events for the coupling strength $k = 0.0075$ and $c_2 = 0.82$

p-values of the DK test as well as CCDF, and the obtained results are showcased in Fig. 10b, c. It demonstrated that DK test values elucidate less than 0.1 for a specific range of rank values. Besides, in the CCDF plot, outbreak events in the tail region are divagated from the straight line fit (dashed red line). Both the DK test and CCDF upheld the existence of dragon-kings dynamics in the conjugately coupled FY model. We have established the existing region of extreme outbreak events with the effect of intrinsic growth rate and carrying capacity values by presenting the two-parameter phase diagram in Fig. 11. The outbreak dynamics appeared for the specific range of system parameter values.

5 Conclusion

In this study, we have explored the formation of extreme outbreak events in both single and different coupled layouts of population models. The system exhibits discrete complex dynamics for the wide region of parameter space. A detailed analysis of the chaotic region discloses the advent of extremely large amplitude dynamics in a limited range of the parameter region. The transition from the chaotic region to abrupt expansions is illustrated using bifurcation diagrams. In addition, the FY model proves a dragon-kings dynamics for all the cases. We have performed rigorous statistical measures namely p-values of the DK test and complementary cumulative distribution function to confirm the appearance of DK behaviors for the critical outbreak dynamics. Finally, our observation illuminated various possibilities for the advent of unusual dynamics in the population models. Moreover, the intensive interaction of one population with another can elicit sudden outbreaks or critical collapse in an ecosystem. In view of this, rigorous understanding between distinct populations with different coupling schemes would be more beneficial to

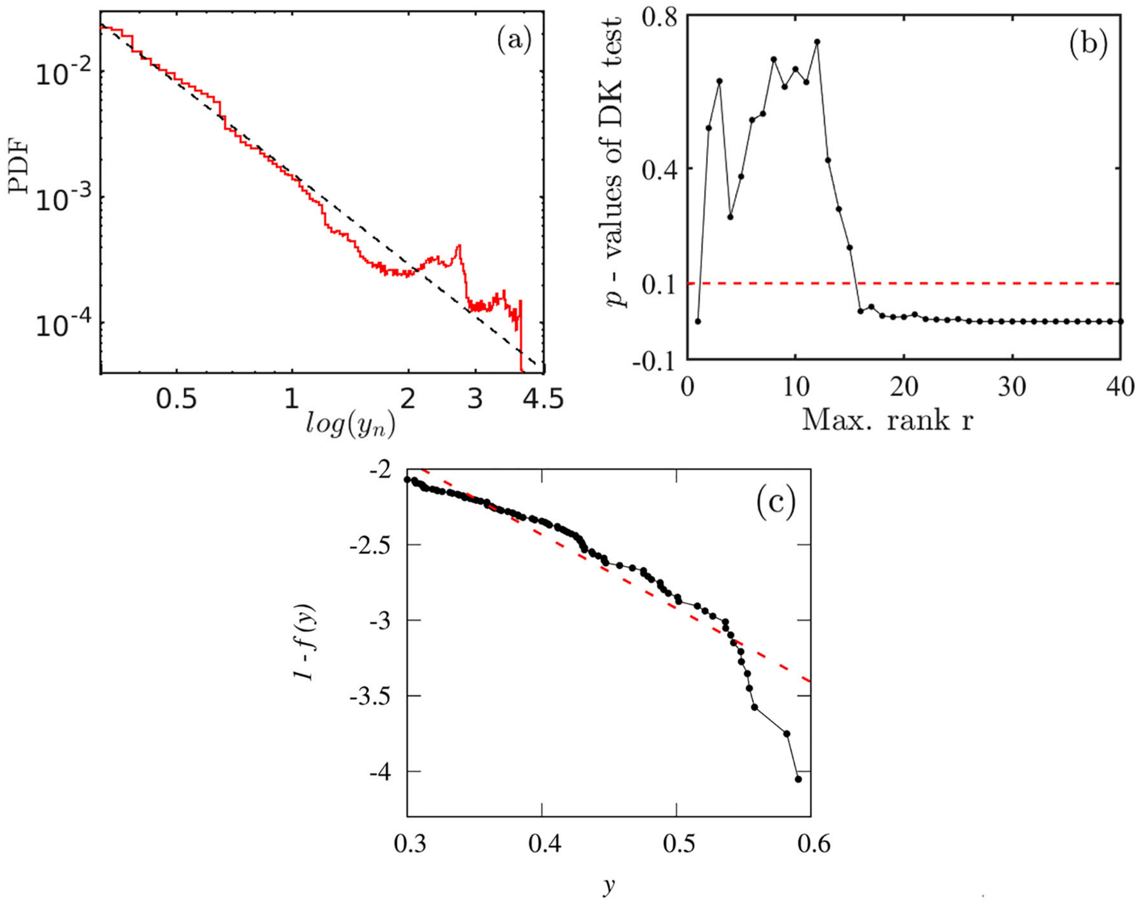
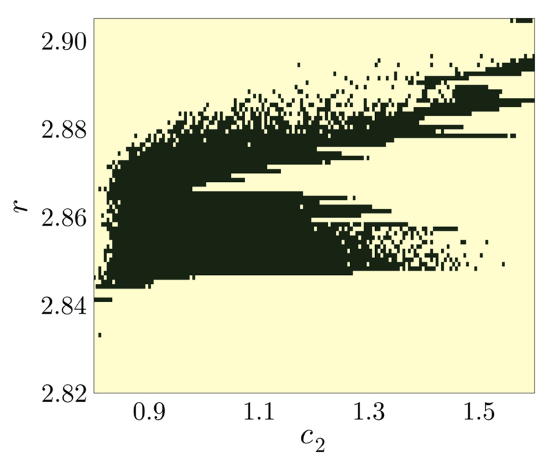


Fig. 10 **a** Extreme outbreak events for $c_2 = 0.82$ signify a dragon-kings distribution. Bounded dynamics fitted with the power law (gray dashed line), and the outbreak events in the tail region deviated from the power law. **b** DK test of p-values less than 0.1 proves the appearance of extreme events in the conjugately coupled FY model. **c** CCDF represents that the atypical events in the tail are diverse from the straight line fit (dashed red line)

Fig. 11 The phase diagram for the conjugately coupled population model: formation of extreme events (dark gray) and other dynamics (light yellow) are distinguished in the two-parameter space



identify a resilient environment in a ecosystem. Consequently, it helps to pinpoint the disasters vulnerable regions in the population models.

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Author contribution statement

SLK and TK suggested the model. All authors helped in the results interpretation and manuscript evaluation. SLK and SK did simulations. All the authors contributed to the system's investigation. SDV, SLK, and SK helped to evaluate and edit the manuscript. All the authors read and approved the final manuscript.

Data availability statement The data that support the findings of this study are available from the corresponding author upon request.

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