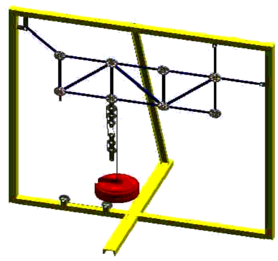
INTERNATIONAL FACULTY OF ENGINEERING



Test 2

DETERMINATION OF FORCES IN THE TRUSS MEMBERS

2.1. Aim of the experiment

The aim of the test is to determine forces in the members of the plane truss experimentally using a strain-gauge measurements technique and by analytical method of joints and method of sections.

2.2. Theoretical backgrounds

2.2.1. Forces in members of the truss

The truss consists of straight slender members connected at joints. In the case of a simple truss, we have the following condition for its proper rigidity (the truss will not collapse – is not a mechanism)

$$p = 2w\text{-}3, \tag{2.1}$$

where p is the total number of truss members and w is the total number of joints.

Loads must be applied to the joints only, and not to the members themselves. In the analysis of the truss, the weights of bars are either omitted or, if required, they are applied to the joints (a half of the weight to each of the bar joints).

The truss presented in Fig.2.1 is a plane structure made of $p = 13$ members, $w = 8$ joints and 3 supporting bars (these are not truss members). If a load is applied at arbitrary joints, the internal forces in the truss members and in the supporting bars (reactions) occur.

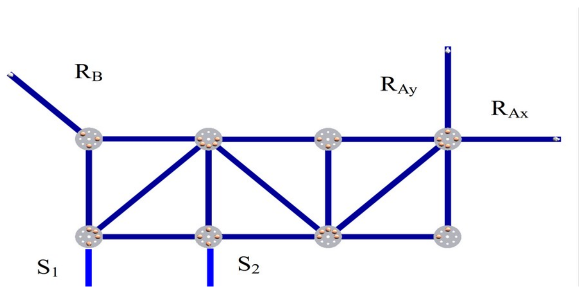


Fig.2.1. Plan truss

Such truss may be considered as a set of two-force members pin-jointed at their ends. The forces in all bars can be determined considering successively the equilibrium of each joint. There are $p+3$ equations available for p members and 3 reactions. This *method of joints* is most effective when the forces in all the members are to be determined.

If the force in only one member or the forces in few members are to be determined, the *method of section* is more efficient. In this method, we analyze the equilibrium of a large portion of the truss, composed of several joints and members separated from the structure by an imaginary section. This section must cut maximum three members thus allowing the calculation of three unknowns for 2-D force system. In this case there are *3* equations available (considering the entire truss as a free body, we have in addition *3* equations to determine the reactions if necessary).

In practice, there are special cases of joints loading. Particularly important is zero-force member.

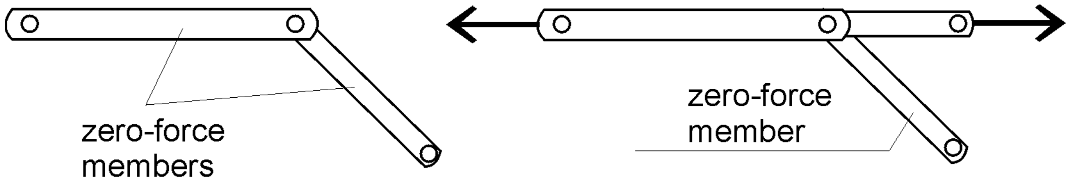


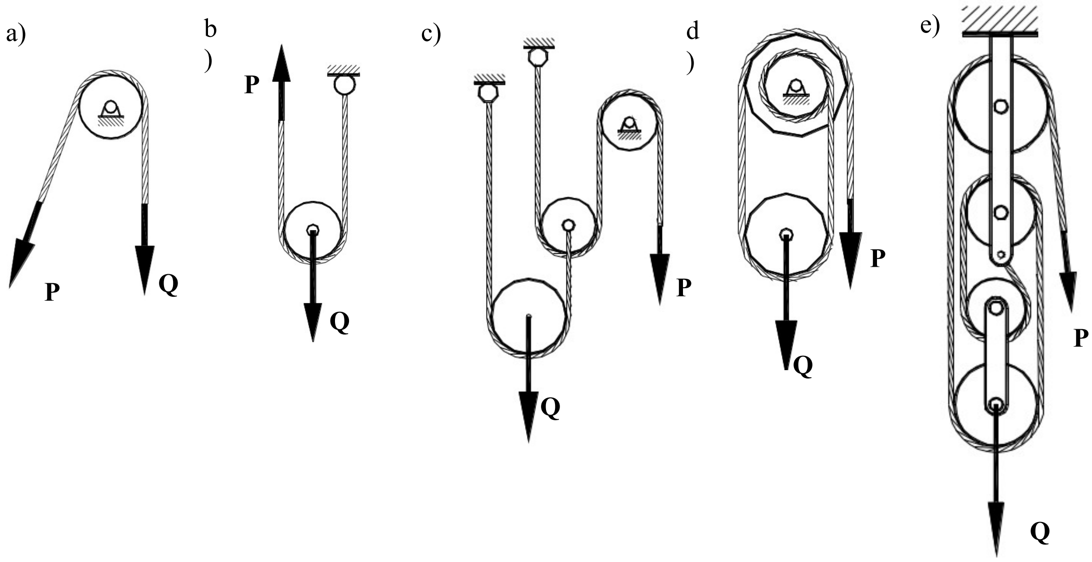
Fig.2.2. Examples of zero-force members

It should be noted, however, that such members are not useless (they are needed to secure the stability of the truss structure under self-weight or if the loading conditions are changed for example).

2.2.2. Rope-and-pulley arrangements

Their main purpose is to transmit and modify input force **P** into the output force **Q**. Several rope-and-pulley arrangements are shown in the Fig. 2.3.

Fig.2.3. Examples of rope-and-pulley arrangements



In the ideal case the virtual work of the applied force **P** is equal to the one of the load **Q**. It should be noted, however, that in real systems the magnitude of the force **P** must be grater because of friction forces and other resistances. For example in the case of the simple pulley (Fig.2.3a) its mechanical efficiency is of 97-99%.

2.2.3. Strain-gauge experimental analysis

Let us consider the rod loaded by two equal and opposite forces **N** and -**N** (Fig.2.4).

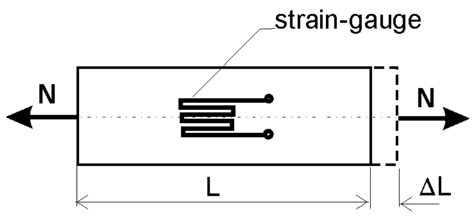


Fig.2.4. Uniaxial tension of the bar and a strain-gauge

If the rod is made from homogeneous, isotropic material its relative elongation (strain) is given by

$$\varepsilon = \frac{\Delta L}{L} = \frac{N}{EF} = \frac{\sigma}{E}, \tag{2.2}$$

where:

ε - longitudinal strain of the bar.
ΔL - elongation of the bar,
L - length of the bar,
N - magnitude of the axial force,
F - area of the cross-section of the bar,
E - Young's modulus,
σ - normal stress in the bar.

The contact between the strain-gauge and the surface of the bar is assumed as ideal (there is no sliding of the gauge with respect to the bar surface).

The variation of the resistance is produced by mechanical and thermal deformation of the bar:

$$\left(\frac{\Delta R}{R}\right) = \left(\frac{\Delta R}{R}\right)_{\varepsilon} + \left(\frac{\Delta R}{R}\right)_{T}. \tag{2.3}$$

Variation of the resistance due to mechanical deformations

The elongation ΔL of the bar produces the deformation of the gauge wire. The electric resistance R of the gauge is expressed as

$$R = \rho \frac{l}{S}$$

where ρ, l, S are: resistivity, length and cross section area of the gauge sensor (for example the diameter for wire sensor is between *20* and *50* μm or the thickness for foil sensor is about *25* μm).

A total differential of the R will be

$$dR = \frac{1}{S} d\rho + \frac{\rho}{S} dl - \frac{\rho l}{S^2} dS$$

and the relative increase ΔR due to elongation of the bar is given by the following

$$\left(\frac{\Delta R}{R}\right)_{\varepsilon} = \frac{\Delta \rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta S}{S}.$$

The state of deformations of the wire is uniaxial tension (stress σ_x only). The elongation of the wire Δl produces not only axial strains ε_x in the x direction but also same contractions of the wire section in the y and z directions.

$$\varepsilon_x = \varepsilon, \qquad \varepsilon_y = -\nu\varepsilon_x = -\nu\varepsilon \qquad \text{and} \qquad \varepsilon_z = -\nu\varepsilon_x = -\nu\varepsilon,$$

where $\varepsilon = \frac{\Delta l}{l}$.

The change of cross section ΔS of the wire due to σ_x stress with accuracy up to second order terms is expressed as

$$\frac{\Delta S}{S} = -2\nu\varepsilon.$$

The mechanical deformation of the bar yields a relative variation of resistance ΔR in the following form

$$\left(\frac{\Delta R}{R}\right)_{\varepsilon} = \left(\frac{\Delta\rho / \rho}{\varepsilon} + 1 + 2\nu\right)\varepsilon. \tag{2.4}$$

For a given type of gauge, the expression in brackets is for substantial range of strain is constant and is called strain-gauge factor k.

Finally, in the range of elastic strains we arrive at the following relation (assuming constant temperature) is

$$\varepsilon = \frac{\Delta l}{l} = \frac{1}{k}\frac{\Delta R}{R}. \tag{2.5}$$

The factor k is determined experimentally and it depends on the material of the gauge. For example, in the case of *constantan* (the alloy of 60 % of Cu and 40 % of Ni) $k = 2,1 - 2,4$, for another alloy called *iso-elastic* the value of $k = 3,2$.

When the factor k is known, the measurement of the changes of the strain-gauge resistance induced by bar deformation due to the axial force acting on the bar can be used for determination of bar strain ε from which the stress in the bar cross-section can be calculated.

Variation of the resistance due to temperature influence

The thermal resistance variation of the gage is caused by:

- material resistance variation due to temperature change

$$\left(\frac{\Delta R}{R}\right)_T = \beta_j \Delta T$$

□$_j$ - coefficient of the thermal resistivity of the gauge,

- thermal dilatation of the gauge

$$\left(\frac{\Delta L}{L}\right)_T = \alpha_j \Delta T,$$

□$_j$ - coefficient of the thermal dilatation of the gauge,

- thermal dilatation of the measured element (for not constrained member)

$$\left(\frac{\Delta L}{L}\right)_T = \alpha_p \Delta T$$

□$_b$ - coefficient of the turmoil dilatation of the measured element.

Finally, the total -influence of the temperature on the relative variation of resistance ΔR is given by the following expression:

$$\left(\frac{\Delta R}{R}\right)_T = \left[\beta_j + k(\alpha_b - \alpha_j)\right]\Delta T. \tag{2.7}$$

The changes of the resistance caused by the force action (mechanical deformations) are very small. Because quite often they are of magnitude comparable to the changes induced by the temperature, it is necessary to introduce compensating systems of strain-gauges.

Wheatstone's bridge system

During resistance measurement using Wheatstone's bridge system there are two methods applied for temperature compensation (Fig. 2.5):

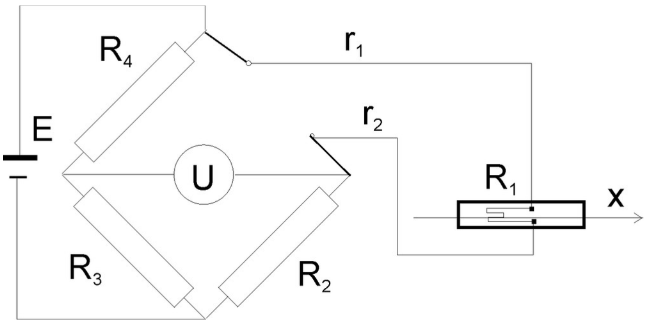
- 1/4 (quarter) Wheatstone's bridge system,
- 1/2 (half) Wheatstone's bridge.

Changes of resistance in the Wheatstone's bridge circuit branches result in the change of the voltage along the bridge diagonal as follows:

$$\Delta U = C\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] \tag{2.6}$$

where C is the constant of the bridge circuit.

- *1/4 bridge*



For 1/4 bridge the formula (2.6) becomes

$$\Delta U = C\left[\frac{\Delta(R_1 + r_1 + r_2)}{R_1 + r_1 + r_2}\right]$$

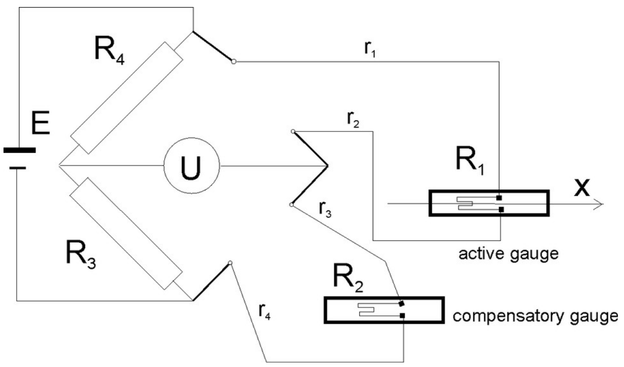
In the case when:

- resistances r_1 and r_2 of connecting wires are identical and equal r,
- strain-gauges and the connecting wires are subjected to identical temperature changes,
- self-compensating gauge is used

we obtain: $\varepsilon = \frac{1}{k}\frac{\Delta U}{C}\left[1 + \frac{2r}{R_1}\right]$.

It is possible to correct this error by some modification of the factor *k*.

- *1/2 bridge*



In the case when:

- resistances r_1 and r_2 of connecting wires are identical and equal r,
- strain-gauges and the connecting wires are subjected to identical temperature changes
- the compensating gauge R_2 is identical with the measuring gauge R_1; its resistance change might be caused by the ΔT only

hence we have

$$\Delta U = C\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right]\frac{R}{R + r}.$$

Finally, the following relation for strain is obtained

$$\varepsilon = \frac{1}{k}\frac{\Delta U}{C}\left[1 + \frac{r}{R}\right].$$

In this way, any thermal influence can be eliminated at all.

Fig.2.5. Strain-gauges connection (Wheatstone bridge)

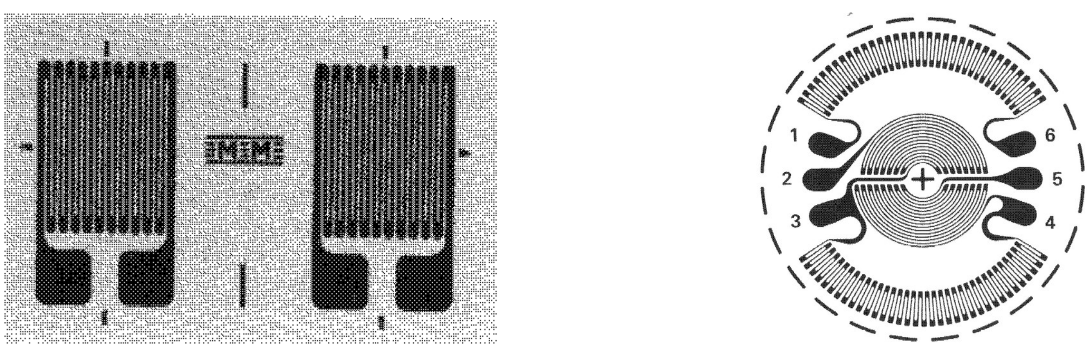


Fig.2.6. Examples of foil strain-gauges

2.3. Description of the test rig

2.3.1. The stand

In Fig.2.7 the test rig used in the experiment is presented.

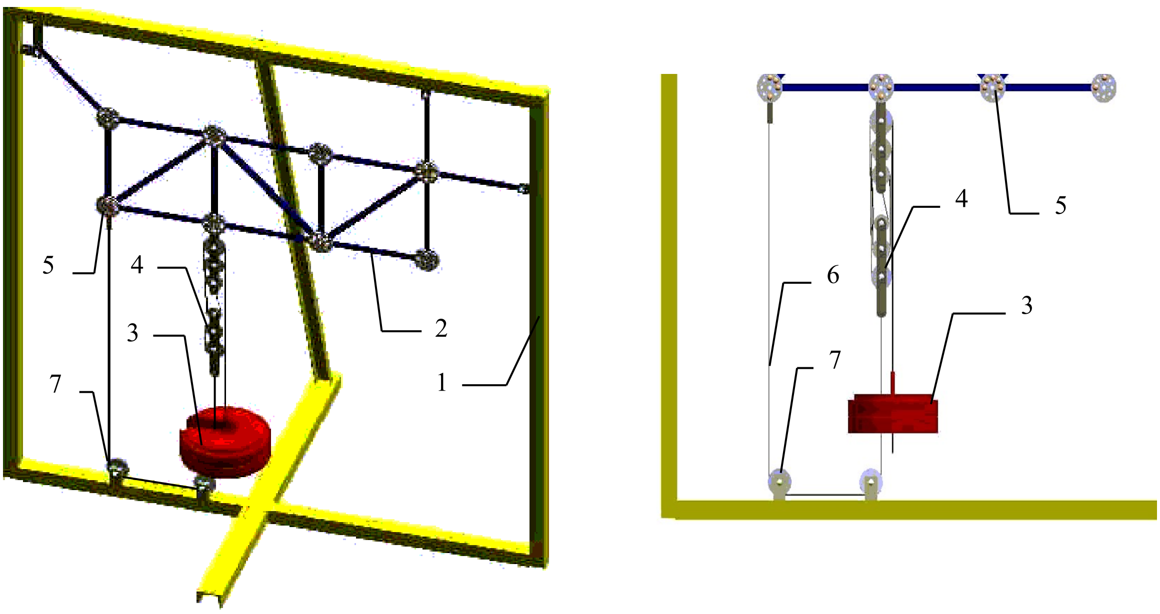
a) b)

Fig. 2.7.View of the experimental stand: a) truss model, b) cable-and-pulley arrangement

The truss (2) is suspended on the main frame (1). Truss members are made from slender steel bars of rectangle cross-section. The bars are connected together in joints (5) by means of small plates and screws (these connections are assumed in the analysis to be pin-jointed). The structure is loaded by the weights (3) applied to the cable-and-pulley arrangement (4).

2.3.2. Measuring equipment

Strain-gauges bonded on opposite surfaces of bars (truss members) are used in order to determine internal forces in these members. The change of the gauge resistances is measured by the Wheatston's bridge. The view of the measuring system is given on the Fig. 2.8. The method of reading depends on a type of a strain-gauge amplifier used. An exact procedure of reading either is given in the separate manual attached to the measuring stand or will be explained by a tutor.



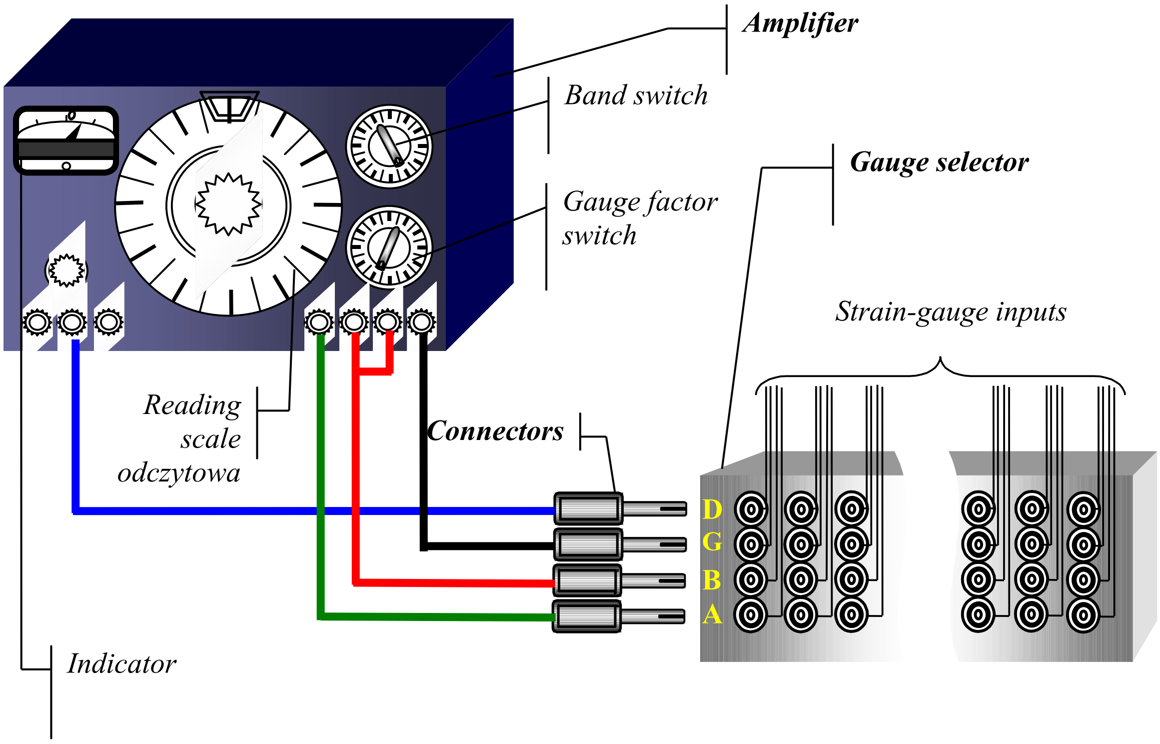


Fig.2.8. View of the measuring equipment

A procedure of reading is given in the separate manual attached to the measuring stand.

2.4. Experiment procedure

Switch on the amplifier, the computer, run the program T_2.EXE and follow commands, which appear, on the screen.

After the preliminary part is accomplished the measured process begins. Before the exercise begins the number of tested truss members (*maximum 6 members are examined*) and the number of loading cases has to be given by tutor.

1. Place the red disk on the pulley block to give a pre-tension of the truss.
2. Equilibrate the bridge and take record of the W_{oi} (i - bars number) for all the examined bars; the values of W_{oi} write down into the tables 2.1 or 2.2 and input them to the computer program.
3. Place quietly the yellow disk on the pulley block.
4. Equilibrate the bridge for all examined bars; the measured values of W_i write down in the tables 2.1 or 2.2 and input them to the computer program.
5. The results of computer calculations write down into the table 2.1 and table 2.2.
6. Next repeat the measurements (steps 4 - 5) for another value of the weight (after adding 2nd yellow disk) on the pulley block.
7. Take off the yellow disks; equilibrate the bridge for all examined bars and the measured values W_{fi} write down into the tables 2.1 or 2.2. Compare the bridge readings W_{fi} for all bars with the W_{oi} before loading of the truss. If the differences between these values are substantial for any bar, the test must be repeated for this bar.
8. Switch off the instruments and the computer.

Leave the work place in order, please.

2.5. Elaboration of experimental results and final report

2.5.1. Auxiliary calculations

Using the measured values of W_{oi} and W_i for the support bars students have to calculate the forces in these bars (R_{AY} , R_B). The results write down into the table 2.2.

The experimental force P_i is determined using the formula

$$P_i = \frac{E\,F\,\Delta W_i}{1000\,k_u}\,, \tag{2.4}$$

where

$$\Delta W_i = W_i - W_{oi}$$

Numerical data are:

Young's modulus $E = 210$ GPa,

Cross-section area of

- the truss members: 1, 2, 4, 5, 6, 8, 9, 10, 12, 13 — $F = 27.2$ mm^2,
- the truss diagonal members: 3, 7, 11 — $F = 61.2$ mm^2,
- the auxiliary bars: 14, 15 — $F = 27.2$ mm^2,
- the support bars: R_{AX}, R_{Ay}, R_B — $F = 27.2$ mm^2,

Strain-gauge arrangement factor $k_u = 2.47$.

Relative differences between experimental (P_i) and theoretical ($P_{i\,t}$) results is calculated from the following expression

$$\delta_i = \frac{P_i - P_{i\,t}}{P_{i\,t}}100\%\,. \tag{2.5}$$

All the calculations must be done taking into account the accuracy of measurements. At the same time, one has to be reasonable in assessing their accuracy. In particular the percentage differences should be calculated with the accuracy to one or two meaningful figures.

Students have to determine also - by ***method of section*** - forces in the truss members indicated by the tutor for one loading condition, for example **G = 50 N**. Necessary values of forces in the auxiliary bars (S_1, S_2) and support bars (R_{AX}, R_{AY}, R_B) have to be calculated using the results obtained in the preliminary part.

2.5.2. Report

The proper care should be taken in elaborating the final report, which must include:

a) subject and short description of the aim of the experiment,
b) results of the preliminary calculations,
c) the tables 2.1 and 2.2 with the experimental results and the calculations,
d) analysis of truss by the method of section (Ritter's method),
e) observations and conclusions.

2.6. Questions (to be asked before test)

1. *Give a definition of a truss and the force in the member.*
2. *Explain the expressions: rigid and non-rigid truss, statically determinate and indeterminate truss.*
3. *Describe the method of joints and the method of section in the analysis of trusses.*
4. *Give examples of zero-force members.*
5. *Derive the basic formula for a strain-gauge (relating the strain and relative change of its resistance).*
6. *Explain the influence of the temperature on strain-gauge measurements.*

Table 2.1. The experimental and theoretical results for the truss members

No of the truss bar	Loading case	Strain-gauge indication		Calculated values				Strain-gauge indication after **unloading** *(red disk only)*
		Before loading *(red disk only)*	**After** loading *(red and one or two yellow disks)*	Relative strain	Experimental force	Theoretical force	Relative difference P_i and P_{it}	
i	G [N]	W_{oi} [$^o/_{oo}$]	W_i [$^o/_{oo}$]	W_i [$^o/_{oo}$]	P_i [N]	P_{it} [N]	δ_i [%]	
1								
4								
5	**n^0 1 =**							
8								
10	………							
12								
13								
i	G [N]		W_i [$^o/_{oo}$]	W_i [$^o/_{oo}$]	P_i [N]	P_{it} [N]	δ_i [%]	W_{fi} [$^o/_{oo}$]
1								
4								
5	**n^0 2 =**							
8								
10	………							
12								
13								

Table 2.2. The experimental and theoretical results for the auxiliary and support bars

No of the bar	Loading case	Strain-gauge indication		Calculated values				Strain-gauge indication after **unloading** *(red disk only)*
		Before loading *(red disk only)*	**After** loading *(red and one yellow disk)*	Relative strain	Experimental force	Theoretical force	Relative difference P_i and P_{it}	
i	G [N]	W_{oi} [$^o/_{oo}$]	W_i [$^o/_{oo}$]	W_i [$^o/_{oo}$]	P_i [N]	P_{it} [N]	δ_i [%]	W_{fi} [$^o/_{oo}$]
14 (S_1)								
15 (S_2)	**n^0 1 =**							
R_{AY}								
R_B	………							