INVESTIGATION OF THE FRICTION PHENOMENON

Exercise 1

1 Aim of the Exercise

The aim of the exercise is to observe the effects of static and kinetic friction forces in simple systems. In particular, it involves experimentally determining the static friction coefficients between a block and a plane, a disk and a plane, and a flexible string and a stationary cylinder.

2 Introduction

The direction and sense of the sliding friction force are defined differently depending on whether the motion of the contacting bodies is about to start or is already occurring. In the first case, the friction force is opposite to the intended displacement, while in the second, it is opposite to the relative velocity of the sliding bodies. The onset of motion can not only change the direction of the friction force but also its magnitude. During this exercise, you will be able to observe the action of friction forces in several qualitatively different situations.

3 Description of the Test Stand

The exercise is conducted at three test rigs. The view of the first one, a slippery slope, used for determining the static friction coefficient between a rigid block and a flat surface, is shown in Fig. 1.

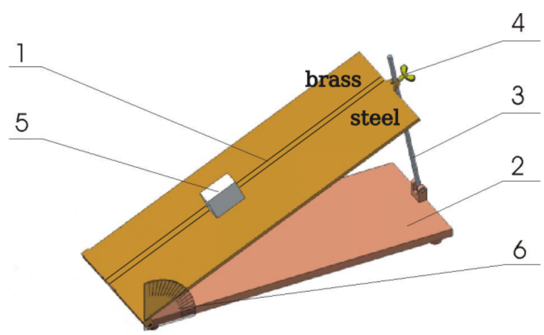

Figure 1: Rig for measuring the static friction coefficient – slippery slope

The plate (1) can be set at any angle to the horizontal base (2) using the guide (3) and clamp with screw (4). The block (5) placed on the plate with two different surfaces. Three such rectangular blocks of the same dimensions and similar masses are to be tested. They differ in type and condition of contact surface – namely: 1 - Teflon, 2 - steel, 3 - rubber. The plate inclination angle is measured using a protractor (6) attached to the stand frame. A laboratory scale is used to determine the mass of the weights.

The main elements of this station shown in Fig. 2 are six stationary pulleys made of different materials and one rotating pulley. Pulleys: (1) - wooden, (2) - textolite, (3) - Teflon, (4) and (5) - brass have the same diameter of 58 mm, while the brass pulley (6) has a diameter of 20 mm. All are fixed to the frame (8). The brass pulley (7) also has a diameter of 58 mm, but thanks to being mounted on a horizontal axis fixed in the frame via a ball bearing, it can rotate. Any selected pulleys can be wrapped with a light, flexible string (9). A weight (10) of mass 100 g is hung on one end, while a container (11) – also of mass 100 g – is attached to the other end of the string. By filling the container with weights, it is possible to cause the string to slide on the stationary pulley or to cause the rotating pulley to turn.

Figure 2: Station for testing string friction and resistance moments

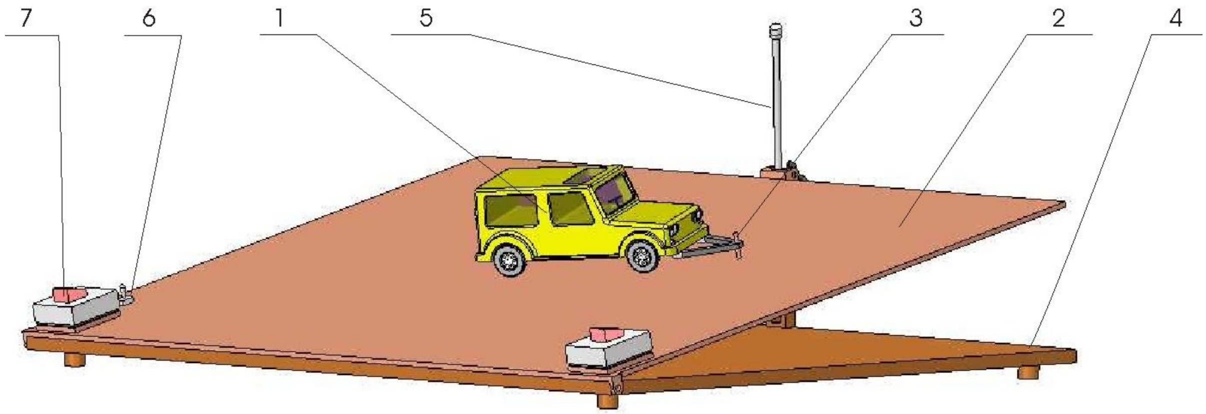

Figure 3: Station for testing static/kinetic friction

Only the rear, driven wheels of the car are in contact with the plate. The front axle is raised, and the car is fixed in a way that allows only rotation of its body around the axis (3) perpendicular to the plate surface. The plate tilt angle can be continuously changed. The guide (5) with clamp allows setting the desired plate inclination relative to the horizontal. The switch (6) is used to change the wheel rotation direction, while the button (7) starts the drive of the rear car wheels.

4 Theoretical Description of the Phenomenon

4.1 Determining the Static Friction Coefficient

Consider the system shown in Fig. 4a consisting of a block of mass m (weight $G = m \cdot g$) placed on a horizontal surface and to which a string with a body of mass M (weight $Q = M \cdot g$) hung on it is attached. Let's examine the equilibrium of the block, on which the vertical gravity force G and horizontal force P act, as well as the reaction forces of the rough substrate N and T (Fig. 4b).

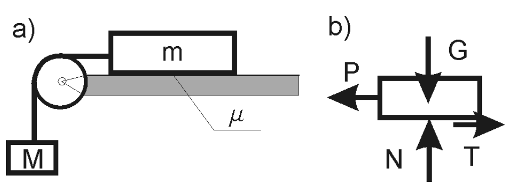

Figure 4: System diagram: block on horizontal plane

Knowing the value of force $P = Q$ at which the block starts moving, the static friction coefficient is determined from the following formula:

$$\mu = \frac{P}{G} = \frac{M}{m}. \tag{1}$$

Now consider the block resting on a surface inclined to the horizontal (Fig. 5a). By gradually increasing the surface inclination angle φ, we can cause the block to move (slide down).

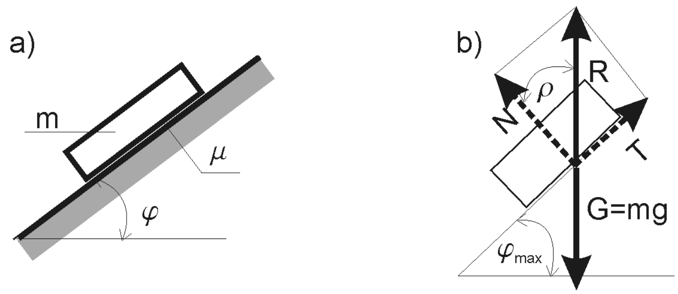

Figure 5: System diagram: block on inclined plane

The maximum value of this angle, for which the block remains at rest, is equal to the static friction angle

$$\mu = \tan\varphi_{\max}. \tag{2}$$

From the above formula, we can determine the static friction coefficient μ.

In case of the rig with flexible rope and weights let us take into account a flexible string wrapping a stationary pulley, as shown in Fig. 6.

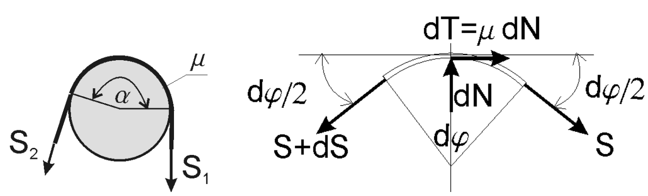

Figure 6: System diagram: flexible rope wrapping a pulley

The relationship between the tension values in the two parts of the string in the limit equilibrium state (the string is about to start sliding on the pulley to the left) is given by the relation

$$\frac{S_2}{S_1} = e^{\mu\alpha} \tag{3}$$

where μ - static friction coefficient, α - wrap angle (expressed in radians), e - base of natural logarithm ($e = 2.718$), S_1 – tension in the resisting part, S_2 – tension in the pulling part.

It should be noted that in the case of slip, the above formula can be used, but the kinetic friction coefficient μ_k should be inserted.

By measuring the values of forces S_1 and S_2, the static friction coefficient can be determined; $S_1 = m_1 g$ and $S_2 = m_2 g$, where g – gravitational acceleration, m_1 and m_2 – masses of the weight and container with weights, respectively

$$\mu = \frac{1}{\alpha}\ln\frac{S_2}{S_1} = \frac{1}{\alpha}\ln\frac{m_2}{m_1}. \tag{4}$$

Knowing the values of forces S_1 and S_2, the bearing resistance moment M_r supporting the rotating pulley 7 can also be determined (Fig. 7a)

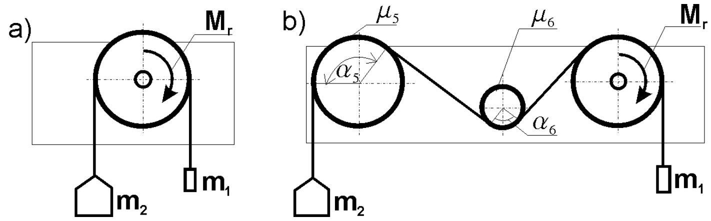

Figure 7: System configuration: a) bearing resistance moment; b) string wrapping stationary pulleys 5, 6 and rotating 7 (see Fig. 2)

$$M_r = (S_2 - S_1)R = (m_2 - m_1)Rg. \tag{5}$$

Using the known friction coefficients μ_5 and μ_6 and the resistance moment M_r, the minimum value of mass m_2 for a known value of mass m_1 can be determined (Fig. 7b referring to the stand from Fig. 2).

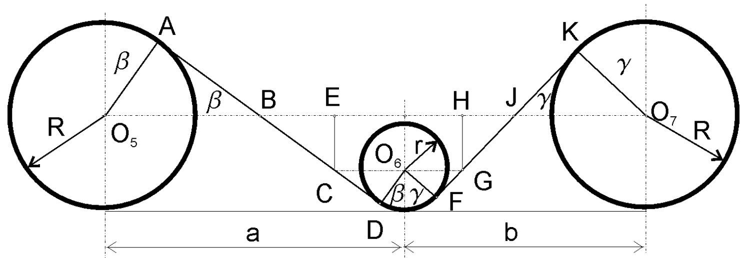

Figure 8: Dimensions and angles of the analyzed system

Given the relationships $O_5B + BE + CO_6 = a$ and $O_6G + HJ + JO_7 = b$, the following equations are obtained to determine β and γ:

$$\frac{R}{\sin\beta} + \frac{(R-r)}{\tan\beta} + \frac{r}{\sin\beta} = a \quad \text{and} \quad \frac{R}{\sin\gamma} + \frac{(R-r)}{\tan\gamma} + \frac{r}{\sin\gamma} = b. \tag{6}$$

The solutions to these equations are:

$$\tan\frac{\beta}{2} = \frac{a - \sqrt{a^2 - 4Rr}}{2r}, \qquad \tan\frac{\gamma}{2} = \frac{b - \sqrt{b^2 - 4Rr}}{2r}. \qquad (1.6) \tag{7}$$

The wrap angles α_5 and α_6 for pulleys 5 and 6 will therefore be

$$\alpha_5 = \frac{\pi}{2} + \beta \qquad \alpha_6 = \beta + \gamma. \tag{8}$$

As a result, the minimum mass m_2 is given by the relation

$$m_2 = \left(m_1 + \frac{M_r}{gR}\right) e^{\mu_5\alpha_5 + \mu_6\alpha_6}. \tag{9}$$

4.2 Analysis of Friction Forces Acting on Car Wheels

As mentioned earlier (in the stand description), only the rear (driven) wheels of the car are in contact with the substrate. The front of the vehicle is slightly raised, and the car is fixed in a way that allows only rotation of its body around an axis perpendicular to the substrate (hinge joint at point D).

The plane ABD (containing the wheel axes) is only slightly deviated from the substrate plane A'D'B, so the difference between angles A'D'B' and ADB (angle $ADB = 2\alpha$) is negligible. The angle α is defined by the following relation resulting from the system geometry:

$$\sin\alpha = \frac{a}{\sqrt{a^2 + (b + b_1)^2}}, \tag{10}$$

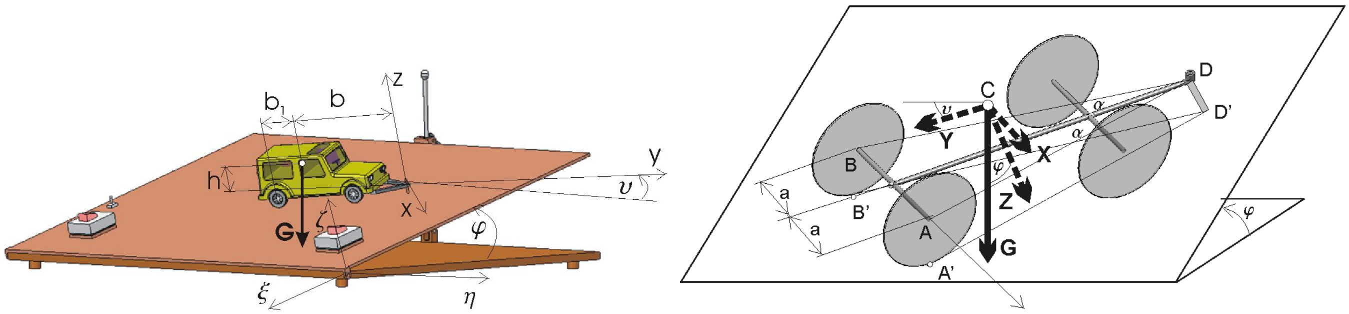

Figure 9: System diagram: dimensions and gravity force components

where: a, b, b_1 as well as r appearing later are dimensions shown in the figure.

The local (moving) reference frame xyz is associated with the car while the frame $\xi\eta\zeta$ is associated with the plate plane. For $\upsilon = 0$, axes x, y, and z are parallel to axes ξ, η, and ζ respectively. The components of the car gravity force G (along x, y, z) are defined as follows:

$$X = G\sin\varphi\cos\upsilon, \quad Y = G\sin\varphi\sin\upsilon, \quad Z = G\cos\varphi. \tag{11}$$

In the further part, we will determine the friction forces occurring at the contact points of the car's rear wheels in two cases: static and dynamic.

Static Friction Forces

Consider the limit equilibrium position of the car with non-rotating rear wheels.

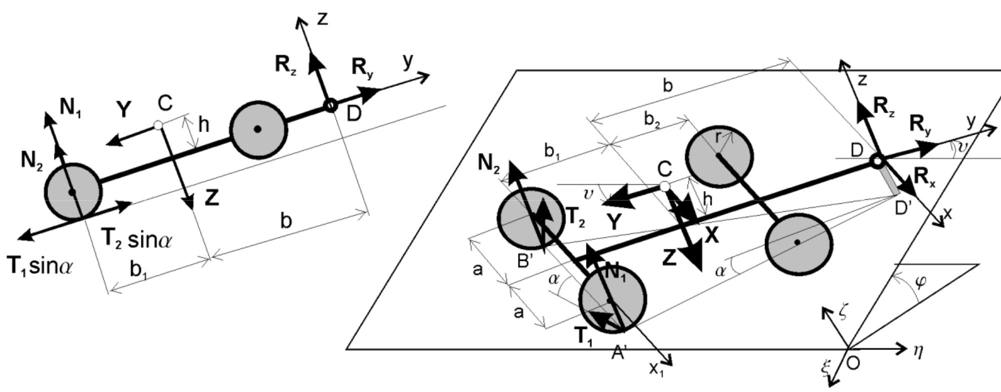

Figure 10: Forces acting on the car in the limit equilibrium position

In the limit of the equilibrium position, the static friction forces T_1 and T_2 are perpendicular to lines A'D' and B'D' respectively. For the system from Fig. 10, the following equilibrium equations are obtained:

$$\sum_i P_{ix} : -T_1\cos\alpha - T_2\cos\alpha + X + R_x = 0, \tag{12}$$

$$\sum_i P_{iy} : -T_1\sin\alpha + T_2\sin\alpha - Y + R_y = 0, \tag{13}$$

$$\sum_i P_{iz} : N_1 + N_2 - Z + R_z = 0, \tag{14}$$

$$\sum_i M_{ix1} : -T_1 r\sin\alpha + T_2 r\sin\alpha - Zb_1 + Yh + R_z(b + b_1) = 0, \tag{15}$$

$$\sum_i M_{iy} : -N_1 a + N_2 a + Xh + T_1 r\cos\alpha + T_2 r\cos\alpha = 0, \tag{16}$$

$$\sum_i M_{iz} : -(T_1 + T_2)(b + b_1)\cos\alpha - (T_1 + T_2)a\sin\alpha + Xb = 0. \tag{17}$$

In the limit equilibrium state, there are additionally two relations regarding friction forces:

$$T_1 = \mu N_1, \quad T_2 = \mu N_2, \tag{18}$$

where μ is the wheel adhesion coefficient. From the above system of equations, the components of reactions at wheel contact points are obtained:

$$N_1 = Z\frac{b(a + \mu r\cos\alpha)}{2a(b + b_1 + \mu r\sin\alpha)} + X\frac{h}{2a}, \quad N_2 = Z\frac{b(a - \mu r\cos\alpha)}{2a(b + b_1 + \mu r\sin\alpha)} - X\frac{h}{2a}. \tag{19}$$

In turn, the rectangular components of the reaction at hinge D will be:

$$\begin{aligned} R_x &= Z\frac{b\mu\cos\alpha}{b + b_1 + \mu r\sin\alpha} - X, \\ R_y &= Z\frac{\mu^2 br\sin\alpha\cos\alpha}{a(b + b_1 + \mu r\sin\alpha)} + X\frac{\mu h\cos\alpha}{a} + Y, \\ R_z &= Z\frac{b_1 + \mu r\cos\alpha}{b + b_1 + \mu r\sin\alpha}. \end{aligned} \tag{20}$$

Solving the above system of equations, the tilt angle φ corresponding to the limit equilibrium state is:

$$\tan\varphi = \frac{\mu b[(b + b_1)\cos\alpha + a\sin\alpha]}{[b + b_1 + \mu r\sin\alpha]b\cos\alpha - [(b + b_1)\cos\alpha + a\sin\alpha]\mu h\sin\alpha}. \tag{21}$$

Kinetic Friction Forces

Now consider the car when its rear wheels start rotating with angular velocity ω_k (Fig. 11).

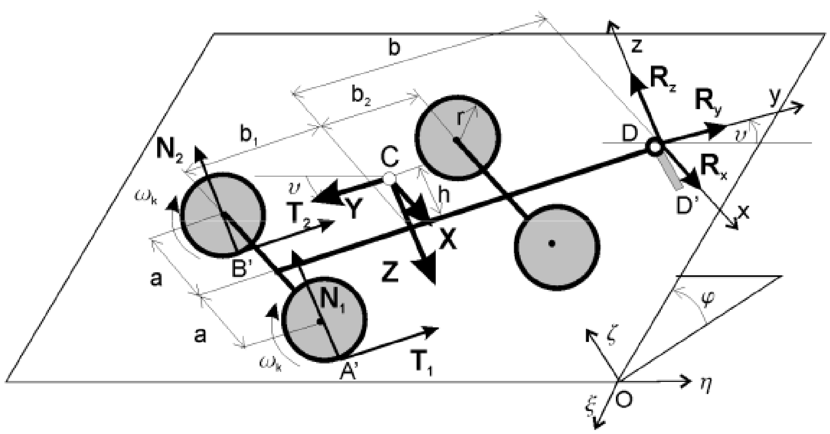

Figure 11: Forces acting on the car at the moment of starting the rear wheels motion

The front of the car is hinge-supported (the car does not have full freedom of motion), so the rotating rear wheels slide on the substrate causing kinetic friction forces at points A' and B' of wheel contact with the plate. In the considered situation, the kinetic friction forces are parallel to the y-axis coinciding with the car's longitudinal axis (compare the current issue with the previously considered case of static friction forces). It should be remembered that the kinetic friction force is always opposite to the relative velocity of the contact point of the sliding bodies (in our case – wheel and plate). Let us denote:

m – car mass: $m = \frac{G}{g}$,

J_z – mass moment of inertia of the car relative to z-axis: $J_z = J_{zc} + mb^2$,

ε – angular acceleration of the car: $\varepsilon = \frac{d^2\theta}{dt^2}$,

p_c – tangential component of the car mass center acceleration: $p_c = \varepsilon b$.

Treating the car as a rigid body rotating around a fixed z-axis (the path of mass center C is a circle of radius b), six following equations are obtained (assuming initial angular velocity of the body $\omega = 0$):

$$\sum_i M_{iz}: \quad J_z\varepsilon = T_1a - T_2a + Xb, \tag{22}$$

$$\sum_i P_{ix}: \quad mp_c = X + R_x, \tag{23}$$

$$\sum_i P_{iy}: \quad m0 = T_1 + T_2 - Y + R_y, \tag{24}$$

$$\sum_i P_{iz}: \quad m0 = N_1 + N_2 - Z + R_z, \tag{25}$$

$$\sum_i M_{ixc}: \quad J_{xc}0 = (T_1 + T_2)(r + h) - (N_1 + N_2)b_1 + R_yh + R_zb, \tag{26}$$

$$\sum_i M_{iyc}: \quad J_{yc}0 = -N_1a + N_2a - R_yh. \tag{27}$$

The kinetic friction forces can be expressed in terms of normal forces:

$$T_1 = \mu_k N_1, \quad T_2 = \mu_k N_2, \tag{28}$$

where μ_k is the kinetic friction coefficient.
By solving the above system of equations, we obtain:
* normal components of wheel-substrate contact forces

$$N_1 = \frac{Zb + Yh}{2(b + b_1 - \mu_k r)} + X\frac{h}{2a} \tag{29}$$

$$N_2 = \frac{Zb + Yh}{2(b + b_1 - \mu_k r)} - X\frac{h}{2a}, \tag{30}$$

* rectangular components of the reaction at hinge D

$$R_x = X\frac{\mu_k hb - J_z}{J_z} \quad R_y = \frac{Y[b + b_1 - \mu_k(r + h)] - \mu_k Zb}{b + b_1 - \mu_k r} \quad R_z = \frac{Z(b_1 - \mu_k r) - Yh}{b + b_1 - \mu_k r}, \tag{31}$$

* mass center acceleration

$$p_c = X\frac{b + \mu_k h}{J_{zc} + mb^2}b, \tag{32}$$

* angular acceleration

$$\varepsilon = X\frac{b + \mu_k h}{J_{zc} + mb^2}. \tag{33}$$

From relation (33), it follows that the X component of gravity force ($X = G\sin\varphi\cos v$) is the cause of the car's rotation around the axis perpendicular to the inclined substrate.

In the special case when $v = 0°$ (car's longitudinal axis parallel to the longer edge of the plate, i.e., across the substrate inclination - then $X = G\sin\varphi$), the following formula is obtained defining the car's angular acceleration:

$$\varepsilon = G\frac{b + \mu_k h}{J_{zc} + mb^2}\sin\varphi. \tag{34}$$

In the second special case, when $v = 90°$ (car's longitudinal axis directed perpendicular to the longer edge of the plate, i.e., along the substrate inclination) component $X = 0$, and thus $\varepsilon = 0$ and car rotation will not occur.

5 Measurement Procedure

5.1 Determining the Static Friction Coefficient

In the first part of the exercise, perform the following actions.

1. Set the plate horizontally and place blocks with steel contact surface on **steel sliding surface**, note the block surface condition in Table 1.
2. Gradually raise the plate until the position when the block starts sliding down.
3. Note in Table 1 the plate inclination angle at the moment of block adhesion break.
4. Perform the measurement three times.
5. Repeat 1–4 for **Teflon contact surface**.
6. Repeat 1–4 for **rubber contact surface**.
7. Repeat 1–5 for for all contact surfaces with **modified conditions** – like moistened with water or soap.

In the second part of the exercise, proceed as follows.

1. Hang the string at half length on one of the pulleys.
2. Gradually fill the container with weights until the string starts moving.
3. Count the mass of the container with weights and note mass m_2 in Table 2.
4. Perform the measurement three times.
5. Then repeat steps 1–4 for each of the stationary pulleys (1–6).

5.2 Investigation of the Car Model Behavior

1. Turn on the car motor power supply and check if the stand base is set horizontally.
2. Place the car on the plate so that its longitudinal axis is parallel to the longer, horizontal edge of the plate.
3. Slowly tilt the plate until the break of car wheel adhesion occurs (this plate tilt angle corresponds to the car's limit equilibrium).
4. Significantly reduce the tilt angle and place the car as before.
5. Set the driving direction switch to position P (wheels rotate as in forward drive).
6. Press the left button activating the rear wheel drive and observe the car's behavior.
7. Switch the driving direction to position T (wheels rotate as in reverse drive) and repeat instructions from point 6.
8. Place the car in any position on the plate and perform steps 3–7.
9. Place the car perpendicular to the plate edge - along the substrate inclination, as shown in the figure and repeat actions 5–7 for any plate tilt angle.
10. Turn off the motor power supply.

Exercise 11

VIBRATIONS OF A SYSTEM WITH 1 DEGREE OF FREEDOM

Group: ____________________________

Team: ____________________________

Date ____________________________

Student names:

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____

Measurements and results: static friction coefficients

Table 1: Measurements of static friction coeff. μ of textile rope on various materials

Material	wrap angle α	m_1	nr	mass m_2	ratio $\frac{m_2}{m_1}$	Aver.	friction coefficient μ (1.4)
	rad	g		g	[−]	[−]	[−]
wood	π	100	1 2 3				
textolite	π	100	1 2 3				
Teflon	π	100	1 2 3				
brass	π	100	1 2 3				
brass	3π	100	1 2 3				
brass	π	100	1 2 3				

Table 2: Measurements of static friction coefficient μ at slippery slope

Block	Floor	run	angle ϕ	average ϕ	μ (1.2)	Remarks
			deg	deg	[–]	
steel	steel	1 2 3				
steel	brass	1 2 3				
Teflon	steel	1 2 3				
Teflon	brass	1 2 3				
rubber	steel	1 2 3				
rubber	brass	1 2 3				
Measurement at modified contact surface						
steel	steel	1 2 3				
steel	brass	1 2 3				
Teflon	steel	1 2 3				
Teflon	brass	1 2 3				
rubber	steel	1 2 3				
rubber	brass	1 2 3				