

INVESTIGATION OF A DYNAMIC VIBRATION ABSORBER

Vibrations and Machine Dynamics – Exercise 9*

1 Aim of the exercise

Suppression of the vibrations of a single-degree-of-freedom system (from Exercise 2) operating with a large vibration amplitude (at its resonant frequency) by means of a dynamic vibration absorber (an added subsystem forming a second degree of freedom). Determination of the range of excitation frequencies in which the absorber effectively reduces the vibration level of the main system.

2 Theoretical introduction

A model of a two-degree-of-freedom vibrating system is considered (Fig. 1). The differential equations of motion of the main mass M and of the mass m of the dynamic absorber have the following form:

$$\begin{aligned} M\ddot{x} + c\dot{x} + (k_1 + k_2 + k_3)x - k_3y &= k_2e \sin \omega t \\ m\ddot{y} + k_3y - k_3x &= 0. \end{aligned} \quad (1)$$

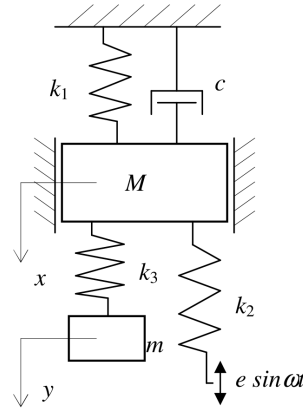


Figure 1: Physical model of the system

After dividing both sides of the equations by the masses one obtains:

$$\begin{aligned} \ddot{x} + \frac{c}{M}\dot{x} + \frac{k_1 + k_2 + k_3}{M}x - \frac{k_3}{M}y &= \frac{k_2e}{M} \sin \omega t \\ \ddot{y} + \frac{k_3}{m}y - \frac{k_3}{m}x &= 0. \end{aligned} \quad (2)$$

Taking into account in equations (2) the notation:

$$2h = \frac{c}{M}; \quad a = \frac{k_1 + k_2 + k_3}{M}; \quad b = \frac{k_3}{M}; \quad q = \frac{k_2e}{M}; \quad r = \frac{k_3}{m}, \quad (3)$$

one obtains the following system of equations:

$$\begin{aligned} \ddot{x} + 2h\dot{x} + ax - by &= q \sin \omega t \\ \ddot{y} + ry - rx &= 0. \end{aligned} \quad (4)$$

*In: LABORATORY OF MECHANICAL VIBRATIONS, K. Marynowski (ed.), Łódź 2002

It is assumed that particular solutions of the system of equations (4) have the form:

$$\begin{aligned} x &= A \sin \omega t + B \cos \omega t \\ y &= C \sin \omega t + D \cos \omega t. \end{aligned} \quad (5)$$

After substitution of the solutions into the equations of motion (4) one obtains

$$\begin{aligned} &-A\omega^2 \sin \omega t - B\omega^2 \cos \omega t + 2h(A\omega \cos \omega t - B\omega \sin \omega t) + \\ &+ aA \sin \omega t + aB \cos \omega t - bC \sin \omega t - bD \cos \omega t = q \sin \omega t \\ &-C\omega^2 \sin \omega t - D\omega^2 \cos \omega t + rC \sin \omega t + \\ &+ rD \cos \omega t - rA \sin \omega t - rB \cos \omega t = 0. \end{aligned} \quad (6)$$

In order that these equations be satisfied at every instant of time, the equations obtained by equating the terms on both sides of the above equations that contain separately the terms with $\sin \omega t$ and $\cos \omega t$ must be satisfied. After rearrangement a system of four algebraic equations arises:

$$\begin{aligned} (a - \omega^2)A - 2h\omega B - bC + 0D &= q \\ 2h\omega A + (a - \omega^2)B + 0C - bD &= 0 \\ -rA + 0B + (r - \omega^2)C + 0D &= 0 \\ 0A - rB + 0C + (r - \omega^2)D &= 0. \end{aligned} \quad (7)$$

To determine the values of the component amplitudes of the displacements A, B, C and D the principal determinant of the system of algebraic equations (7) is calculated:

$$W = \begin{vmatrix} (a - \omega^2) & -2h\omega & -b & 0 \\ 2h\omega & (a - \omega^2) & 0 & -b \\ -r & 0 & (r - \omega^2) & 0 \\ 0 & -r & 0 & (r - \omega^2) \end{vmatrix}, \quad (8)$$

whose value is given by:

$$W = [(a - \omega^2)(r - \omega^2) - br]^2 + 4h^2\omega^2(r - \omega^2)^2. \quad (9)$$

Applying Cramer's formulae the successive determinants are calculated:

$$W_A = \begin{vmatrix} q & -2h\omega & -b & 0 \\ 0 & (a - \omega^2) & 0 & -b \\ 0 & 0 & (r - \omega^2) & 0 \\ 0 & -r & 0 & (r - \omega^2) \end{vmatrix} = q(r - \omega^2)(a - \omega^2)(r - \omega^2) - br, \quad (10)$$

$$W_B = \begin{vmatrix} (a - \omega^2) & q & -b & 0 \\ 2h\omega & 0 & 0 & -b \\ -r & 0 & (r - \omega^2) & 0 \\ 0 & 0 & 0 & (r - \omega^2) \end{vmatrix} = -2qh\omega(r - \omega^2)^2. \quad (11)$$

The amplitudes of the component vibrations of the main mass M are given by the formulae:

$$A = \frac{W_A}{W}; \quad B = \frac{W_B}{W}. \quad (12)$$

The vibration amplitude of the mass of the main system is obtained by geometric summation of the constants A and B

$$A_1 = \sqrt{A^2 + B^2} = \frac{\sqrt{W_A^2 + W_B^2}}{W}. \quad (13)$$

After taking (9)–(11) into account in formula (13) one has

$$A_1 = \frac{q\sqrt{(r - \omega^2)^2} \sqrt{[(a - \omega^2)(r - \omega^2) - br]^2 + 4h^2\omega^2(r - \omega^2)^2}}{[(a - \omega^2)(r - \omega^2) - br]^2 + 4h^2\omega^2(r - \omega^2)^2}, \quad (14)$$

hence:

$$A_1 = q \frac{|r - \omega^2|}{\sqrt{[(a - \omega^2)(r - \omega^2) - br]^2 + 4h^2\omega^2(r - \omega^2)^2}}. \quad (15)$$

In the absence of the dynamic vibration absorber (absence of the mass m and the spring k_3), i.e. $b = r = 0$, whence $a = (k_1 + k_2)/M = \alpha^2$, one obtains the well-known formula

$$A_0 = q \frac{|-\omega^2|}{\sqrt{(a - \omega^2)^2\omega^4 + 4h^2\omega^2\omega^4}} = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}} \quad (16)$$

that determines the vibration amplitude of a linear single-degree-of-freedom system. The phase-shift angle between the excitation and the motion of the mass M is given by the formula:

$$\begin{aligned} \varphi_1 = \arctan \frac{B}{A} &= \frac{-2qh\omega(r - \omega^2)^2}{q(r - \omega^2)[(a - \omega^2)(r - \omega^2) - br]} = \\ &= \frac{-2h\omega(r - \omega^2)}{(a - \omega^2)(r - \omega^2) - br}. \end{aligned} \quad (17)$$

2.1 Example of analytical determination of the amplitude A_1 and of the phase-shift angle φ_1 for specified system parameters

Adopting the following values of the dimensionless parameters of the system under consideration:

$$4 \times 2h = \frac{a}{2} = b = r = q = 1, \quad (18)$$

one obtains the expression for the vibration amplitude of the mass M in the form:

$$A_1 = \frac{|1 - \omega^2|}{\sqrt{[(2 - \omega^2)(1 - \omega^2) - 1]^2 + \frac{\omega^2}{16}(1 - \omega^2)^2}}, \quad (19)$$

and the formula for the phase-shift angle:

$$\varphi_1 = \arctan \frac{-\frac{\omega}{4}(1 - \omega^2)}{(2 - \omega^2)(1 - \omega^2) - 1}. \quad (20)$$

For the main system without the dynamic vibration absorber one has respectively:

$$A_0 = \frac{1}{\sqrt{(1 - \omega^2)^2 + \frac{\omega^2}{16}}}; \quad \varphi_0 = \arctan \frac{-\omega}{4(1 - \omega^2)}. \quad (21)$$

The dependence of the amplitude values A_1 and A_0 on the excitation frequency is shown in Fig. 2. On this figure the region of beneficial action of the absorber has been shaded.

From the analysis of the formula for the amplitude A_1 and from Fig. 2 it follows that for an excitation frequency equal to the natural frequency of the absorber ($\omega^2 = r$) complete extinction of the vibrations of the main mass M occurs. For devices operating at a constant frequency the absorber should be tuned to that operating frequency. If the excitation frequency is not equal to the natural frequency of the absorber, the damping is incomplete and outside the beneficial frequency interval the addition of the dynamic absorber worsens the dynamic state of the system.

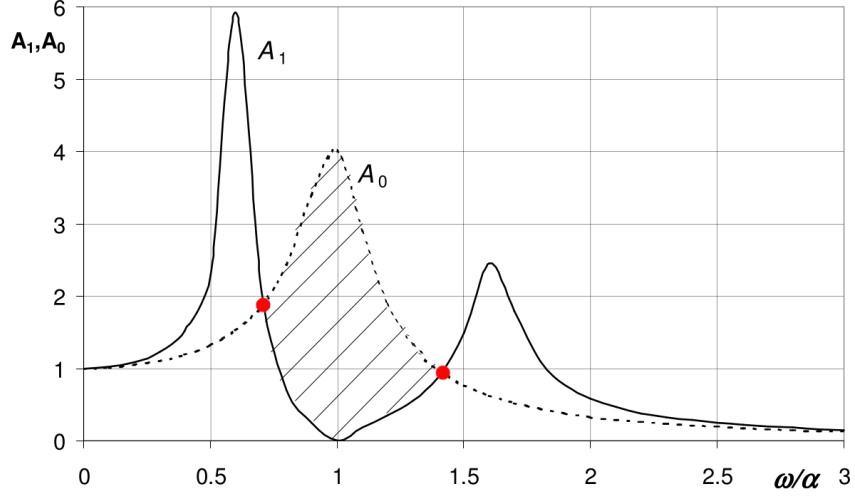


Figure 2: Comparison of the resonance curves of a single-degree-of-freedom system (dashed line) with the system with an added dynamic vibration absorber (second degree of freedom, solid line)

The excitation frequencies ω for which the vibration amplitudes of the main system with the absorber and without the absorber are equal can be calculated by equating the amplitudes A_1 and A_0 . Making use of formulae (15) and (16) one obtains:

$$q \frac{|r - \omega^2|}{\sqrt{[(a - \omega^2)(r - \omega^2) - br]^2 + 4h^2\omega^2(r - \omega^2)^2}} = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}}. \quad (22)$$

where: $a = \frac{k_1+k_2+k_3}{M} = \frac{k_1}{M} + \frac{k_2}{M} + \frac{k_3}{M} = \alpha^2 + \frac{q}{e} + b$.

The comparison of the amplitudes in formula (22) can be written in the following form:

$$\begin{aligned} (r - \omega^2)^2 [(\alpha^2 - \omega^2)^2 + 4h^2\omega^2] &= \\ &= \left[\left(\alpha^2 - \omega^2 + \frac{q}{e} + b \right) (r - \omega^2) - br \right]^2 + 4h^2\omega^2(r - \omega^2)^2. \end{aligned} \quad (23)$$

The values of the excitation frequency ω determined from equality (23) define the interval of beneficial action of the dynamic vibration absorber (in Fig. 2 – between the red points).

3 Experimental stand

The construction of the vibrating system hereafter called the main system has been presented in detail in the description of the experimental stand in Exercise 2. To the main system a dynamic absorber consisting of a mass m_r and a flat spring of stiffness k_3 has been attached. The schematic diagram of the vibrating system together with the absorber is shown in Fig. 3. The method of measuring amplitudes and excitation frequencies has been described in Exercise 2.

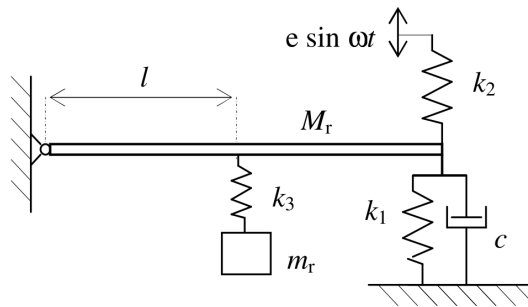


Figure 3: Schematic diagram of the experimental stand

4 Procedure

1. Experimental determination of the resonance curve of the main system.
 - (a) For successive values of the angular velocities given in the measurement table the values of the vibration amplitudes of the system without the dynamic vibration absorber should be measured.
 - (b) On the basis of the data from the measurement table an experimental resonance diagram for the system without the absorber should be plotted.
 - (c) The coordinates of the peak of the resonance diagram (A_m, ω_m) should be determined.
2. The calculation of the natural frequency of the main system is performed according to formulae (10) given in the description of Exercise 2.
3. Selection of the dynamic vibration absorber for a prescribed frequency. The forced vibrations of the main system of frequency equal to its natural frequency α should be suppressed. Then the natural frequency of the dynamic vibration absorber α_t must be equal to the natural frequency of the main system α . The length l of the spring of the dynamic absorber must therefore be chosen so that the condition

$$\alpha_t = \alpha. \quad (\text{a})$$

is satisfied. Attention should be paid to the fact that with a change of the spring length l the stiffness of the spring k_{3r} and the reduced mass m_r participating in the motion change. Both these quantities influence the value of α_t . The values of k_{3r} and m_r can be obtained by employing, for example, Rayleigh's method in the calculations

$$k_{3r} = \frac{3EJ}{l^3}; \quad m_r = m + \frac{1}{5}\mu l, \quad (\text{b})$$

where:

E – modulus of longitudinal elasticity of the material of the flat spring,

J – moment of inertia of the cross-section of the bent spring,

$\mu = \frac{m_b}{l}$ – mass per unit length of the beam (spring).

Since the natural frequency of the dynamic vibration absorber is given by the relation

$$\alpha_t^2 = \frac{k_{3r}}{m_r}, \quad (\text{c})$$

then after substitution of (b) into (c) and taking condition (a) into account, one obtains after rearrangements

$$\frac{1}{5}\alpha^2\mu l^4 + \alpha^2 m l^3 - 3EJ = 0. \quad (\text{d})$$

Equation (d) is a fourth-degree equation with the unknown l and during the performance of the exercise is solved by a numerical method. As a result one obtains the sought length l of the flat spring of the dynamic vibration absorber.

For the solution a spreadsheet and the built-in Goal Seek function can be used. It is sufficient to enter in successive cells the values of the parameters used in equation (d) – μ , m , E , J . The value of the cell l with an arbitrary initial value, e.g. 0.5 (metre), is the cell that the said function will change (utilise) in order to seek a zero value in the last cell, which will contain the formula of expression (d). The result of the action of seeking a zero value of that cell will be the solution of equation (d), and therefore the value of l suggested for setting the length of the spring.

The material from which the spring constituting the dynamic absorber has been made is silicon bronze, whose value of Young's modulus E should be found in engineering tables. A way of verifying the value assumed in this way may be the experimental determination of such a length l_{min} for which the vibrations of the main mass are minimal and the use of this value as a parameter for control by means of the Goal Seek function with the variable set as the value of the parameter E .

4. Experimental determination of the resonance curve for the main system with the dynamic vibration absorber. It is performed analogously to the measurements for obtaining the resonance curve without the dynamic absorber (item 1).
5. From a comparison of both resonance diagrams the values of the excitation frequencies ω_1 and ω_2 are read for which the vibration amplitudes of the main system with the dynamic absorber and without it are equal. The agreement of the results obtained with formula (3.23) should be checked.

5 Literature

1. Kaliski S.: *Vibrations and Waves*. PWN, Warsaw 1966.
2. Parszewski Z.: *Dynamics and Vibrations of Machines*. WNT, Warsaw 1982.
3. Kapitaniak T.: *Introduction to Vibration Theory*, Publishing House of the Technical University of Łódź, Łódź 1992.

LABORATORY OF
MECHANICAL
VIBRATIONS

Exercise 9
INVESTIGATION OF A DYNAMIC VIBRATION
ABSORBER

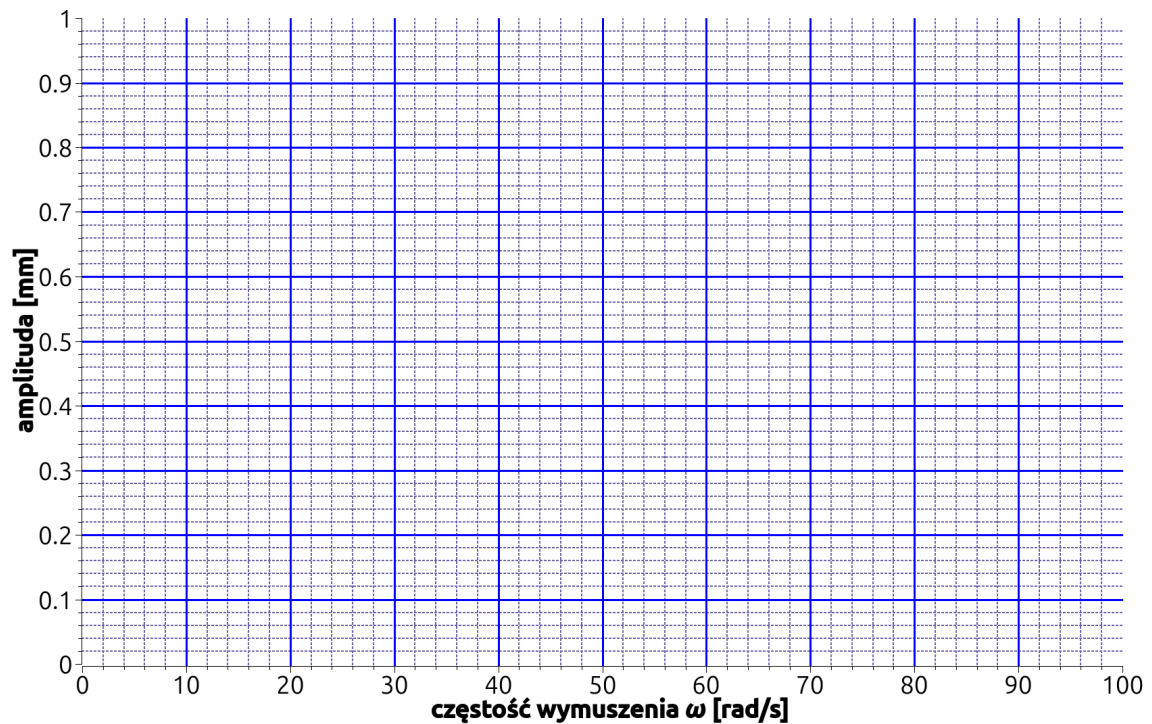
Group: _____ date _____ Name
Team: _____

and surname:

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Measurement results

Vibration frequencies		System without absorber	System with absorber
Measurement No. i	Angular velocity of the motor ω_i [rad/s]	Vibration amplitude A_0	Vibration amplitude A_1
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			



Natural frequency of the main system according to formula (13) (Ex. 2):

$$\alpha^2 = \frac{\omega_m^2}{\sqrt{1 - \left(\frac{x_0}{A_m}\right)^2}} = \text{_____} \text{ [rad/s]}$$

Calculation of the active length l of the flat spring of the absorber: $l = \text{_____}$ [m]

Frequency range of effectiveness of the absorber:

$$\omega_1 = \text{_____} \quad \omega_2 = \text{_____} \quad \Delta\omega = \text{_____}$$