

HARMONIC ANALYSIS OF VIBRATION SIGNALS

Vibrations and Machine Dynamics – Exercise 8*

1 Aim of the exercise

Analysis of the vibration signals of a machine and determination of the harmonic components of these signals.

2 Introduction

2.1 Measurement signals

In order to perform correct measurements, and therefore to select appropriate sensors and apparatus, one must be aware of the types of vibration signals that are encountered.

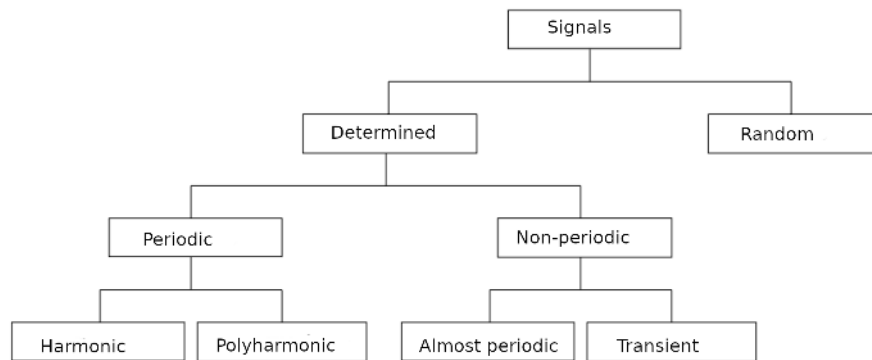


Figure 1: Classification of signals

Deterministic signals – their waveforms can be described uniquely by means of mathematical functions, and this description may not contain random quantities.

Random signals (stochastic signals) are those that cannot be uniquely described by means of mathematical functions; their values at every instant are random variables, and their properties are described by means of statistical characteristics. They are divided into stationary and non-stationary signals.

Periodic signals – for every instant of time t the equality $x(t + T) = x(t)$ holds, where T is the period of the signal.

Non-periodic signals – signals for which the periodicity condition is not satisfied.

Harmonic signals – in the time domain they can be described by a harmonic function: $x(t) = A \cos(\omega_0 t + \varphi)$, where A is the amplitude of the signal (e.g. in mm, m/s, m/s²), $\omega_0 = 2\pi f_0$ is the circular frequency (pulsation), and φ is the phase shift.

Polyharmonic signals – are linear combinations of at least two harmonic signals whose frequencies must be integer multiples of a certain fundamental frequency.

Transient signals – signals that begin and end at the zero level (impulse signals, e.g. impacts).

Stationary signals – the statistical characteristics (among others the mean value, the mean-square value) are not functions of time.

2.2 Vibration analysis

Variable forces of different frequencies act on the object under investigation during its operation:

$$P_1 \sin(\omega_1 t + \phi_1), P_2 \sin(\omega_2 t + \phi_2), \dots, P_k \sin(\omega_k t + \phi_k) \quad (1)$$

and as a result of their action the vibration waveform of the object is complex. In order to determine the origin of the exciting forces and their influence on the vibrations of the object, the vibration signals $x(t)$ obtained from the vibration sensor must be decomposed into harmonic components:

$$x(t) = \sum_{i=1}^k A_i \sin(\omega_i t + \beta_i) \quad (2)$$

* Author: B. Jagiello, in: LABORATORY OF MECHANICAL VIBRATIONS, K. Marynowski (ed.), Łódź 2002

For this purpose a spectral (frequency) analysis of the time signals of vibration obtained from the measurements must be performed. Signal analysis can be carried out in an analogue, digital or mixed manner.

2.2.1 Analogue analysers

Analogue signal processing can be performed by means of electronic spectrum analysers. This may be a set of filters of different pass-band frequencies or tunable narrow-band filters.

In analogue analysers, electrical band-pass filters are most frequently used. These filters pass the components of the signal whose frequencies lie in the pass band B of the filter (Fig. 2).

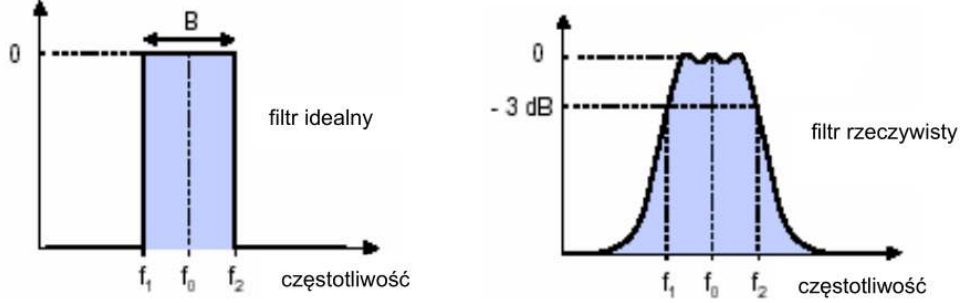


Figure 2: Characteristics of an ideal and a real filter

Here the characteristics of an ideal and a real band-pass filter are presented. An ideal band-pass filter should have attenuation equal to zero in the pass band and infinitely large outside that band; thus the characteristic of an ideal filter is rectangular. The attenuation of filters is given in decibels.

$$N[\text{dB}] = 10 \log \left(\frac{U_{\text{out}}^2}{U_{\text{in}}^2} \right) = 20 \log \left(\frac{U_{\text{out}}}{U_{\text{in}}} \right) \quad (3)$$

U_{out} and U_{in} denote the output and input signals of the filter.

The characteristics of real filters approach those of ideal filters when they have a flat portion of the characteristic in the pass band and the steepest possible slopes. Band-pass filters are defined by means of the centre frequency f_0 and the bandwidth $B = f_2 - f_1$ determined by the cut-off frequencies: the lower f_1 and the upper f_2 at which the signal power is halved, i.e.

$$\frac{U_{\text{out}}^2}{U_{\text{in}}^2} = \frac{1}{2} \quad (4)$$

Then the attenuation of the signal is

$$N[\text{dB}] = 10 \log \frac{1}{2} \approx -3 \text{ dB} \quad (5)$$

and the gain $k = \frac{U_{\text{out}}}{U_{\text{in}}}$ decreases from the value $k = 1$ (the average gain level in the pass band) to $k = \frac{1}{\sqrt{2}} \approx 0.707$ (Fig. 3). For the frequency analysis of vibration signals two types of filters are used:

- of constant absolute bandwidth, e.g. 3 Hz, 100 Hz, etc.
- of constant percentage bandwidth, referred to the centre frequency f_0 , 3%, 10%, 30%, (Fig. 3).

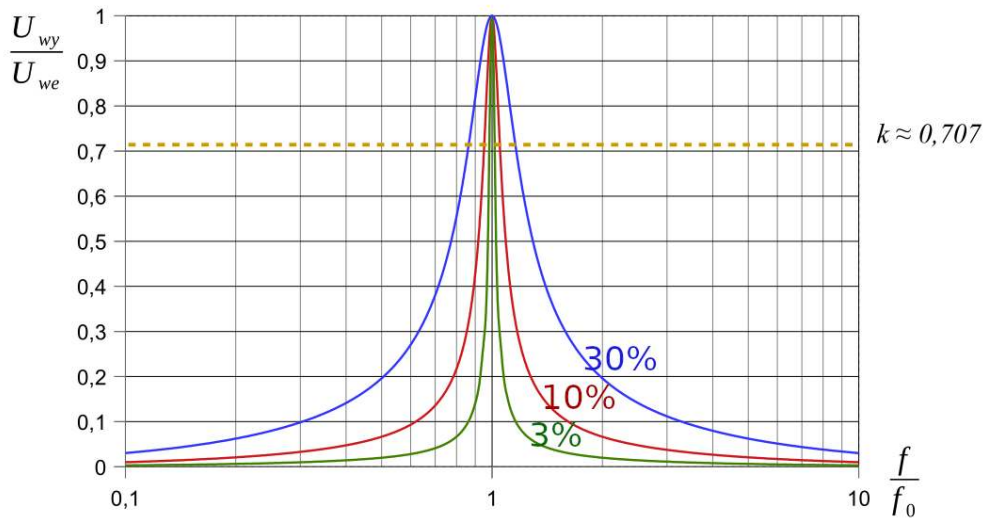


Figure 3: Characteristics of a narrow-band filter of width 3%, 10% and 30%

The centre frequency of the band and the cut-off frequencies of these filters are related by the formula:

$$f_0 = \sqrt{f_1 \cdot f_2} \quad (6)$$

The narrower the bandwidth of the filter, the more detailed information can be obtained from the analysed waveform, but the longer is then the analysis time.

2.2.2 Digital analysers

Fourier series A periodic waveform that can be physically realised (satisfies the Dirichlet conditions) can be represented in the form of a Fourier series consisting, in the general case, of a constant component a_0 and a sum of trigonometric functions of different pulsations:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_1 t + b_n \sin n\omega_1 t) \quad (7)$$

where a_0 is the constant component, a_n, b_n are the Fourier-series coefficients that can be determined analytically, ω_1 is the fundamental pulsation, $\omega_1 = 2\pi f = 2\pi/T$, where T is the period of the function, and n is a natural number.

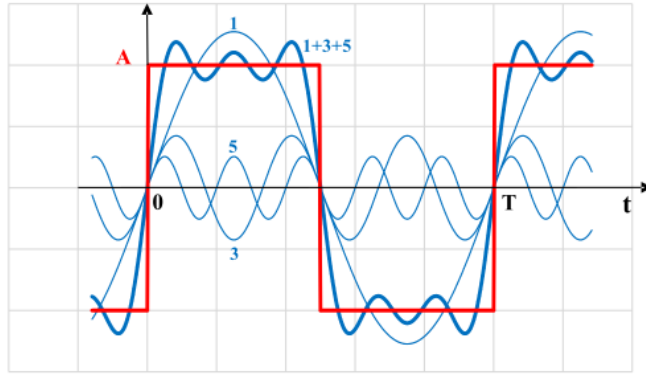


Figure 4: Approximation of a rectangular waveform by means of the first three terms of the Fourier series (the first, third and fifth harmonics)

Consider a periodic signal in the form of a rectangular wave (Fig. 4). After determination of the Fourier-series coefficients it can be represented by means of an infinite trigonometric series of odd harmonics (1, 3, 5, 7, ...) of decreasing amplitudes:

$$x(t) = \frac{4A}{\pi} \left(\sin \omega_1 t + \frac{1}{3} \sin 3\omega_1 t + \frac{1}{5} \sin 5\omega_1 t + \frac{1}{7} \sin 7\omega_1 t + \dots \right) \quad (8)$$

This is a representation of the signal in the time domain. The figure shows the first three components of equation (8) and their sum. Examples of approximations of various signals by means of a finite number of components can be found on the page: wikipedia.org/wiki/Fourier_series

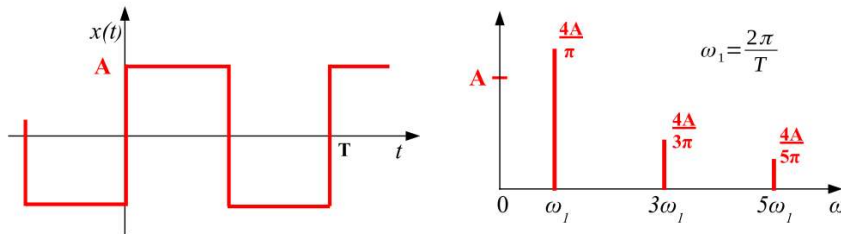


Figure 5: Amplitude spectrum of a rectangular signal

Periodic signals can also be represented graphically in the frequency domain. On the abscissa axis the frequency f or the pulsation $\omega = 2\pi f$ is adopted. On the ordinate axis the amplitudes or amplitude ratios are shown in the form of lines (Fig. 5). The length of the lines is proportional to the values of the amplitudes of the corresponding harmonics present in the analysed signal. Such a plot is called an amplitude spectrum or a frequency spectrum. The spectra of periodic signals have a discrete character (Fig. 6), whereas the spectra of non-periodic signals (e.g. an impulse signal or a stochastic signal) have a continuous character (Fig. 7, Fig. 8).

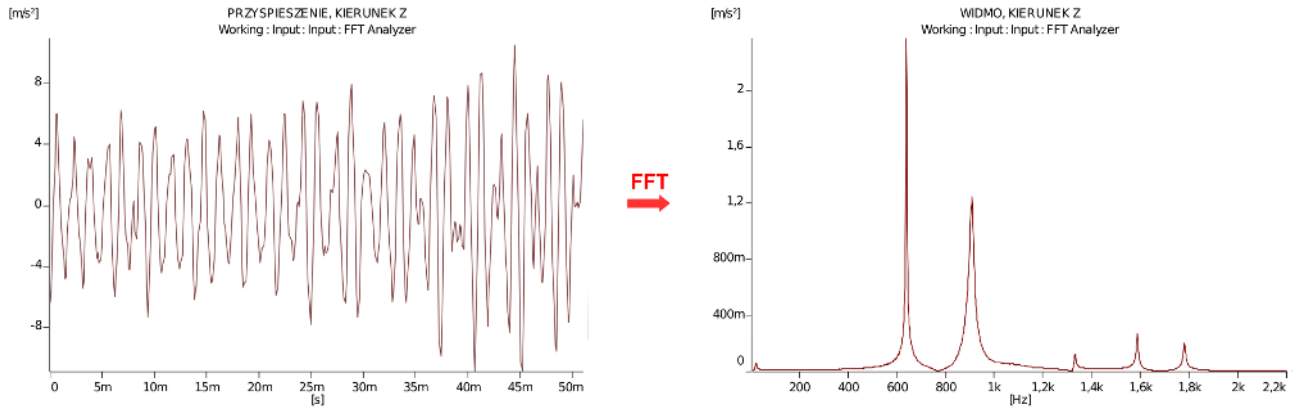


Figure 6: Fourier transform of a vibration-acceleration signal, transition from the time domain to the frequency domain

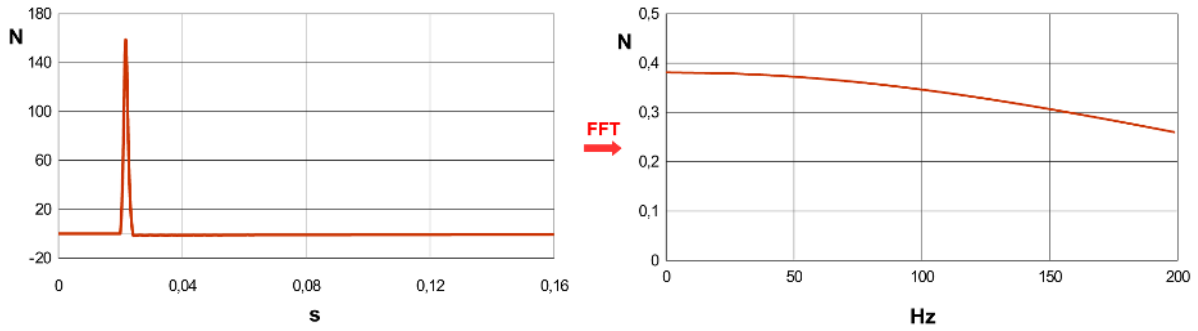


Figure 7: Time history and amplitude spectrum of an impulse signal

2.2.3 Fourier transform

For digital processing of signals the Fourier transform is most frequently used; it is the basic tool in the analysis of stationary signals. The result of the Fourier transform is a function called the **Fourier transform**. The Fourier transform decomposes a signal (e.g. a vibration signal) into sinusoidal components of different frequencies:

$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi ft} dt \quad (9)$$

An optimised computer version of the Fourier transform, the so-called **Fast Fourier Transform (FFT)**, permits a rapid and accurate evaluation of the harmonic components of a given signal. As a result one obtains a set of data in the form of frequency values and the corresponding amplitudes in the selected frequency range for analysis.

This result can be presented on a plot where, in general, the ordinate axis denotes the amplitudes of the individual components (harmonics), while the abscissa axis gives their frequencies. The Fourier transform therefore serves to convert a function dependent on time into a function dependent on frequency, i.e. it enables the **transition from the time domain to the frequency domain**.

Fourier transforms have many applications, first of all in signal processing, but also in image processing, sound compression in the MP3 format, the solution of important partial differential equations (Poisson's equation), or numerical FEM analyses (part of the dynamic problems requires transitions between the time and frequency domains).

For an impulse signal (Fig. 7) the amplitude spectrum has a continuous character. The theoretical impulse $\delta(t)$ (Dirac function) contains signals of all frequencies from $-\infty$ to $+\infty$ of equal amplitude equal to 1. At the instant $t = 0$ all spectral components are in the same phase. It is precisely this concentration that enables the formation of the impulse, see [1]. A short-duration impulse caused, for example, by the impact of a so-called instrumented hammer (possessing a built-in force sensor) contains components of equal amplitude over a wide frequency range. Thus the system is excited by all frequencies, which makes it possible to obtain the so-called dynamic characteristics (dynamic stiffness or receptance) by means of an FFT analyser.

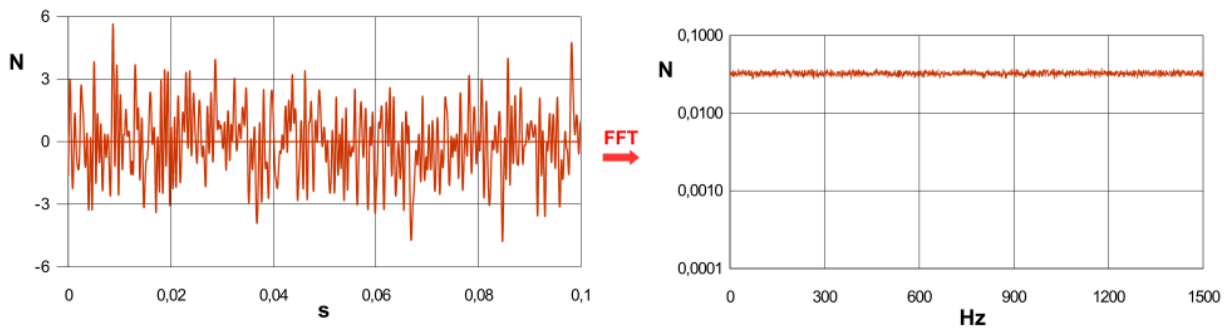


Figure 8: Time history and amplitude spectrum of a stochastic signal

A stochastic signal is a signal whose values at every instant are random variables. Of great practical importance is a completely unordered stochastic signal containing all frequencies of equal amplitude. Its energy is uniformly distributed over the entire frequency band. By analogy with optical spectra it is called white noise. If a signal from a noise generator is supplied to an exciter (which produces a force proportional to the input signal), one has excitation of the object under investigation by a force containing all frequencies (Fig. 8), which permits determination of the dynamic properties of the object under investigation.

3 Experimental stand

One of the objects whose vibrations will be analysed is a reciprocating compressor. The vibrations of the compressor during its normal operation have a complex waveform. This is the result of the superposition of the effects of several exciting forces of different frequencies that are associated with the operation of the device. The analysis of the vibration signal from the measuring sensor will be performed in the exercise in two ways:

1. Direct analysis of the vibration waveform of the object under investigation by means of the analogue narrow-band analyser SM32 located in the SM231 vibration meter of the RFT company.
2. Computer analysis with the use of the Fast Fourier Transform, by means of the PULSE measuring system (Brüel & Kjær) or the SCADA system (LMS).

4 Procedure

4.1 Analysis of the vibrations of the object under investigation by means of the analogue narrow-band analyser

1. Select an appropriate sensor (piezoelectric, electrodynamic, transformer or laser) depending on the object under investigation and the conditions of the vibration measurement and place it on the object under investigation.
2. Set the switch of the measured quantity type to the position a, v or x (acceleration, velocity or displacement).
3. Using the range switch of the signal level of the SM231 meter, set the highest value.
4. Set the BANDWIDTH switch of the SM32 analyser to the position LIN (the entire frequency spectrum).
5. Switch the vibration meter on to the mains and start the object under investigation.
6. By changing the range of the level meter, bring the pointer of the meter to a deflection above 1/3 of the range. Record the meter readings.
7. In order to perform the analysis, set the BANDWIDTH switch of the analyser to the position 30% and tune the frequency f_0 with the measuring potentiometer until distinct deflections of the pointer are obtained. Record the levels of the spectral components of the vibrations under investigation and the frequencies at which they occur. For a more accurate determination of the frequency, change the bandwidth to 3%.

4.2 Spectral analysis of periodic signals, of a white-noise signal and of an impulse signal

1. Supply to the FFT analyser periodic signals (sine, rectangular, triangular) and a white-noise signal from a function generator. Record the time histories and the spectra of these signals.
2. Record the time history and the spectrum of the impulse force signal from an impact hammer.

4.3 Vibration analysis with the use of the Fast Fourier Transform

1. Place the vibration sensor on the object under investigation.
2. Start the appropriate computational programme in the PULSE or SCADA system.
3. Start the object under investigation.
4. Perform the vibration analysis and record the results.

4.4 Report on the completed exercise

1. Enter the measurement results from the analogue analyser into the report.
2. Print the recorded waveforms from the digital analyser.
3. Compare the results from the analogue and digital analysers.
4. On the basis of the investigations performed, determine the sources of the vibrations, i.e. the origin of the exciting forces.
 - Under the figures describe what the waveforms recorded during the exercise represent.
 - On the basis of the FFT waveform of the vibration acceleration of the computer housing, calculate the displacement amplitude for the largest value of the acceleration amplitude.
 - Determine the source of these vibrations.
 - On the basis of the FFT waveform of the vibration velocity of the two-piston compressor, calculate the displacement amplitudes for the two largest values of the velocity amplitudes.
 - Determine the source of these vibrations and justify the values of the frequencies at which they occur. Use the attached photographs of the nameplate of the motor and of the belt transmission. The transmission ratio of the belt drive is 3.5.
5. Compare both methods of vibration analysis.

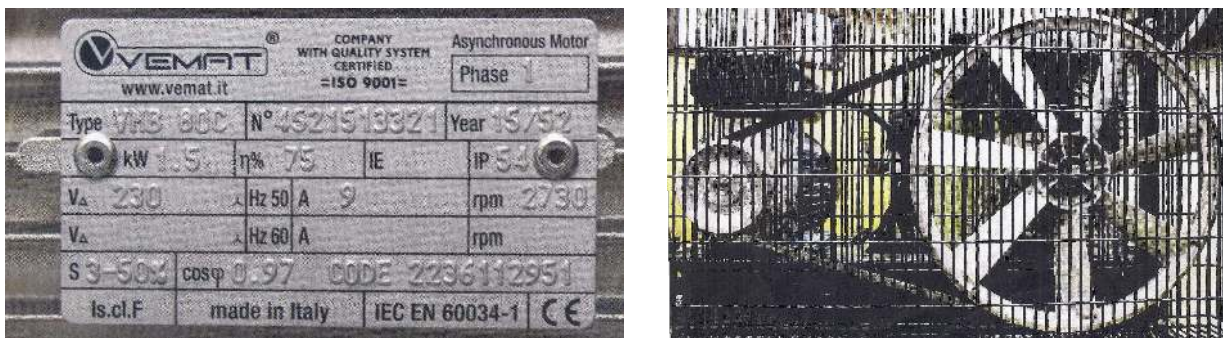


Figure 9: Nameplate of the compressor motor and its belt transmission, transmission ratio $k = 3.5$

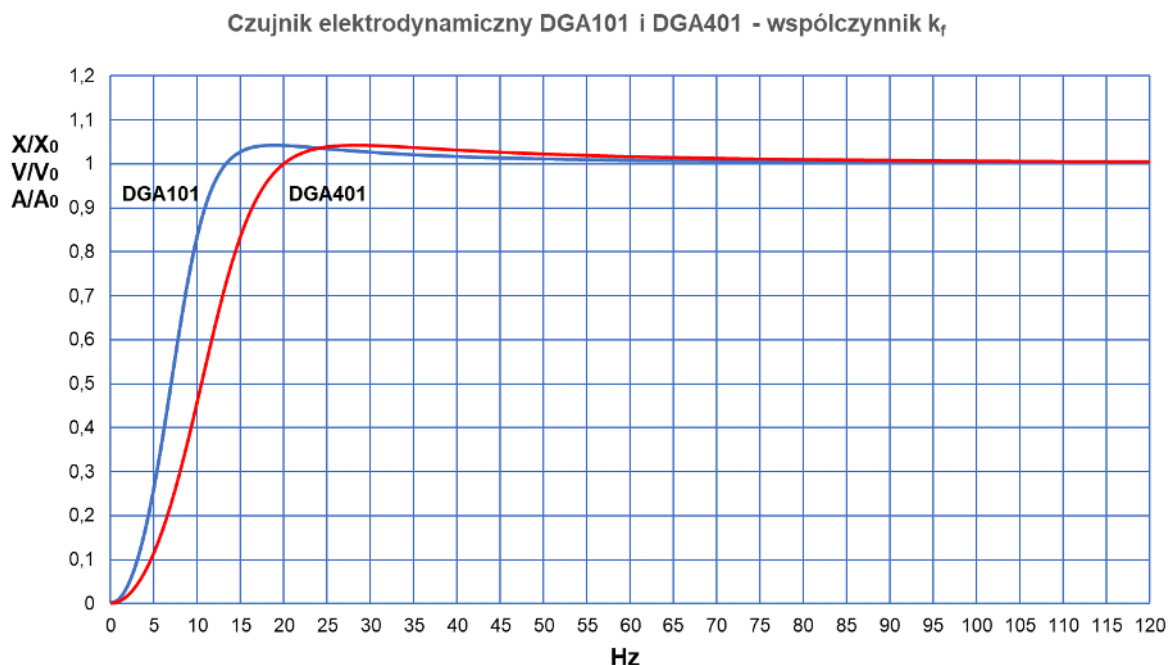


Figure 10: Characteristics of DGA sensors

Literature

- [1] Hagel R., Zakrzewski J.: *Dynamic Measurements*, WNT, Warsaw 1984.
- [2] Otnes R.K., Enochson L.: *Numerical Analysis of Time Series*, WNT, Warsaw 1978.
- [3] Catalogues of the Bruel & Kjaer company.

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Exercise 8

HARMONIC ANALYSIS OF VIBRATION SIGNALS

Group: _____
Team: _____

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Name and surname:

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