

EXPERIMENTAL DETERMINATION OF MOMENTS OF INERTIA BY THE PENDULUM METHOD

Vibrations and Machine Dynamics – Exercise 1A*

Aim of the exercise

The aim of the first part of the exercise is to determine the location of the center of mass of a connecting rod and the moment of inertia of this connecting rod with respect to an axis passing through the center of mass and parallel to the bearing axes. The aim of the second part of the exercise is to determine the moment of inertia of a crankshaft with respect to its axis of rotation and to determine the value of the shear modulus of the material from which the torsion bar is made.

Part One

Determination of the location of the centre of mass and the moment of inertia of the connecting rod as shown in Fig. 1.

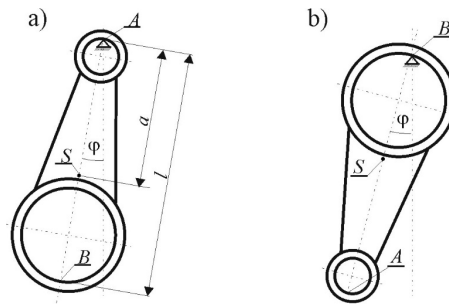


Figure 1

The connecting rod suspended as shown in Fig. 1a) constitutes a physical pendulum whose equation of motion has the form:

$$B_A \ddot{\phi} + mga\phi = 0, \text{ where} \quad (1)$$

- B_A – moment of inertia with respect to the axis of rotation,
- m – mass,
- g – gravitational acceleration equal to 9.81 m/s^2 ,
- a – distance between the axis passing through the centre of mass S and the axis of rotation,
- ϕ – angle of rotation.

Consider why this particular equation describes the motion of the system; how was it derived?

*In: LABORATORY OF MECHANICAL VIBRATIONS, K. Marynowski (ed.), Łódź 2002

The period of free oscillations of this pendulum is:

$$T_A = 2\pi\sqrt{\frac{B_A}{mga}} \quad (2)$$

Do you know how to solve equation (1) and derive this formula?

The connecting rod suspended as shown in Fig. 1b) constitutes another physical pendulum with the equation of motion:

$$B_B\ddot{\phi} + mg(l - a)\phi = 0 \quad (3)$$

where:

- B_B – moment of inertia with respect to the axis of rotation,
- l – distance between the axes of rotation.

The period of free oscillations of this pendulum is:

$$T_B = 2\pi\sqrt{\frac{B_B}{mg(l - a)}} \quad (4)$$

According to Steiner's theorem on moments of inertia

$$B_B = B_S + ma^2, \quad B_A = B_S + m(l - a)^2 \quad (5)$$

where B_S – moment of inertia with respect to the axis passing through the centre of mass.

Substituting equations (5) into equations (2) and (4) yields the system of two equations:

$$T_A = 2\pi\sqrt{\frac{B_S + ma^2}{mga}}, \quad T_B = 2\pi\sqrt{\frac{B_S + m(l - a)^2}{mg(l - a)}} \quad (6)$$

From this system of equations it is easy to obtain the formulae for the sought quantities a and B_S :

$$a = \frac{g(T_A^2 + T_B^2) - 8\pi^2 l}{4\pi^2(T_B^2 - T_A^2)/l + 8\pi^2}, \quad B_S = \frac{mgaT_A^2}{4\pi^2} - ma^2 \quad (7)$$

Procedure – Part 1

- Determine the duration of 50 oscillations of the connecting rod suspended as shown in Fig. 1a. Dividing this time by 50, determine the oscillation period T_A .
- Determine the duration of 50 oscillations of the connecting rod suspended as shown in Fig. 1b. Dividing this time by 50, determine the oscillation period T_B .
- Calculate a and B_S from formulae (7), using the determined values of T_A and T_B together with the previously known values: $l = 0.27$ m, $m = 1.85$ kg.

Part Two

Determination of the moment of inertia of the crankshaft with respect to the axis of rotation and determination of the value of the shear modulus of the material from which the torsion bar is made.

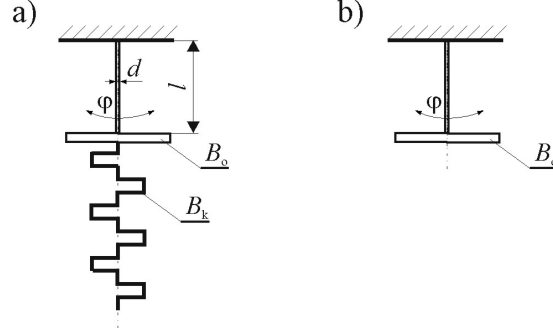


Figure 2: Figure 2.

The crankshaft of moment of inertia B_k , suspended together with a flywheel of moment of inertia B_o on a torsion bar of length l and diameter d (Fig. 2a), constitutes a physical pendulum with the equation of motion:

$$(B_k + B_o)\ddot{\phi} + k_s\phi = 0 \quad (8)$$

where:

- k_s – torsional stiffness coefficient of the torsion bar,
- ϕ – angle of rotation.

The period of free oscillations of such a pendulum is:

$$T_c = 2\pi\sqrt{\frac{B_k + B_o}{k_s}} \quad (9)$$

The flywheel of moment of inertia B_o suspended on the same torsion bar (Fig. 2b) constitutes a physical pendulum with the equation of motion:

$$B_o\ddot{\phi} + k_s\phi = 0 \quad (10)$$

The period of free oscillations of this pendulum is:

$$T_o = 2\pi\sqrt{\frac{B_o}{k_s}} \quad (11)$$

From strength of materials the formula for the torsional stiffness coefficient of the torsion bar is known:

$$k_s = \frac{G\pi d^4}{32l} \quad (12)$$

From equations (9), (11) and (12) it is easy to obtain the formulae for the sought moment of inertia B_k and for the shear modulus G of the torsion-bar material:

$$G = \frac{4\pi^2 B_o l}{T_o^2 I_o}, \quad B_k = B_o \left(\frac{T_c^2}{T_o^2} - 1 \right) \quad (13)$$

Procedure – Part 2

- Determine the duration of 20 oscillations of the crankshaft suspended as shown in Fig. 2a). Dividing this time by 20, determine the oscillation period T_c .
- Determine the duration of 20 oscillations of the flywheel suspended as shown in Fig. 2b). Dividing this time by 20, determine the oscillation period T_o .
- Calculate G and B_k from formulae (13), using the determined values of T_c and T_o , the previously measured values: $l = 0.59\text{ m}$, $d = 0.005\text{ m}$, and the calculated value of the flywheel moment of inertia $B_o = 0.0707\text{ kg}\cdot\text{m}^2$.

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Exercise 1A

Experimental determination of moments of inertia by the pendulum method

Group: _____

Team: _____

date _____

Name and surname:

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Determination of the moment of inertia and centre of gravity of the connecting rod

Results of three measurements of the time of 50 oscillations of the connecting rod suspended at point A:

$T_{A1} =$ _____ s $T_{A2} =$ _____ s $T_{A3} =$ _____ s

Oscillation period:

$$T_A = \frac{T_{A1} + T_{A2} + T_{A3}}{150} = \text{_____ s}$$

Results of three measurements of the time of 50 oscillations of the connecting rod suspended at point B:

$T_{B1} =$ _____ s $T_{B2} =$ _____ s $T_{B3} =$ _____ s

Oscillation period:

$$T_B = \frac{T_{B1} + T_{B2} + T_{B3}}{150} = \text{_____ s}$$

Calculations of the distance of the centre of mass from point A and of the moment of inertia of the connecting rod:

$$a = \frac{g(T_B^2 - 4\pi^2 l^2 / g) - 8\pi^2 l}{g(T_A^2 + T_B^2) - 8\pi^2 l} = \text{_____ m}$$

$$B_S = \frac{mgaT_A^2}{4\pi^2} - ma^2 = \text{_____ kg}\cdot\text{m}^2$$

Calculation of the measurement error:

$$\frac{\Delta B_S}{B_S} \times 100\% = \text{_____ } \%$$

Determination of the moment of inertia of the crankshaft with respect to the axis of rotation and determination of the shear modulus of the material

Results of three measurements of the time of 20 oscillations of the crankshaft and flywheel:

$$T_{C1} = \text{_____ s} \quad T_{C2} = \text{_____ s} \quad T_{C3} = \text{_____ s}$$

Oscillation period:

$$T_C = \frac{T_{C1} + T_{C2} + T_{C3}}{60} = \text{_____ s}$$

Results of three measurements of the time of 20 oscillations of the flywheel:

$$T_{O1} = \text{_____ s} \quad T_{O2} = \text{_____ s} \quad T_{O3} = \text{_____ s}$$

Oscillation period:

$$T_o = \frac{T_{O1} + T_{O2} + T_{O3}}{60} = \text{_____ s}$$

Moment of inertia of the crankshaft:

$$B_k = B_0 \left(\frac{T_C^2}{T_o^2} - 1 \right) = \text{_____ kg}\cdot\text{m}^2$$

Calculation of the measurement error:

$$\frac{\Delta B_k}{B_k} \times 100\% = \text{_____ } \%$$

Calculation of the shear modulus:

$$I_0 = \frac{\pi d^4}{32} = \text{_____ m}^4$$

$$G = \frac{4\pi^2 B_0 l}{T_0^2 I_0} = \text{_____ Pa}$$

Remarks and conclusions: