

ISOLATION OF A FOUNDATION FROM MACHINE VIBRATIONS

Vibrations and Machine Dynamics – Exercise 6*

1 Aim of the exercise

The aim of the exercise is to reduce the amplitude of foundation vibrations caused by the vibrations of a single-degree-of-freedom vibrating system mounted on that foundation.

2 Theoretical introduction

Rotating machines with unbalance are typical sources of vibration. If these devices are mounted rigidly on their base, the vibrations will be transmitted to the surroundings. The transmitted vibrations may cause noise, harmful effects on the human body or overloading of nearby instruments. Vibration isolation can minimise such undesirable interactions. Visco-elastic elements are usually placed between the vibrating machine and its base. This is so-called **active isolation** or force isolation, when one wishes to prevent the transmission of force to the base. In the case where one wishes to isolate a device from a vibrating base, one speaks of **passive isolation** or displacement isolation.

2.1 Permissible vibrations acting on the human body

In order to ensure appropriate conditions for persons working in the vicinity of machines, isolation of machine vibrations is carried out, among other measures. Technical standards describe the principles of measuring mechanical vibrations in order to assess their influence on the human body. These standards also present the permissible values of vibration parameters. These parameters are the permissible accelerations of whole-body vibration, of local vibration (acting on the hands) and the permissible working time in the event that these limits are exceeded.

For isolation purposes the vibration-damping properties of rubber are very important. They cause absorption of vibration energy and facilitate a smooth passage through resonance. Depending on the type of rubber, the value of the dimensionless damping coefficient h/α lies in the range from 0.04 to 0.1 (where: h – visco-elastic damping coefficient referred to mass in [rad/s], α – natural frequency in [rad/s]). In contrast to steel, rubber does not corrode and has a high fatigue resistance. This does not apply to low temperatures, because rubber pads can be used in temperatures from -30° to $+75^\circ\text{C}$.

2.2 Transmissibility

A single-degree-of-freedom vibrating system is considered. A harmonic exciting force $F \sin \omega t$ acts on a mass m (Fig. 1). Between the vibrating mass and its base there is a spring of stiffness coefficient k and a viscous damper of damping coefficient c .

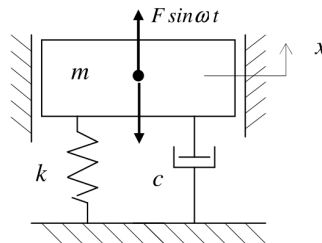


Figure 1

The equation of motion of the model under consideration can be written in the following form:

$$m\ddot{x} + c\dot{x} + kx = F \sin \omega t. \quad (1)$$

*In: LABORATORY OF MECHANICAL VIBRATIONS, K. Marynowski (ed.), Łódź 2002

After rearrangements the equation of motion takes the form:

$$\ddot{x} + 2h\dot{x} + \omega_n^2 x = q \sin \omega t, \quad (2)$$

where:

$$\frac{c}{m} = 2h; \quad \frac{k}{m} = \alpha^2; \quad \frac{F}{m} = q. \quad (3)$$

A particular solution of the equation can be expected in the form:

$$x = A \sin(\omega t - \phi), \quad (4)$$

where the displacement amplitude A is given by the formula:

$$A = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}}, \quad (5)$$

and the phase-shift angle ϕ between the excitation and the displacement is determined by the following relation:

$$\phi = \arctg \frac{2h\omega}{\alpha^2 - \omega^2}. \quad (6)$$

The force acting on the base consists of the force in the spring and the force in the damper. The force in the spring is:

$$S = kx = kA \sin(\omega t - \phi). \quad (7)$$

The force in the damper is equal to:

$$R = c\dot{x} = cA\omega \cos(\omega t - \phi). \quad (8)$$

The maximum total force acting on the base is expressed as:

$$P_{\max} = \sqrt{S_{\max}^2 + R_{\max}^2} = \sqrt{(kA)^2 + (c\omega A)^2}. \quad (9)$$

Substituting the relations and rearranging, one obtains:

$$P_{\max} = \frac{F \sqrt{1 + 4 \frac{h^2}{\alpha^2} \frac{\omega^2}{\alpha^2}}}{\sqrt{(1 - \frac{\omega^2}{\alpha^2})^2 + 4 \frac{h^2}{\alpha^2} \frac{\omega^2}{\alpha^2}}}. \quad (10)$$

The ratio of the maximum total force acting on the base $P_{\max}$ to the amplitude of the exciting force F is called the transmissibility and in the further considerations will be denoted by ν . The transmissibility can be determined in the form:

$$\nu = \frac{P_{\max}}{F} = \frac{\sqrt{1 + 4 \frac{h^2}{\alpha^2} \frac{\omega^2}{\alpha^2}}}{\sqrt{(1 - \frac{\omega^2}{\alpha^2})^2 + 4 \frac{h^2}{\alpha^2} \frac{\omega^2}{\alpha^2}}}. \quad (11)$$

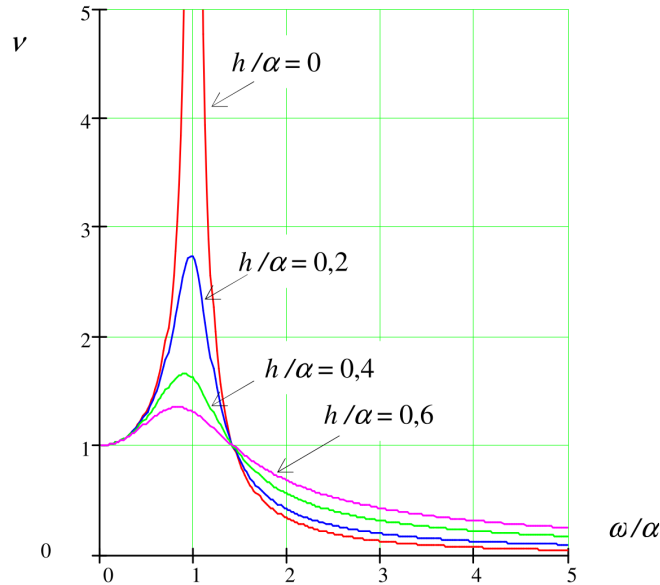


Figure 2: Plots of the transmissibility ν for different values of damping

Figure 2 presents plots of the transmissibility ν as a function of the excitation frequency ratio $\gamma = \omega/\alpha$ (the ratio of the dimensional excitation frequency ω to the natural frequency α) for different values of the dimensionless damping coefficient h/α .

Depending on the type of rubber, the value of the dimensionless damping coefficient h/α lies in the range from 0.04 to 0.1. In order to obtain, on the basis of Fig. 2, a transmissibility value of about 0.1 for these values of damping, the dimensionless excitation frequency $\gamma = \omega/\alpha$ should be higher than 3.

2.3 Example of selection of an isolator for active vibration isolation on the basis of its static characteristic

In this numerical example a system consisting of a fan and a driving motor is considered. The total mass of the system is $m_1 = 200$ kg. The rotational speed of the motor is $n = 1450$ rpm. The system is mounted on a supporting structure of mass $m_2 = 100$ kg. The base should be isolated from the vertical vibrations of the system by means of four rubber isolators. After isolation the value of the transmissibility should not be higher than $\nu = 0.1$.

Solution

The exciting frequency is equal to:

$$\omega = \frac{2\pi n}{60} = 152.05 \quad [\text{rad/s}] \quad (\text{a})$$

The gravitational force acting on one isolator is:

$$Q = \frac{m_1 + m_2}{4}g = 735.75 \quad [\text{N}] \quad (\text{b})$$

It is assumed that the rubber isolators will operate with a linear static characteristic. Two types of isolators, designated A and B according to their hardness, have been selected for isolation of the base of the system. The static characteristics of isolators of type M200 are shown in Fig. 3.

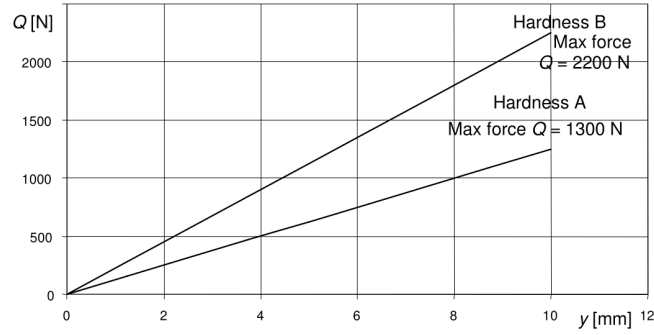


Figure 3: Static characteristics of isolators of type M200

From the characteristic curve of the isolator of type M200 of hardness A the static deflection under the load Q can be read. This deflection is $y_{st} = 5.89$ mm. The natural frequency of such an isolator can then be determined:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{y_{st}m}} = \sqrt{\frac{g}{y_{st}}} = \sqrt{\frac{9.81}{5.89 \cdot 10^{-3}}} = 40.81 \quad [\text{rad/s}] \quad (\text{c})$$

The ratio of the excitation frequency ω to the natural frequency α is:

$$\gamma = \frac{\omega}{\alpha} = \frac{152.05}{40.81} = 3.72 \quad (\text{d})$$

Nevertheless, the value of the transmissibility should be checked for the limiting (lim) value of the ratio h/α of rubber isolators:

$$\nu = \frac{\sqrt{1 + 4 \left(\frac{h}{\alpha}\right)_{\lim}^2 \frac{\omega^2}{\alpha^2}}}{\sqrt{\left(1 - \frac{\omega^2}{\alpha^2}\right)^2 + 4 \left(\frac{h}{\alpha}\right)_{\lim}^2 \frac{\omega^2}{\alpha^2}}} = \frac{\sqrt{1 + 4 \cdot 0.1^2 \cdot 3.72^2}}{\sqrt{(1 - 3.72^2)^2 + 4 \cdot 0.1^2 \cdot 3.72^2}} = 0.097 < 0.1 \quad (\text{e})$$

3 Experimental stand

The schematic diagram of the measuring device is shown in Fig. 4. The exciter (a loudspeaker device supplying energy) 1 is mounted on its base, which is represented by a rigid plate 3. The base is to be isolated from the vertical vibrations of the loudspeaker device by means of four rubber isolators 2. The plate 3 is connected to the building structure by means of four supporting pads 4 of known stiffness. The vibrations of the exciter 1 and of the plate 3 are measured by means of a piezoelectric sensor – an accelerometer in positions 5 or 6. The sensor is connected to a set of vibration meters and an oscilloscope 7.

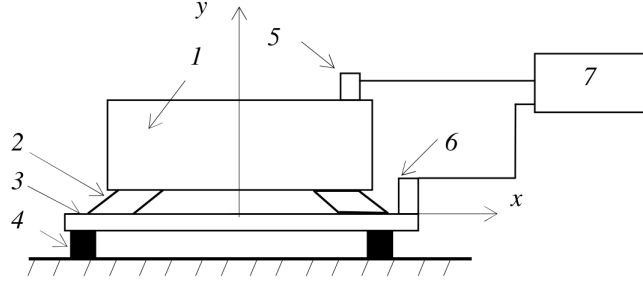


Figure 4

3.1 Physical model

Figure 5 presents the physical model of the system of Fig. 4. A harmonic force exciting its vibrations in the direction y acts on the element that constitutes the exciter of mass m . The exciter itself is connected to the foundation by means of a vibration isolator that can be described mathematically as a linear combination of a spring of stiffness coefficient k and a linear damper of damping coefficient c .

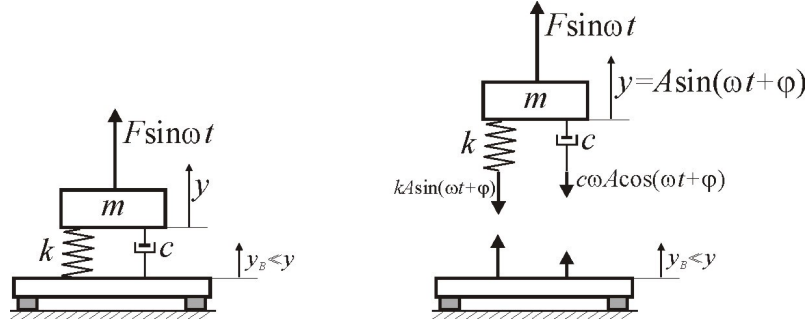


Figure 5

As a result of the vibrations of the exciter, forces arise in the spring and the damper that are transmitted to the foundation, causing its motion in the direction y_B . The system visible has two degrees of freedom; we shall, however, treat it as two subsystems, each with one degree of freedom, for the sake of simplification of the reasoning (and of the formulae).

If it is assumed that the amplitude of the foundation vibrations is small in comparison with the amplitude of the exciter vibrations, the mathematical model describing the motion of the exciter can be written as follows:

$$m\ddot{y} + c\dot{y} + ky = F \sin \omega t \quad (12)$$

A particular solution of this equation is assumed in the form

$$y = A_s \sin \omega t + A_c \cos \omega t \quad (13)$$

After substitution of (13) into (12) and performance of simple rearrangements one obtains

$$A_s = \frac{F(k - \omega^2 m)}{(k - \omega^2 m)^2 + \omega^2 c^2}, \quad A_c = \frac{-F\omega c}{(k - \omega^2 m)^2 + \omega^2 c^2} \quad (14)$$

Writing the particular solution (13) in the form

$$y = A \sin(\omega t + \beta) \quad (15)$$

one can state that the amplitude of the exciter vibrations is given by the formula

$$A = \sqrt{A_s^2 + A_c^2} = \frac{F}{\sqrt{(k - \omega^2 m)^2 + \omega^2 c^2}} \quad (16)$$

The force transmitted by the spring k to the foundation is given by the formula

$$R_k \sin(\omega t + \beta) = ky = kA \sin(\omega t + \beta) \quad (17)$$

The force transmitted by the damper c to the foundation is given by the formula

$$R_c \cos(\omega t + \beta) = c\dot{y} = \omega cA \cos(\omega t + \beta) \quad (18)$$

From the above formulae it follows that the total force transmitted to the foundation by the spring and by the damper is expressed by the formula:

$$R \sin(\omega t + \gamma) = \sqrt{R_k^2 + R_c^2} \sin(\omega t + \gamma) = \sqrt{k^2 A^2 + \omega^2 c^2 A^2} \sin(\omega t + \gamma) = F \sqrt{\frac{k^2 + \omega^2 c^2}{(k - \omega^2 m)^2 + \omega^2 c^2}} \sin(\omega t + \gamma) \quad (19)$$

The quality of isolation of the foundation from the vibrations of the exciter is assessed by determining the so-called transmissibility, i.e. the ratio of the amplitude of the force exciting the foundation vibrations R to the amplitude of the force generated by the exciter F . As follows from formula (19) it is equal to

$$\nu = \frac{R}{F} = \sqrt{\frac{k^2 + \omega^2 c^2}{(k - \omega^2 m)^2 + \omega^2 c^2}} \quad (20)$$

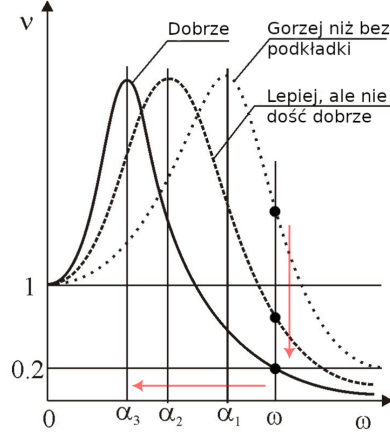


Figure 6

Figure 6 presents the variation of the transmissibility ν as a function of the circular frequency of the exciting force ω for three different values of the circular frequency of free vibrations of the exciter α , determined by the approximate (in view of the existence of damping) formula $\alpha = \sqrt{\frac{k}{m}}$.

Analysis of Fig. 6 indicates that reduction of the transmissibility to an acceptably low value $\nu = 0.2$ requires reduction of the circular frequency α to a correspondingly low value α_3 . This in turn requires the use, for supporting the exciter, of a different structure – a spring – of a sufficiently low value of the stiffness coefficient k . Determination of the transmissibility on the basis of formula (9) is not possible when the parameters k , m , c are unknown. It can be determined experimentally on the basis of measurements of the amplitudes of displacement or acceleration of the foundation, performed with the isolator and without the isolator, Fig. 7.

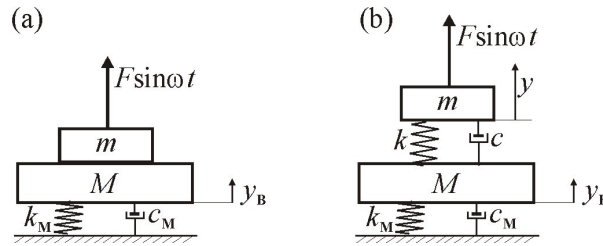


Figure 7

3.2 Measurement I

Let us assume that the mass of the exciter is many times smaller than the mass of the foundation. This means that after attachment of the exciter to the foundation without a vibration isolator (as in Fig. 3a), the force $F \sin \omega t$ generated by the exciter will act directly on the foundation. The mathematical model describing the motion of the foundation can be written as follows:

$$M \ddot{y}_B + c_B \dot{y}_B + k_B y_B = F \sin \omega t \quad (21)$$

In the manner shown above one can state that the amplitude of the foundation vibrations is given by the formula:

$$A_F = \frac{F}{\sqrt{(k_B - \omega^2 M)^2 + \omega^2 c_B^2}}. \quad (22)$$

3.3 Measurement II

In order to reduce the amplitude of the force acting on the foundation, an isolator is introduced between the exciter and the foundation, as in Fig. 2b). Consequently, the foundation on elastic supports can be treated as a single-degree-of-freedom system on which the force $R \sin(\omega t + \gamma)$ defined by equation (8) acts. The mathematical model describing the motion of the foundation can be written as follows:

$$M\ddot{y}_B + c_B\dot{y}_B + k_By_B = R \sin(\omega t + \gamma) \quad (23)$$

The amplitude of the foundation vibrations is now given by the formula:

$$A_R = \frac{R}{\sqrt{(k_B - \omega^2 M)^2 + \omega^2 c_B^2}}. \quad (24)$$

It is readily seen that the transmissibility

$$\nu = \frac{R}{F} = \frac{A_R}{A_F} = \frac{\omega^2 A_R}{\omega^2 A_F}. \quad (25)$$

It is therefore equal to the ratio of the vibration amplitudes, and also to the ratio of the acceleration amplitudes, which can be measured by means of an accelerometer on the experimental stand.

3.4 Procedure:

1. Attach the yellow exciter (with a loudspeaker inside, excited by a function generator through an amplifier) to the foundation **without any damping pad**. In this arrangement it is assumed that the transmissibility is $\nu = 1$. Measure the amplitude of the foundation vibrations, equal to the amplitude of the exciter vibrations A_F .
2. Place the exciter on the foundation, using a damping pad of **cork**. Measure the amplitude of the foundation vibrations A_{R1} . By varying the excitation frequency, establish the value of the resonant frequency α_1 .
3. Place the exciter on the foundation, using a damping pad of **rubber**. Measure the amplitude of the foundation vibrations A_{R2} . By varying the excitation frequency, establish the value of the resonant frequency α_2 .
4. Attach the exciter to the foundation, using 4 **vibration isolators** (dampers). Measure the amplitude of the foundation vibrations A_{R3} . By varying the excitation frequency, establish the value of the resonant frequency α_3 .
5. Calculate the values of the transmissibility ν for the three types of isolation.

Data for calculations

Mass of the machine model $m = 3.58$ kg. We assume a dimensionless damping coefficient for rubber of value $\xi = 0.05$ for calculation of the damping value h , c_{kr} – critical damping. Measurement path of the reading on the RIGOL DS1102 oscilloscope: sensitivity of the B&K 4396 accelerometer: 1 mV = 0.101 m/s². Amplification factor of the accelerometer signal $k = 10$ [-].

$$\xi = \frac{c}{c_{kr}}$$

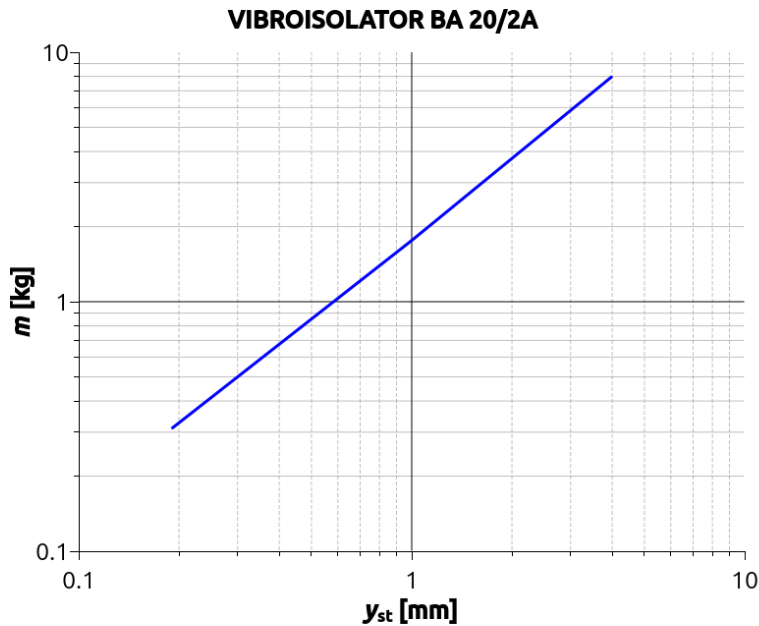


Figure 8: Vibration isolator BA 20/2A – manufacturer's characteristic

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Exercise 6

**ISOLATION OF A FOUNDATION FROM MACHINE
VIBRATIONS**

Group: _____
Team: _____

date _____

Name and surname:

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Table 1: Measurement results

Type of damping	Amplitude of base vibrations	Resonant frequency α	Transmissibility ν
Without damping	$A_F =$	Theoretically infinitely large	$\nu = A_F/A_F = 1$
Cork pad	$A_{R1} =$	$\alpha_1 =$	$\nu = A_{R1}/A_F =$
Rubber pad	$A_{R2} =$	$\alpha_2 =$	$\nu = A_{R2}/A_F =$
Vibration isolators	$A_{R3} =$	$\alpha_3 =$	$\nu = A_{R3}/A_F =$

Auxiliary calculations

Force loading one vibration isolator:

$$Q = \text{_____} [\text{N}]$$

Static deflection according to the manufacturer's characteristic graph

$$y_{st} = \text{_____} [\text{mm}]$$

Natural frequency of the system with vibration isolators:

$$\alpha = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{y_{st} m}} = \sqrt{\frac{g}{y_{st}}} = \text{_____} [\text{rad/s}]$$

Ratio of the excitation frequency to the free-vibration frequency:

$$\gamma = \frac{\omega}{\alpha} = \text{_____} [-]$$