

MEASUREMENT OF VIBRATION PARAMETERS

Vibrations and Machine Dynamics – Exercise 2*

1 Aim of the exercise

To become familiar with the technique of measuring mechanical vibration parameters and with the apparatus used for these measurements.

2 Theoretical introduction

Mechanical vibration is a type of motion in which a body (system) moves between two extreme positions, passing through the equilibrium position.

Vibrations occurring in machines and equipment are excited by driving forces or by resistance forces (e.g. during cutting), as well as by impacts, changes in load, changes in external conditions, and as a result of clearances and irregularities. In addition to forced vibrations associated with machine motion, self-excited vibrations may also occur, which can arise, for example, during the operation of cutting tools.

Excessive vibrations cause accelerated wear of machines, material fatigue, harmful effects on people, and noise. They are often an indication of damage to machines and equipment.

Purposeful applications of vibrations include, for example, vibrators, pneumatic hammers, exciters for strength testing or modal analysis, and reference vibration sources for sensor calibration.

2.1 Vibration measurement

The aims of vibration measurement are:

- to check whether the amplitudes of vibrations at specified frequencies do not exceed permissible values,
- to establish the causes of resonant vibration excitation in certain parts of a machine,
- to damp or isolate vibration sources,
- to monitor the dynamic condition of machines,
- to obtain data for the development or verification of computer models of structures.

When measuring vibratory motion we may be interested in the time history of displacement x , velocity v or acceleration a . Since these quantities are related by the dependencies:

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad (1)$$

it is sufficient to measure one of them; the remaining quantities can be obtained by integration or differentiation.

In the case of harmonic vibratory motion:

$$x = x_0 \cos \omega t \quad v = -\omega x_0 \sin \omega t \quad a = -\omega^2 x_0 \cos \omega t. \quad (2)$$

The operations mentioned above can be readily performed analytically. Integration or differentiation operations are most often carried out directly during measurements, using electrical integrating or differentiating circuits. From this one might conclude that it is immaterial which of the quantities characterising vibratory motion is measured directly. This is not entirely correct, however, because the process of electrical integration or differentiation (especially if repeated twice) reduces the accuracy with which the amplitude of the quantity of interest is determined and introduces a certain phase shift between the measured waveform and that obtained at the output of the circuit. Therefore, when performing measurements one must be aware of which mechanical quantity the primary electrical signal obtained from the sensor is proportional to.

In the exercise the following sensors will be used:

- inductive transformer sensors, which produce a signal proportional to **displacement**,
- electrodynamic seismic sensors – a signal proportional to **velocity**,
- piezoelectric seismic sensors – a signal proportional to **acceleration**,
- laser sensors – a signal proportional to **displacement or velocity**.

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2.2 Vibration measurement with transformer sensors

In transformer sensors with variable mutual inductance, use is made of the dependence of the electromotive force induced from the primary winding into the secondary windings on the mutual inductance coefficient. The sensor shown in Fig. 1 has a primary winding with z_1 turns and two secondary windings with an equal number of turns z_2 , wound on a cylindrical insulating sleeve. Inside the sleeve a ferromagnetic core moves, connected via a rod to the vibrating object. The primary winding is supplied with a sinusoidally varying voltage (usually of frequency from 50 Hz to 50 kHz) and generates a sinusoidally varying magnetic flux $\phi = \Phi \sin \omega t$ that induces sinusoidally varying electromotive forces E_1 and E_2 in the secondary windings. The secondary windings are connected in opposition (differentially), so the output voltage U_{out} is equal to the difference of the electromotive forces E_1 and E_2 .

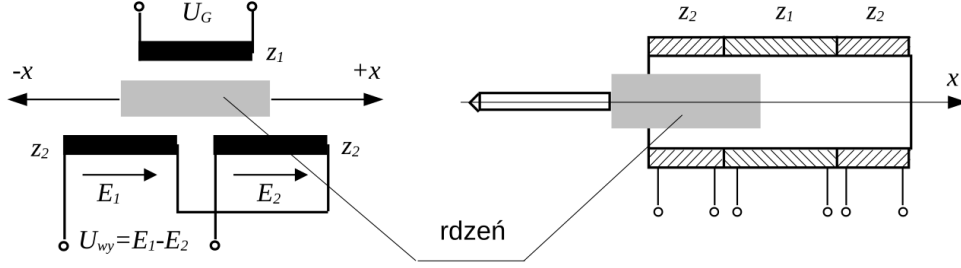


Figure 1: Inductive transformer sensor

The instantaneous value of the electromotive force induced in the secondary windings is:

$$e = -z_2 \frac{d\phi}{dt} \quad (3)$$

In the mid-position of the core the amplitude of the (sinusoidally varying) electromotive force $E_1 = E_2 = \omega z_2 \Phi$, and because the windings are connected differentially, $U_{\text{out}} = E_1 - E_2 = 0$. Displacement of the core, for example to the left, causes an increase of the flux penetrating the left coil by $\Delta\Phi$, while the flux penetrating the right coil decreases by $\Delta\Phi$.

The output voltage from the sensor is therefore:

$$U_{\text{out}} = E_1 - E_2 = 2\omega z_2 \Delta\Phi \quad (4)$$

The change in flux $\Delta\Phi$ is proportional to the change in magnetic reluctance, which is proportional to the displacement of the core x . Finally one obtains:

$$U_{\text{out}} = kx \quad (5)$$

It should be noted that the output voltage U_{out} from the sensor is a sinusoidally varying voltage of the frequency of the supply voltage of the primary winding. The voltages for the two directions of core displacement differ in phase by 180 degrees, Fig. 2.

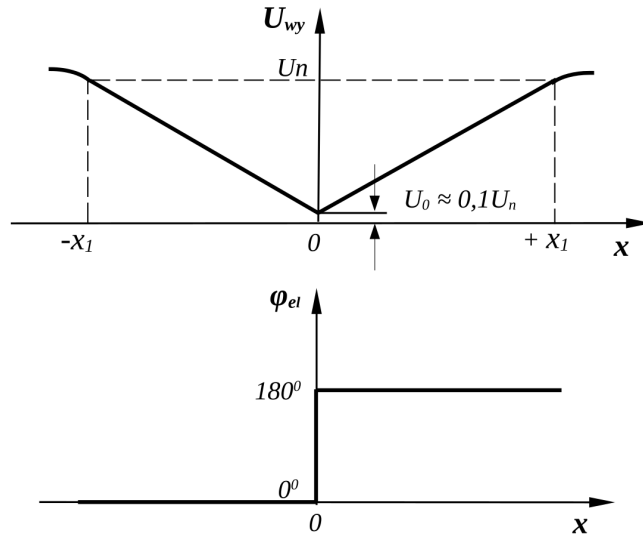


Figure 2: Dependence of the amplitude of the output voltage U_{out} and of the phase-shift angle ϕ_{el} (between the output voltage U_{out} and the supply voltage U_G) on the core displacement x

At the core position corresponding to $x = 0$ a certain small residual (unbalance) voltage U_0 appears at the sensor output, caused by the unavoidable magnetic and electrical asymmetry of the system.

The simplest measuring circuit is shown in Fig. 3. A generator supplies the sensor with a sinusoidally varying voltage of constant amplitude and constant frequency. The output voltage from the sensor, which is a measure of the core displacement, is measured with an ordinary a.c. voltmeter.

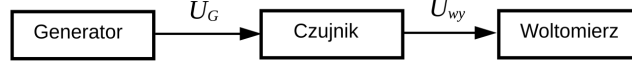


Figure 3: Displacement measurement with a transformer sensor using an a.c. voltmeter

In the circuit of Fig. 3 the voltage U_{out} measured with an ordinary a.c. voltmeter has the same value for both directions of core displacement ($+x$ and $-x$), so the characteristic is ambiguous. In such a circuit, for unambiguous displacement measurement only half of the measuring range of the sensor can be used – one of the two branches of its characteristic, Fig. 2.

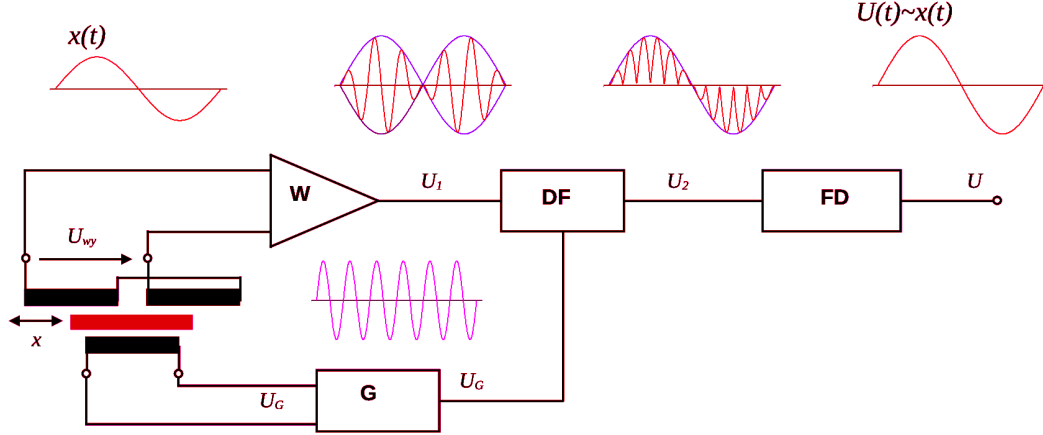


Figure 4: Block diagram of a meter with phase-sensitive rectification

The use of a phase-sensitive rectifier (Fig. 4) permits unambiguous measurement for both directions of core displacement. Generator G supplies the primary winding of the sensor and the phase-sensitive rectifier with a sinusoidal voltage of constant amplitude and frequency. Amplifier W amplifies the voltage U_{out} from the sensor. In the phase-sensitive rectifier DF the voltage U_I from the amplifier is rectified taking into account the phase of this voltage relative to the generator voltage U_G . The voltage U_2 assumes positive and negative values depending on the direction of core displacement from the mid-position; it nevertheless contains undesirable ripple at the frequency of the generator voltage. The low-pass filter FD “passes” only signals of low frequencies (relative to the generator frequency); in the signal from the rectifier this is the low frequency of the envelope that reproduces the core-displacement signal.

For correct reproduction of the core motion the generator frequency should be several times higher than the frequency of the measured vibrations. At the output of the circuit one obtains a d.c. voltage whose polarity depends on the direction of core displacement from the mid-position, while at the same time the measuring range is twice as large as in the previous circuit.

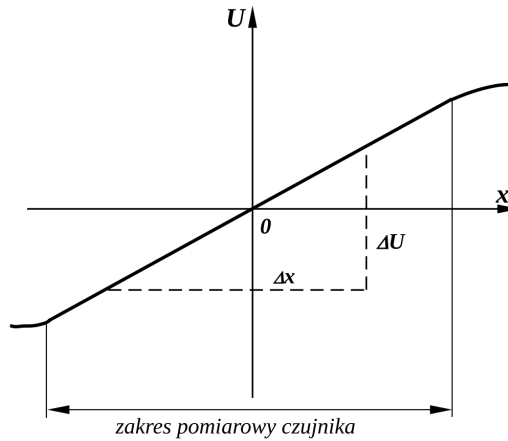


Figure 5: Static characteristic of the measuring system with a phase-sensitive rectifier

The sensitivity k of the measuring system is defined as the ratio of the voltage increment ΔU for a core-

displacement change of Δx (Fig. 5):

$$k = \frac{\Delta U}{\Delta x} \quad (6)$$

It can be determined from the static characteristic $U = f(x)$.

Features of differential transformer sensors:

- the electrical signal is proportional to displacement,
- possibility of measuring both static and dynamic displacements (vibrations),
- measuring range of displacements – from micrometres to several tens of centimetres,
- high sensitivity – up to about 100 V/mm,
- linearity error – 0.1...1%,
- residual (unbalance) voltage – below 1% of the rated output voltage,
- simple construction,
- the maximum frequency $f_{\max}$ of vibrations that can be measured with transformer sensors is limited by the frequency of the supply voltage (5 to 10 times lower than the generator frequency f_c).

The moving rod (connected to the sensor core) is in contact with the vibrating object under test, so the vibrations of the object relative to the reference frame in which the sensor housing is fixed are measured. Under real vibration-measurement conditions it is difficult to obtain a stationary reference frame, which may limit the applications of these sensors.

2.3 Vibration measurement with electrodynamic sensors

Electrodynamic sensors operate on the principle of induction of an electromotive force in a winding moving in the field of a permanent magnet:

$$e = Blv \quad (7)$$

B – magnetic induction produced by the permanent magnet,

l – length of the winding (depends on the number of turns),

v – velocity of the winding in the magnetic field.

Electrodynamic sensors can measure **vibration velocity** directly. Electrodynamic sensors are generating sensors, i.e. they require no power supply and themselves produce an electromotive force. They are constructed as relative-vibration sensors and absolute-vibration sensors – **seismic sensors**. A seismic sensor is a single-degree-of-freedom mechanical system possessing a seismic mass m , a spring of stiffness k and a damper of damping coefficient c .

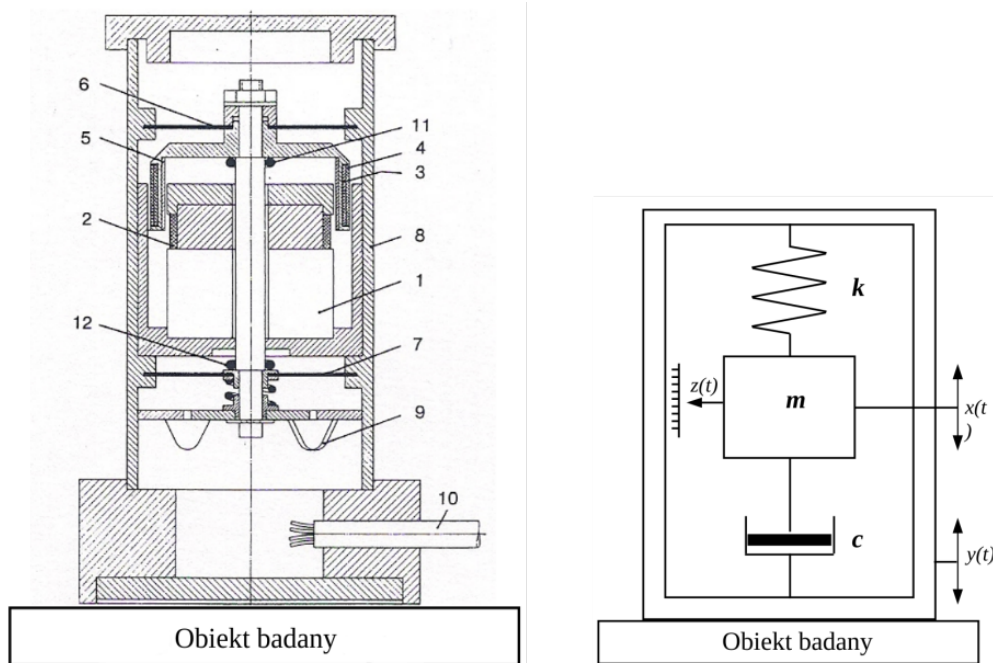


Figure 6: Construction and mechanical-system model of the electrodynamic seismic sensor PR 9266. Notation: 1 – permanent magnet, 2 – correction coil, 3 – measuring coil, 4 – additional damping coil, 5 – damping cylinder, 6, 7 – membrane springs, 8 – housing, 9 – leads, 10 – screened cable, 11, 12 – stops

In electrodynamic seismic sensors the coil most often constitutes the seismic mass m suspended on a spring of stiffness k ; damping c is most often realised electrically. As a result of housing vibration the coil moves relative to the magnet rigidly connected to the housing, although solutions with a moving magnet also exist. In both cases the electromotive force is proportional to the **relative velocity** of the coil and the magnet. The sensor is placed on the object under test whose vibrations $y(t)$ we wish to measure, Fig. 6. Here:

$x(t)$ – coordinate describing the absolute motion of mass m relative to a fixed frame (relative to the earth) – absolute motion of the coil,
 $y(t)$ – coordinate describing the motion of the sensor housing, i.e. the motion of the object under test – the vibrations we wish to measure,
 $z(t)$ – relative motion of the mass and the housing (in the sensor – relative motion of the coil and the magnet).
 On the basis of d'Alembert's principle the equation of motion of the mass can be written in the form:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \quad (8)$$

The output (measuring) signal usually depends on the relative motion $z(t)$ of the mass and the housing:

$$z(t) = x(t) - y(t) \quad (9)$$

Equation of motion (12) can be rewritten in the form:

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \quad (10)$$

or in the form:

$$\ddot{z} + 2h\dot{z} + \alpha^2 z = -\ddot{y} \quad (11)$$

where:

$$\alpha^2 = \frac{k}{m}, \quad 2h = \frac{c}{m}, \quad \varepsilon = \frac{h}{\alpha} \quad (12)$$

If the excitation (object vibration) is harmonic: $y = y_0 \sin \omega t$, the solution has two components: a transient component of circular frequency of damped oscillations $\lambda = \sqrt{\alpha^2 - h^2}$ that decays at a rate dependent on the relative damping ε , and a steady-state component of forced oscillations:

$$z = z_0 \sin(\omega t - \phi) \quad (13)$$

The amplitude and phase of the relative oscillations are given by:

$$z_0 = y_0 \frac{\omega^2}{\sqrt{(\alpha^2 - \omega^2)^2 + (2h\omega)^2}}, \quad \phi = \arctan \frac{2h\omega}{\alpha^2 - \omega^2} \quad (14)$$

The ratio of the amplitude of relative oscillations z_0 to the amplitude of object (housing) oscillations y_0 is the sensitivity k of the transducer:

$$k = \frac{z_0}{y_0} = \frac{\omega^2}{\sqrt{(\alpha^2 - \omega^2)^2 + (2h\omega)^2}} \quad (15)$$

Presenting the sensor characteristic in dimensionless form – i.e. the ratio of the amplitudes of relative oscillations z_0 and of the exciting (measured) oscillations y_0 as a function of excitation frequency or of the ratio ω/α – is advantageous because:

- a) the sensor characteristics are identical irrespective of whether one considers amplitudes of displacement, velocity or acceleration of the measured harmonic vibrations, since:

$$\frac{z_0}{y_0} = \frac{z_0 \omega}{y_0 \omega} = \frac{z_0 \omega^2}{y_0 \omega^2} \quad (16)$$

- b) the characteristics enable rapid correction of the measurement error introduced by the sensor, expressed in dimensionless units or in %.

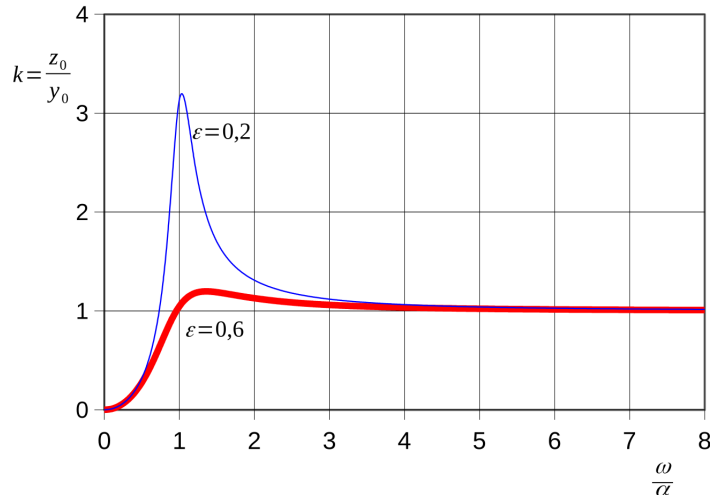


Figure 7: Frequency amplitude characteristic of a seismic sensor

The sensitivity coefficient k of the transducer depends on the ratio ω/α (frequency of the measured vibrations and natural frequency of the vibrating system of the seismic sensor). The sensor introduces no amplitude error only when $k = z_0/y_0 = 1$ (Fig. 7); in practice this occurs when the frequency ω of the measured vibrations is at least (2.5 to 3) times higher than α . The seismic mass (coil) is then practically stationary relative to the earth, and the measured relative oscillations of the coil and the magnet are practically the oscillations of the object relative to the earth, i.e. the **absolute vibrations of the object**.

By appropriate choice of damping a wider range of flat amplitude characteristic can be obtained. The most advantageous value is the coefficient $\varepsilon = h/\alpha = 0.6$. Taking into account the amplitude correction and the phase-shift-angle correction permits extension of the measuring range of the sensor into the region of frequencies close to, and even lower than, the frequency α .

Features of electrodynamic seismic sensors:

- the electrical signal is proportional to vibration velocity, so the sensors can be calibrated only dynamically,
- they require no reference frame (mounted directly on the object),
- they require no power supply,
- they are of simple construction,
- high sensitivity can be obtained by increasing the number of turns of the coil,
- low resistance of the coil as a voltage source (of the order of 10 k Ω) permits cooperation with any voltmeter,
- large dimensions, relatively large mass of the order of 0.5 kg,
- necessity of determining the frequency of the measured vibrations in order to apply a correction to the sensor readings,
- in typical sensors the natural frequency is 10...20, and even 80 Hz.

In the Philips PR 9266 sensor damping is realised electrically by means of eddy currents induced during motion in a copper cylinder located in the same air gap. An additional correction winding enables compensation of changes in the damping coefficient at high temperatures or compensation of the static deflection of the springs when measuring vertical vibrations – by supplying a constant current from an external source to the winding.

2.4 Vibration measurement with piezoelectric sensors

In piezoelectric sensors use is made of the well-known piezoelectric effect exhibited by certain crystals, e.g. quartz (SiO_2), tourmaline, Rochelle salt, barium titanate. This effect, discovered by the Curie brothers in 1880, consists in the appearance of ionic electric charges on the faces of crystals deformed under the action of mechanical loads, Fig. 8.

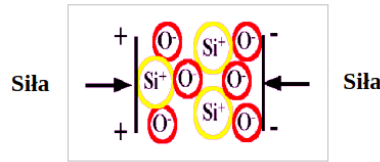


Figure 8: Piezoelectric effect in the crystal lattice of quartz

Charges appear under compressive, bending and shear stresses, Fig. 9. Piezoelectric sensors are used for the measurement of forces, accelerations and pressures.

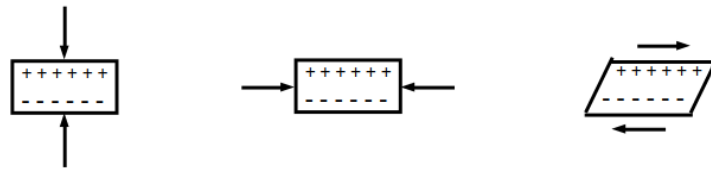


Figure 9: Longitudinal, transverse and shear piezoelectric effects

In the sensor shown in Fig. 10a the sensing element consists of two piezoelectric plates on which a mass m rests, pressed by a spring of stiffness k in order to ensure permanent contact of the seismic mass with the piezoelectric material during vibration. Thin metal electrodes collect the charges that appear on the faces of the piezoelectric material.

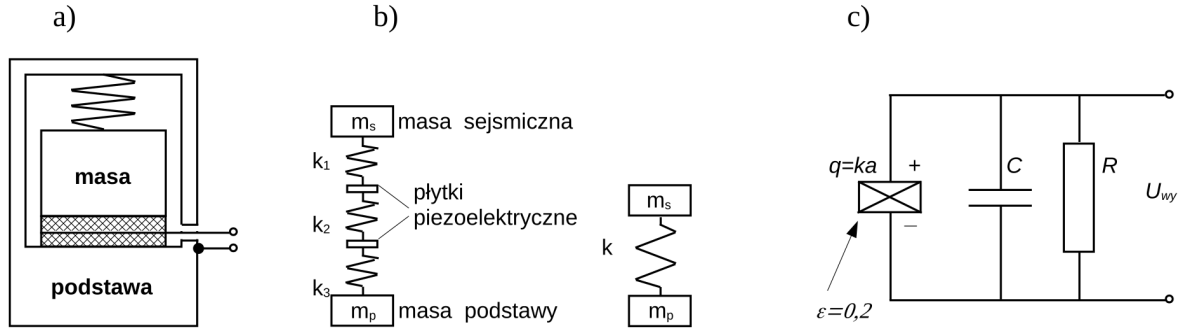


Figure 10: Piezoelectric acceleration sensor: a) schematic diagram, b) equivalent mechanical model, c) equivalent electrical model

If an acceleration a acts on the sensor, caused for example by vibration, the charge that appears on the faces of the piezoelectric material (proportional to the inertia force) is a measure of the acceleration:

$$q = kF = kma \quad (17)$$

Piezoelectric sensors are generating sensors; they produce a signal only when a change of load of the piezoelectric material occurs (e.g. a change of acceleration in a vibration sensor).

The piezoelectric acceleration sensor whose construction and equivalent model are shown in Fig. 10 can be treated as a lightly damped mass-spring system with one degree of freedom, described by the classical second-order differential equation whose solution has the form:

$$k_1 = \frac{a_y}{a_x} = \frac{\sqrt{\alpha^4 + (2h\omega)^2}}{\sqrt{(\alpha^2 - \omega^2)^2 + (2h\omega)^2}} \quad (18)$$

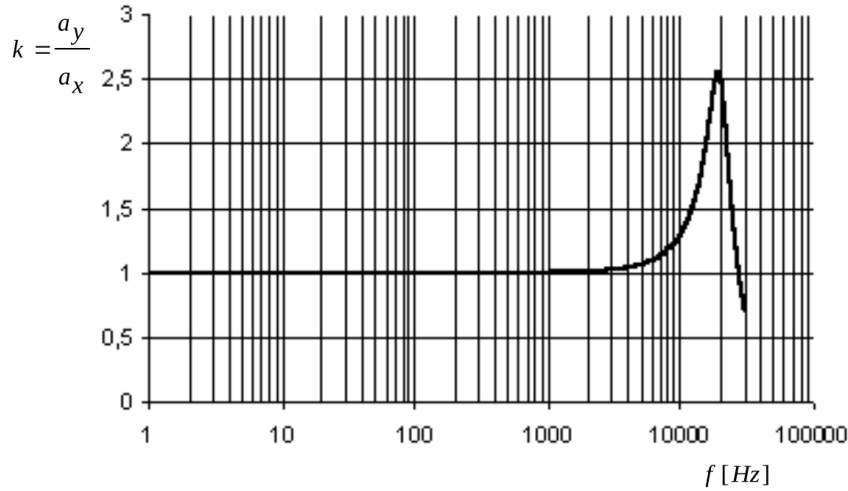


Figure 11: Example of the characteristic of a seismic piezoelectric sensor

There are two basic types of piezoelectric transducers:

- with **charge output** – characterised by the high output impedance of the piezoelectric element and generally requiring an external charge or voltage amplifier for further utilisation of the signal. Moreover, the high impedance of a piezoelectric sensor necessitates the use of special low-noise cables in order to limit charge variations in the cable caused by motion, bending, compression and tension (triboelectric effect), and to limit noise caused by radio and electromagnetic interference.

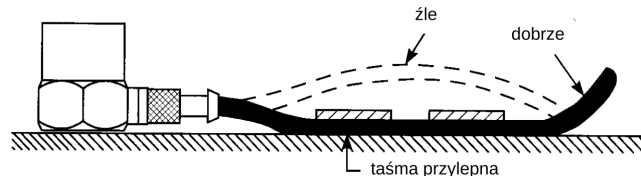


Figure 12: Method of cable mounting intended to limit the influence of the triboelectric effect

An important issue is keeping the cables clean and preventing deterioration of their insulation, especially under difficult field conditions during measurements. Piezoelectric sensors impose special requirements on the measuring system concerning both the resistance and the input capacitance of the apparatus used. The values of these parameters determine the frequency range of vibrations that can be measured with these sensors as well as the sensitivity of the measuring systems. In practice the upper limit of the frequency range of measured vibrations is 0.3...0.5 of the resonant frequency – the error then does not exceed 12%. At the frequency $f = 0.3 f_0$ the error is about 10% (Fig. 13); the lower frequency limit is determined by the electrical parameters of the sensor and of the devices connected directly to it.

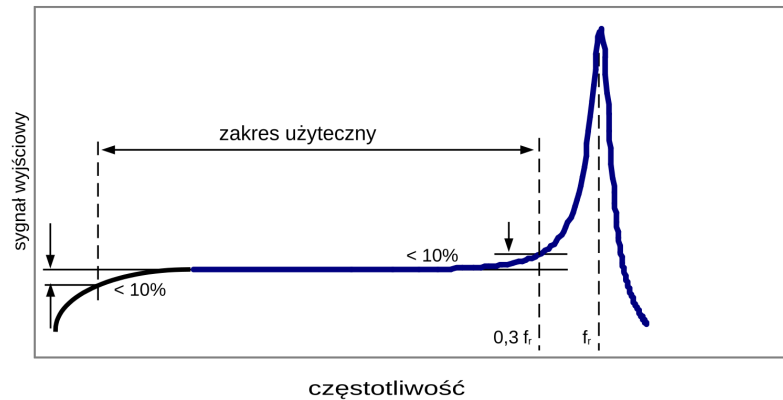


Figure 13: Usable frequency range of a piezoelectric sensor

- b) with an **internal amplifier** – these contain a built-in integrated amplifying circuit supplied from an external source.

Features of piezoelectric sensors with a built-in amplifier:

- they do not require special cables – an ordinary two-wire or coaxial cable is sufficient,
- constant sensitivity independent of cable length,
- supply from a current source permits the use of the same two-wire cable also for transmitting the signal from the sensor,
- low output impedance – below 100 Ω ,
- compact construction,
- the cost of a measuring channel is lower than that of an external amplifier,
- limited temperature range (<125°C for ordinary designs, <150°C for special designs).

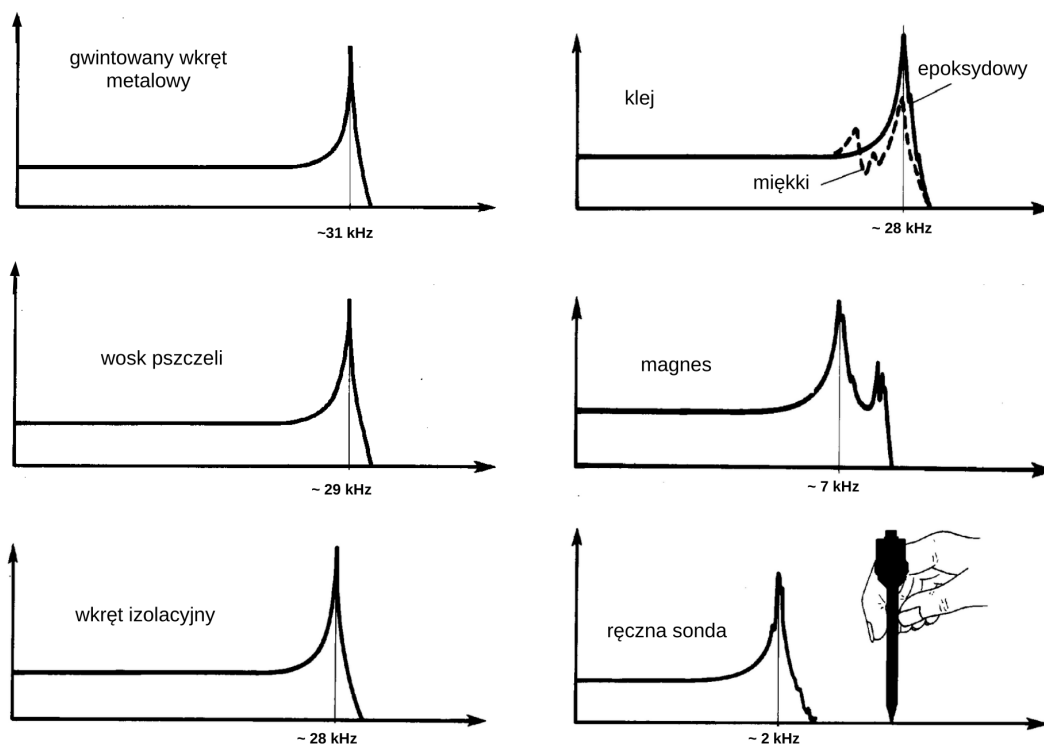


Figure 14: Influence of the method of sensor mounting on the resonant frequency

Piezoelectric acceleration sensors, owing to their advantages, are at present the most frequently used sensors for vibration measurement. They have no moving parts and are therefore durable. They are characterised by a large dynamic range and a wide frequency range. They are relatively inexpensive and reliable in use, and easy to calibrate. They can be mounted in any orientation. Owing to their small dimensions and mass (from 0.2 g) they enable vibration measurements on very small objects. This is quite important, because the mass of the sensor should not exceed 1/10 of the mass of the object. Sensors are produced for general applications, precision sensors, miniature sensors, sensors for high- or low-temperature measurements, shock sensors, and triaxial sensors. Triaxial sensors enable measurement of accelerations in three mutually perpendicular directions.

An essential factor for correct performance of measurements is the method of mounting the sensors on the object under test. Improper (loose) mounting lowers the resonant frequency and therefore narrows the usable frequency range of the sensor, Fig. 14. The optimum method is mounting the sensor by means of screws. The mounting surface should be clean and coated with a thin layer of grease. Other methods of sensor mounting include attachment with beeswax, adhesives or double-sided adhesive tape. On a flat magnetic surface magnets are used for mounting. For preliminary measurements that do not require high accuracy a hand-held probe may be used. Some modern piezoelectric transducers are available with the TEDS (Transducer Electronic Data Sheet) function – electronic technical data of the transducer. A memory circuit built into the transducer contains information about the transducer – its identifier, sensitivity, expiry date of calibration data. In the measuring system automatic detection of the data and their entry into the system take place. This avoids possible errors that can occur when settings are entered “manually” and also facilitates and accelerates preparation of measurements.

2.5 Laser sensors

In vibration measurements it is important that the measuring instrument should not influence the object under test. This is of particular importance when measuring vibrations of objects of small dimensions and masses, where the mass of sensors placed on the object may alter the measurement conditions. In many cases mounting of a sensor is altogether impossible because of the operating conditions of the objects under test.

Laser sensors enable non-contact vibration measurement on practically any surface of the object under test, measurement from a large distance and under difficult ambient conditions, e.g. at high temperatures. Moreover, they can be used for low-frequency vibrations whose measurement by means of accelerometers or electrodynamic seismic sensors is difficult and often impossible. Optical **laser distance meters** (laser triangulation distance meters) and **laser vibrometers** utilising the Doppler effect are employed.

Laser triangulation distance meters enable measurement of both static and dynamic displacements.

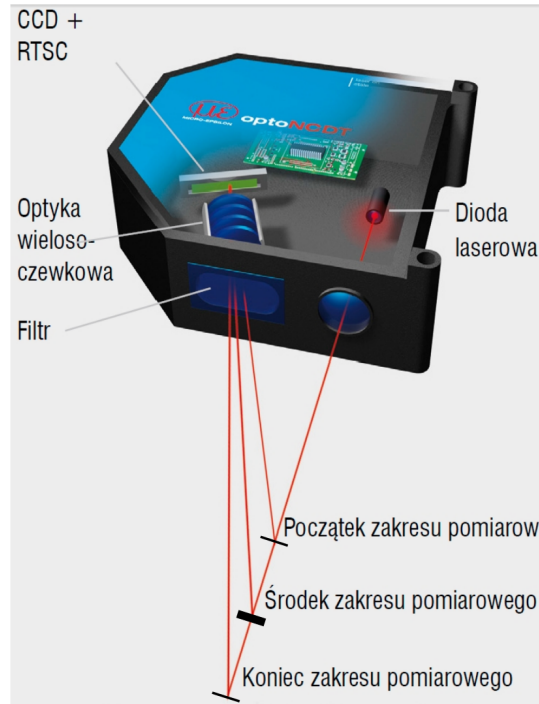


Figure 15: Laser triangulation sensor of the firm Micro-Epsilon

An optical laser distance meter consists of a light source – transmitter – and a receiver – objective (behind which a CCD array is located) of known mutual position and parallel optical axes. The receiver is separated from the transmitter by a fixed distance called the base B and is inclined at an angle α . The light beam reflected from the object at a distance r falls on the CCD detector behind the receiver objective. Knowing the focal length of the objective f and the place of incidence of the light beam on the detector dx we can determine the distance from the

object:

$$r = \frac{B}{\frac{dx}{f} + \frac{1}{\tan \alpha}} \quad (19)$$

The operating principle of a **laser vibrometer** consists in comparing the laser beam reflected from the object under test and reaching the photodetector with a known reference signal. When the wave source or the receiver moves, the frequency of the light changes as a result of the Doppler effect. The frequency of the wave increases (the colour of the light shifts towards violet) when the source and the receiver approach each other, and decreases (the colour of the light shifts towards red) when the wave source and the receiver move apart. By measuring this frequency difference we are able to read the velocity of the object under test directly. Further digital processing of the signal yields information on vibration amplitude, velocity, acceleration and the frequency spectrum.

The frequency of the laser light of the head is expressed by the relation of the speed of light c and the laser wavelength λ :

$$f_0 = \frac{c}{\lambda} \quad (20)$$

The relation between the Doppler frequency f_D , the wavelength λ and the object velocity v is given by the formula:

$$f_D = \frac{2v}{\lambda} \quad (21)$$

The laser vibrometer of the firm Polytec used in the exercise enables non-contact measurement of vibrations of frequency from 0 to 22 kHz, from a distance of 0.2 m to 30 m.

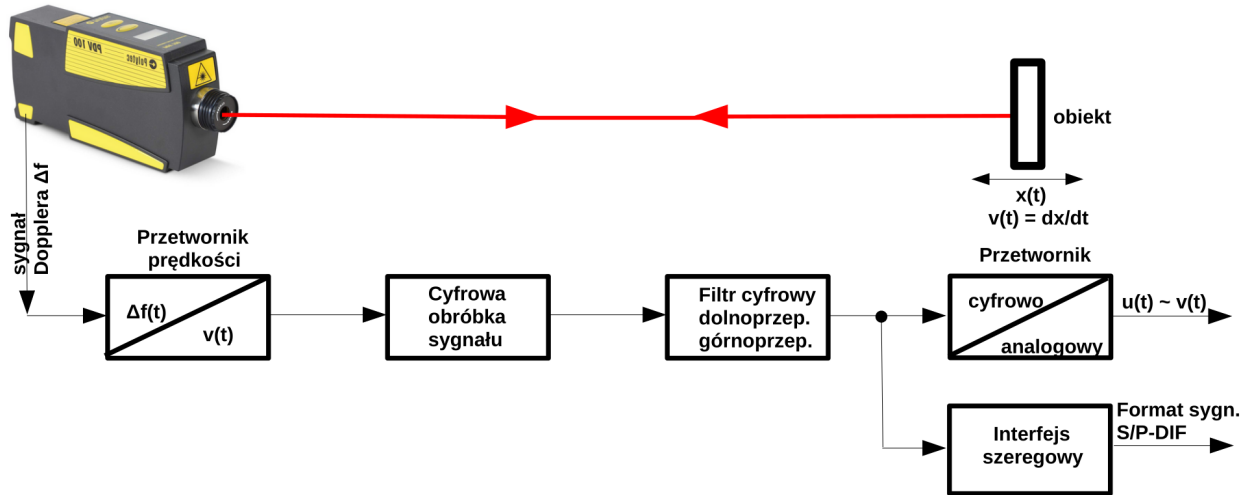


Figure 16: Schematic diagram of velocity measurement with the PDV-100 meter of the firm Polytec

3 Procedure

Carry out vibration measurements on an object selected by the instructor:

1. select an appropriate sensor,
2. calibrate the sensor,
3. measure the vibration parameters (amplitude, frequency),
4. for an electrodynamic sensor apply the correction.

Literature

1. Parszewski Z.: *Dynamics and Vibrations of Machines*, WNT, Warsaw 1992,
2. Hagel R., Zakrzewski J.: *Dynamic Metrology*, WNT, Warsaw 1984,
3. Catalogues of the firms Philips, Brüel & Kjær, PCB, Polytec.

LABORATORY OF MECHANICAL VIBRATIONS

Exercise 2

MEASUREMENT OF VIBRATION PARAMETERS

Group: _____
Team: _____

date _____

Name and surname:

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

1. Measurement of beam vibrations with an electrodynamic sensor:

Type DGA 101

$$k = 30 \text{ mV/mms}^{-1}$$

No.	T	f	ω	U	v	k_f	v_{true}	x_{true}	a_{true}
of meas.	ms	Hz	rad/s	mV	mms ⁻¹	-	mms ⁻¹	mm	ms ⁻²
1									
2									

2. Simultaneous measurement of vibrations with an electrodynamic and a laser sensor:

Type DGA 401

$$k = 30 \text{ mV/mms}^{-1}$$

No.	T	f	ω	U	v	k_f	v_{true}	x_{true}	a_{true}
of meas.	ms	Hz	rad/s	mV	mms ⁻¹	-	mms ⁻¹	mm	ms ⁻²
1									
2									

Type PDV 100

$$k = 25 \text{ mms}^{-1}/\text{V}$$

No.	T	f	ω	U	v	x	a
of meas.	ms	Hz	rad/s	mV	mms ⁻¹	mm	ms ⁻²
1							
2							

Calculations. (Compare the vibration velocity of the object measured with the electrodynamic sensor and with the laser sensor.)

Czujnik elektrodynamiczny DGA101 i DGA401- współczynnik k_f

