IDENTIFICATION OF THE PARAMETERS OF A SINGLE-DEGREE-OF-FREEDOM VIBRATING SYSTEM

Vibrations and Machine Dynamics – Exercise 3*

1 Aim of the exercise

The aim of the exercise is to determine the values of the damping coefficient, the stiffness coefficient and the amplitude of excitation of a single-degree-of-freedom vibrating system.

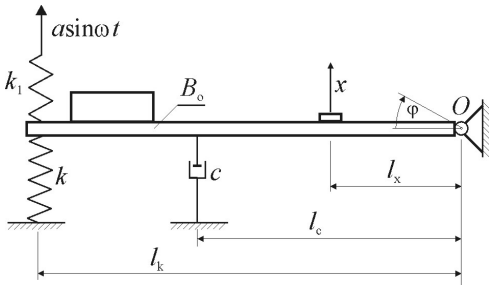


Figure 1: Schematic diagram of the vibrating system

Fig. 1 shows a vibrating system composed of the following elements:

- a rigid beam connected to the base by a revolute joint O,
- a set of springs of stiffness coefficient k,
- an oil damper of damping coefficient c,
- a set of springs of stiffness coefficient k_1.

The set of springs of stiffness coefficient k_1 is connected to an eccentric on the shaft of the driving motor; rotation of the motor shaft constitutes a kinematic excitation of the motion of the upper ends of these springs, described as $a \sin \omega t$. As a result of the excitation the beam is deflected from the equilibrium position by an angle ϕ. The deflection of the beam is measured by a linear displacement sensor that determines the displacement x of a point of the beam located at a distance l_x from the axis of rotation. The equation of linear motion of the physical model of the system under investigation is as follows:

$$B_0\ddot{\phi} + cl_c^2\dot{\phi} + (k + k_1)\phi = k_1 l_k a \sin \omega t \tag{1}$$

where: B_0 – moment of inertia of the beam with respect to the axis of rotation. Dividing equation (1) by B_0 and multiplying by l_x yields:

$$l_x\ddot{\phi} + \frac{cl_c^2}{B_0} l_x\dot{\phi} + \frac{k + k_1}{B_0} l_x\phi = \frac{k_1 l_k a l_x}{B_0} \sin \omega t \tag{2}$$

*Author K. Czołczyński, In: Laboratory of Mechanical Vibrations, K. Marynowski (ed.), Łódź 2002

Denoting:

$$l_x\ddot{\phi} = \ddot{x}, \quad \frac{cl_c^2}{B_o} = 2h, \quad \frac{k+k_1}{B_o} = \alpha^2, \quad l_x\phi = x, \quad \frac{k_1 l_k a l_x}{B_0} = q \tag{3}$$

equation (1) can be written in the form:

$$\ddot{x} + 2h\dot{x} + \alpha^2 x = q\sin\omega t \tag{4}$$

where:

$2h$ – damping,
α^2 – circular frequency of free oscillations of the system,
q – kinematic excitation.

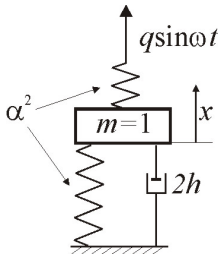


Figure 2: Physical model of the system

A particular solution of the differential equation (4), describing the forced oscillations of the physical model of the system under investigation shown in Fig. 2, is the function:

$$x(t) = A\sin(\omega t + \beta), \tag{5}$$

where A is the amplitude of the forced oscillations and β is the phase angle between the kinematic excitation and the oscillations of the physical model. The amplitude A as a function of the excitation frequency ω is given by the formula:

$$A(\omega) = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}} \tag{6}$$

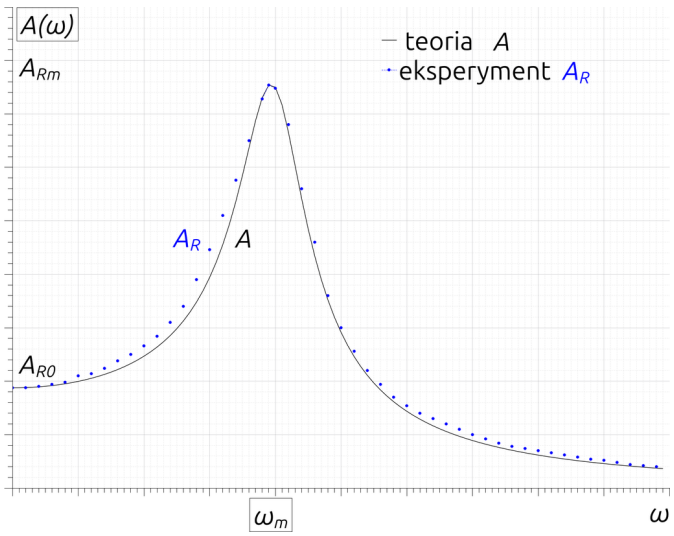


Figure 3: Resonance curves

and its graph has the shape of the resonance curve shown by the continuous line in Fig. 3. Determination of the values of the parameters α, h and q in equation (6) is a task that can be solved in the manner described below.

By measuring the amplitude of oscillations of the real system at various values of the circular frequency of excitation one obtains the real resonance curve $A_R(\omega)$, shown in Fig. 3 as (blue) points. The value of the amplitude of oscillations of the real system for a small (practically equal to zero) value of the circular frequency ω is denoted by A_{R0}. The amplitude attains its maximum value A_{Rm} at the resonance frequency ω_m. The theoretical resonance curve shown in Fig. 3 by the continuous line results from calculations of the amplitude of oscillations $A(\omega)$ by means of formula (6).
It is assumed that both resonance curves must satisfy three conditions:

1. For a circular frequency ω close to zero the amplitudes A and A_R are the same:

$$A(0) = \frac{q}{\sqrt{(\alpha^2 - 0^2)^2 + 4h^2 \cdot 0^2}} = \frac{q}{\alpha^2} = A_{R0} \tag{7}$$

2. For the resonance circular frequency ω_m the amplitudes A and A_R are also the same:

$$A(\omega_m) = \frac{q}{\sqrt{(\alpha^2 - \omega_m^2)^2 + 4h^2\omega_m^2}} = A_{Rm} \tag{8}$$

3. At the frequency ω_m the amplitude A attains its maximum value (just as A_R does), therefore:

$$\frac{\partial A}{\partial \omega}(\omega = \omega_m) = \frac{q[-4\omega(\alpha^2 - \omega^2) + 8h^2\omega]}{2\sqrt{((\alpha^2 - \omega_m^2)^2 + 4h^2\omega^2)^3}} = 0 \tag{9}$$

The system of equations (7), (8), (9) can easily be transformed into the form:

$$q = \frac{\omega_m^2 A_{R0}}{\xi}, \quad \alpha^2 = \frac{\omega_m^2}{\xi}, \quad 2h = \omega_m \sqrt{\frac{1}{2\xi} - 1} \tag{10}$$

where:

$$\xi = \sqrt{1 - \frac{A_{R0}^2}{A_{Rm}^2}} \tag{11}$$

which permits calculation of the unknown values of the parameters α, h and q. Knowledge of these values allows identification of the **parameters of the real system** on the basis of the formulae:

$$c = \frac{2hB_0}{l_c^2}, \quad k_1 = \frac{qB_0}{al_k l_x}, \quad k = B_0\alpha^2 - k_1 \tag{12}$$

Measurements and calculations performed earlier made it possible to establish the values of the remaining required parameters:

$$B_0 = 1.38 \text{ kgm}^2, \quad a = 0.003 \text{ m}, \quad l_c = 0.541 \text{ m}, \quad l_k = 0.54 \text{ m}, \quad l_x = 0.24 \text{ m}\,. \tag{13}$$

Procedure:

1. Measure the amplitude of oscillations A_R for various values of the angular velocity ω; the number of measurements is about 15. Establish the value of ω_m at which the amplitude of oscillations attains its maximum value A_{Rm}. Establish the value of the amplitude A_{R0} at an angular velocity close to zero. Enter the results in the table.

2. Calculate the values of the parameters α, h and q using formulae (10).

3. Calculate the values of the amplitude A of the theoretical resonance curve using formula (6) for those values of ω at which A_R was measured.

4. Draw the real and the theoretical resonance curves.

5. Calculate the values of the parameters of the real system k, c and k_1 using formulae (12) and the values of the parameters given in formula (13).

LABORATORY OF MECHANICAL VIBRATIONS

Exercise 3

IDENTIFICATION OF THE PARAMETERS OF A SINGLE-DEGREE-OF-FREEDOM VIBRATING SYSTEM

Group: _____ **date** _____
Team: _____

Name and surname:

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Calculation results:

$\omega_m =$ _____ [rad/s] $A_{Rm} =$ _____ [m] $A_{R0} =$ _____ [m]

Parameters of the physical model:

$\xi = \sqrt{1 - \frac{A_{R0}^2}{A_{Rm}^2}} =$ _____ [ ] $q = \frac{\omega_m^2 A_{R0}}{\xi} =$ _____ [ ]

$\alpha^2 = \frac{\omega_m^2}{\xi} =$ _____ [ ] $2h = \omega_m \sqrt{\frac{1}{2\xi} - 1} =$ _____ [ ]

Amplitude of oscillations of the physical model

$A = \frac{q}{\sqrt{(\alpha^2 - \omega^2)^2 + 4h^2\omega^2}} =$ _____ [m]

Parameters of the real system:

$c = \frac{2hB_0}{l_c^2} =$ _____ [rad/s] $k_1 = \frac{qB_0}{al_k l_x} =$ _____ [N/m] $k = B_0\alpha^2 - k_1 =$ _____ [N/m]

Measurement results:

ω	A_R	A

