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STABILITY ANALYSIS OF AXIALLY MOVING SYSTEM BY USING A ROTOR'S MODEL

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Abstract

The vibration and stability of two dimensional axially moving plate have been investigated. On the base of a simple rotor system description a general velocity proportional damping force is added to the moving plate equation of motion. Approximate solution is obtained using the Galerkin method. Numerical results are presented which show the contributions of axial velocity, the wheel support system, external and internal damping to stability of a moving paper web.

1. Introduction

The class of axially moving continua, in the form of a thin, flat rectangular materials encompasses such systems as band saw blades, power transmission belts, magnetic tapes, plastic sheets and paper webs. Above a critical speed, the axially moving material experiences divergent or flutter instability. For instance, flutter of a paper web degrades quality, increases defects and can lead to breakage of the web. Thus, characterisation of the vibration and dynamic stability of such systems is requisite for the analysis and optimal design of technological devices.

From the dynamic point of view the translating systems and rotating systems like rotors form together the class of gyroscopic systems. Vibration and stability problems of rotating systems received considerable attention in design of rotors within the last century. One can find great many references in this field (e.g. [1] - *review*).

On the other hand a lot of earlier works on dynamic problems of translating materials focused on dynamic investigations of string-like and beam-like axially moving systems (e.g. [4] - *review*, [5]). In the case of two-dimensional, axially moving web, the exact dynamic solutions, satisfying the non-linear, coupled equations governing the web's motion, probably cannot be determined in closed form. Recent works in this field use approximate solutions and focus on undamped cases [6], [7].

The aim of this paper is stability analysis of axially moving plate when internal and external damping is taken into account. The damping model is defined on the base of simple rotor system description.

2. Mathematical model of gyroscopic system

The vibrations of rotating or translating elastic systems are described by a system of ordinary differential equations of the matrix form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{1}$$

Equations of the form (1) are obtained by direct lumping of the mass, damping and stiffness properties of the elastic system, or by discretization of the partial differential equations derived based on distributed properties. The important features of these systems is that the matrices **C** and **K** depend on the constant velocity of rotation and translation of the elastic system. These velocity dependent matrices arise due to coriolis and centripetal acceleration effects.

2.1. Rotating shaft

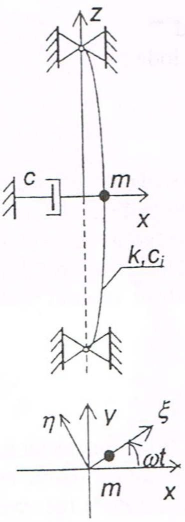

Fig. 1. Rotor system.

A rotor system where the mass m is mounted on the vertical shaft of the flexural stiffness k is shown in Fig.1. The rectangular stationary coordinate system xyz whose z axis coincides with the bearing center line is adopted. The movable coordinate system $\xi\eta z$ is connected with the mass-point m. The angular velocity of the rotor ω is constant. The motion of the mass is damped by external damping which is characterized by viscous damping coefficient c_e and by internal damping, which is characterized by the coefficient c_i.

It is well-know the equations of transverse vibrations of rotating mass m in the movable coordinate system [1]

$$\begin{aligned} m\ddot{\xi} - 2m\omega\dot{\eta} + \beta_1\dot{\xi} - \beta_2\omega\eta + (k - m\omega^2)\xi = 0 \\ m\ddot{\eta} + 2m\omega\dot{\xi} + \beta_1\dot{\eta} + \beta_2\omega\xi + (k - m\omega^2)\eta = 0 \end{aligned} \tag{2}$$

where:

$$\beta_1 = c_i + c_e; \quad \beta_2 = c_e$$

Thus, one can define the matrices in (1)

$$\mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}; \ \mathbf{C} = \begin{bmatrix} \beta_1 & -2m\omega \\ 2m\omega & \beta_1 \end{bmatrix}; \ \mathbf{K} = \begin{bmatrix} k - m\omega^2 & -\beta_2\omega \\ \beta_2\omega & k - m\omega^2 \end{bmatrix}; \ \mathbf{q} = \begin{bmatrix} \xi \\ \eta \end{bmatrix} \tag{3}$$

2.2. Axially moving plate

The transverse vibrations of transmission belts, band saws and paper tapes can be represented using an axially moving plate model as shown in Fig.2.

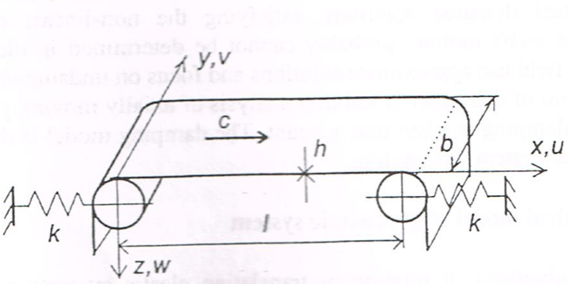

Fig. 2. Axially moving plate.

The equation of the transverse vibration for the undamped case is given by [3]

$$\rho h w_{,tt} + 2\rho h c w_{,xt} + (\kappa\rho h c^2 - P) w_{,xx} + D w_{,xxxx} + 2(\nu D + D_1) w_{,xxyy} + D w_{,yyyy} = 0 \quad (4)$$

where:

- w - the transverse displacement of the plate;
- ρ - density of the plate;
- h - thickness of the plate;
- c - constant axial velocity;
- κ - rolls support constant, $0 \le \kappa \le 1$;
- P - initial static tension in x-direction;
- $D = (Eh^3)/[12(1-\nu^2)]$ - flexural stiffness of the plate,
- $D_1 = Gh^3/6$ - plate's stiffness coefficient;
- E - Young's modulus of the plate,
- G - modulus of non-dilatational strain of the plate,
- ν - Poisson's ratio.

The boundary conditions at the free edges $y = 0$ and $y = b$, are

$$w_{,yyy} + (2-\nu) w_{,xxy} = 0 \; ; \quad w_{,yy} + \nu w_{,xx} = 0 \quad (5)$$

and at the simply supported edges $x = 0$ and $x = l$, are

$$w = 0 \; ; \quad w_{,xx} + \nu w_{,yy} = 0 \quad (6)$$

On the base of the rotor's equations (2) a general velocity proportional damping force of the form $(\beta_1 w_{,t} + \beta_2 c w_{,x})$ is added to the left-hand side of (4) in the modelling of actual damping

$$\rho h w_{,tt} + 2\rho h c w_{,xt} + (\rho h c^2 - P) w_{,xx} + D w_{,xxxx} + 2(\nu D + D_1) w_{,xxyy} + \\ + D w_{,yyyy} + \beta_1 w_{,t} + \beta_2 c w_{,x} = 0 \quad (7)$$

To reduce (7) to the form (1) one can utilize the Galerkin's method and a finite series representation of the transverse displacement

$$w(x,y,t) = \sum_{i=1}^{m} \sum_{j=1}^{n} \phi_i(y) \phi_j(x) g_j(t) \quad (8)$$

where: $\phi_i(y)$, $\phi_j(x)$ - approximating functions satisfies boundary conditions,
$g_j(t)$ - the generalized coordinates.

Assuming approximating functions ϕ and utilizing Hamilton's Principle the elements of the matrices in (1) can be evaluated.

$$m_{ij} = \rho A \int_0^l \int_0^b \phi_i \, \phi_j \, dx \, dy$$

$$c_{ij} = \int_0^l \int_0^b (2\rho A c \phi_i \, \phi_{,j'x} + \beta_1 \phi_i \, \phi_j) dx \, dy \quad (9)$$

$$k_{ij} = \int_0^l \int_0^b [(\kappa \rho A v^2 - P) \phi_i \, \phi_{j'xx} + D \phi_i \, \phi_{j'xxxx} + D \phi_i \, \phi_{j'xxxx} - 2(\nu D + D_1) \phi_{i'xx} \phi_{j'yy} + \\ + D \phi_i \, \phi_{j'yyyy} + c\beta_2 \phi_i \phi_{j'x}] dx \, dy$$

The resulting n second order ordinary differential equations can be obtained. These equations are approximate in nature because of a finite series representation in

(6). For $n = 3$, $m = 1$, sinusoidal approximating function and plane stress the **M**, C and **K** matrices in (1) have the following form

$$\mathbf{M} = \begin{bmatrix} \rho A\frac{l}{2} & 0 & 0 \\ 0 & \rho A\frac{l}{2} & 0 \\ 0 & 0 & \rho A\frac{l}{2} \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} \beta_1\frac{l}{2} & -\frac{8\rho Ac}{3} & 0 \\ \frac{8\rho Ac}{3} & \beta_1\frac{l}{2} & -\frac{24\rho Ac}{5} \\ 0 & \frac{24\rho Ac}{5} & \beta_1\frac{l}{2} \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} \frac{\pi^4}{2l^3}D - \frac{\pi^2}{2l}(\kappa\rho Ac^2 - P) & -\frac{4c\beta_2}{3} & 0 \\ \frac{4c\beta_2}{3} & \frac{8\pi^4}{l^3}D - \frac{2\pi^2}{l}(\kappa\rho Ac^2 - P) & -\frac{12c\beta_2}{5} \\ 0 & \frac{12c\beta_2}{5} & \frac{81\pi^4}{2l^3}D - \frac{9\pi^2}{2l}(\kappa\rho Ac^2 - P) \end{bmatrix} \tag{10}$$

The complex eigenvalues of the system are determined in the form

$$\lambda_j = \sigma_j + i\omega_j \qquad j = 1, 2 \ldots n\,;\; i = \sqrt{-1} \tag{11}$$

and can be obtained from the eigenvalue problem associated with the equation (1).

3. Numerical analysis

Numerical calculations of natural frequencies of the moving plate system have been carried out for the following data of an actual paper web [6]: length $l = 1.194\ m$, width $b = 0.597\ m$, thickness $h = 0.3$ mm, static tension $P = 32.835\ N$, density $\rho = 133.33\ kg/m^3$, Young's modulus $E = 5\ 10^9\ N/m^2$, Poisson's ratio $\nu = 0.3$.

At first to test the accuracy of the computational method the eigenvalues of undamped moving steel string ($\rho = 7800\ kg/m^3$, $P = 15\ N$, $l = 1\ m$, $d = 0.001\ m$) have been calculated for $n = 3$ and compared with the exact solution [2]

$$\omega_i = \frac{i\pi}{l}\sqrt{\frac{P}{\rho A}}(1 - \frac{\rho Ac^2}{P}) \tag{12}$$

Results of calculations are shown in Fig.3. For the first eigenvalue the discrepancy is less then 1%. Thus, three approximating functions have been used in next numerical calculations.

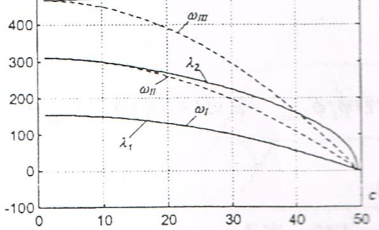

Fig. 3. Eigenvalues of the moving string.

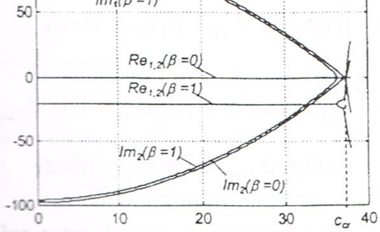

Fig. 4. Eigenvalues of the moving paper web.

For the moving paper web the plots of first two eigenfrequencies of the undamped system ($\beta = 0$) and for the case when only external damping is taken into account ($\beta_1 = \beta_2 = \beta = 1$) are shown in Fig.4. For the undamped system the axial velocity c diminishes absolute values of all imaginary eigenvalues until they vanish at the critical value c_{cr}. For damped case ($\beta = 1$) conjugate complex eigenvalues of the system appear and for the critical axial velocity value the real part of the first eigenvalue passes through zero. That means divergence type of instability.

The effects of internal and both external and internal damping on dynamic behaviour of the moving paper web are shown in Fig.5 and Fig.6, respectively. In wide considered range of damping significant differences of critical axial velocity value have not been observed. All above plots have been obtained for the rolls support constant value $\kappa = 1$.

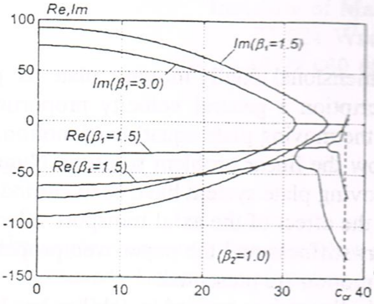

Fig. 5. First eigenvalues of the paper web.

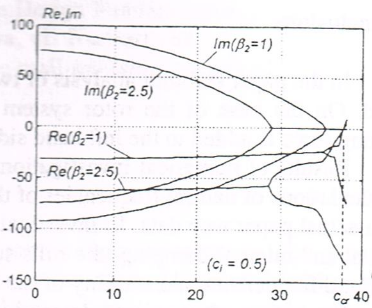

Fig. 6. First eigenvalues of the paper web.

Furthermore, the effect of the rolls support stiffness on the critical axial velocity was investigated. For the rolls support stiffness $\kappa = 0$, the rolls are free to displace relative to each other under web tension variation. For $\kappa = 1$, the two rolls are rigidly fixed with respect to each other, eliminating web tension increase with the axial transport speed c. For $0 < \kappa < 1$, the rolls support system has finite stiffness and the axial tension decreases with c. The plot of the critical axial velocity of the undamped paper web c_{cr} versus the rolls support stiffness value κ is shown in Fig. 7. The critical velocity is considerably dependent on the rolls support stiffness.

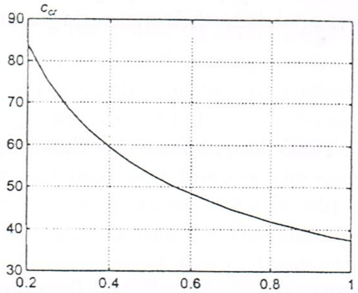

Fig. 7. Critical axial velocity of the web.

The effects of the moving paper web properties such as Young's modulus value E and thickness of the web h on the critical axial velocity c_{cr} are illustrated in Figures 8 and 9, respectively. The values of E and h were changed in the range $\pm 10\%$ with respect to their nominal values, which have been presented in the beginning of this Chapter. The range $\pm 10\%$ corresponds to measuring accuracy of the paper web properties. In the considered ranges the critical axial velocity is nearly independent on Young's modulus and thickness of the moving paper web.

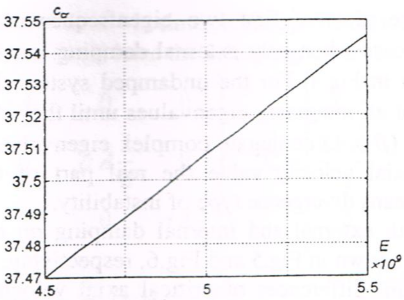

Fig. 8. Critical axial velocity of the web.

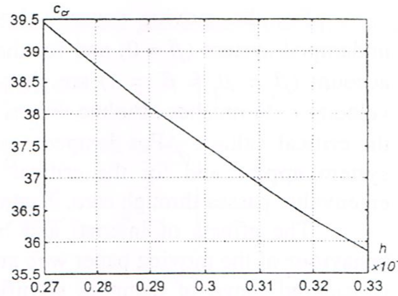

Fig. 9. Critical axial velocity of the web.

4. Conclusions

In the paper stability analysis of two dimensional axially moving plate is presented. On the base of the rotor system description a general velocity proportional damping force is added to the left hand side of the moving plate equation of motion.

Results of numerical investigations show the linear problem solution. Numerical calculations of natural frequencies of the moving plate system have been carried out for an actual paper web data. In investigations the effect of the axial transport velocity, external and internal damping, the rolls support stiffness and the paper web properties on natural frequencies and stability of the web motion are presented.

For both undamped and damped system divergence type of instability has been observed. In the case of undamped system the lowest natural frequency decrease with increasing the axial velocity at the rate mainly dependent on the rolls support stiffness. In wide considered ranges of damping coefficients significant differences of critical axial velocity have not been observed. The effects of the paper web properties such as Young's modulus and thickness in the range corresponds to their measuring accuracy on the critical axial velocity may be neglected.

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