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DYNAMICS OF THE IMPACT FORCE GENERATOR

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Abstract

During the last years the interest of scientists in multibody mechanical systems in which the phenomenon of impact occurs has been still growing. Numerous investigations are carried out on the possibility of employing this phenomenon to increase the efficiency of operation of industrial machines or to eliminate those phenomena which are undesirable during machine operation. In the present paper a principle of operation of the impact force generator being an element of the rotor of the heat exchanger has been presented. Step disturbances of the rotational velocity of the rotor caused by the generator are aimed at intensification of the heat exchange process.

Introduction

In numerous industrial machines the impact of their movable parts is either the basic principle of their operation or the effect which improves their operating efficiency. The classic examples of such machines or devices are: a pneumatic hammer, impact dampers, or heat exchangers. One of many research projects conducted at the Technical University of Łódź deals with a construction of a new heat exchanger characterised by a higher coefficient of efficiency than that of heat exchangers being now produced [1]. One of the factors that contribute to intensification of the heat exchange process are disturbances in the rotational velocity of the rotor of the heat exchanger. These disturbances can have a character of forced torsional vibrations of the rotating rotor, which causes that the angular velocity of the rotor oscillates around a constant average value. These oscillations can have (depending on the assumed parameters of their generator) a harmonic, quasi-periodical or even chaotic character [2].

Another type of disturbances which can result in intensification of the heat exchange are step disturbances of the rotational velocity. The simplest way to generate such disturbances is to employ the phenomenon of impact which causes, according to the Newton's hypothesis, step variations of the velocity of the bodies impacting on each other.

Below a physical and mathematical model of the generator of step disturbances of the angular velocity of the rotor has been presented and a principle of its operation has been discussed. Two selected aspects of the generator operation have been shown: the self-synchronisation phenomenon and the influence of the damping coefficient on the velocity of the rotor.

Physical model of the generator

The object of considerations is a system composed of two main parts (Figure 1):
- a rotor equipped with a fender, driven by an electric engine,

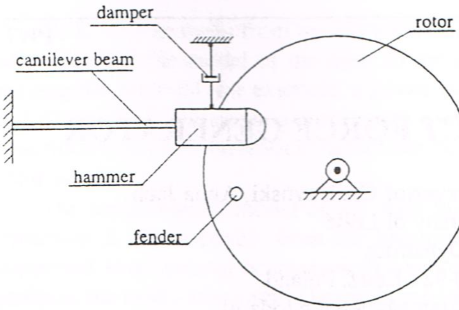

Fig. 1. Schematic diagram of the generator

- a hammer in the form of a cylinder with a semicircular end, mounted on the end of a cantilever beam.

During the operation of the generator, the rotor fender impacts on the hammer, which causes vibrations of the hammer on one hand and the desired step variations of the rotational velocity of the rotor on the other hand. The hammer vibrations are suppressed by a viscous damper. The mass of the cantilever beam and the damper has been neglected. The drive engine is connected to the rotor by means of a massless elastic shaft. The aim of this shaft is to minimise undesirable variations of the engine rotational velocity.

Equations of motion

Equations of motion of the hammer, written in the Cartesian system of coordinates which is connected with the centre of gravity of the hammer in the static equilibrium position, are as follows:

$$\begin{bmatrix} m & 0 \\ 0 & B \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\phi} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} y - a\phi \\ \phi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (1)$$

where:

$$[K_{ij}] = \begin{bmatrix} \dfrac{12EI}{l_s^3} & \dfrac{-6EI}{l_s^2} \\ \dfrac{-6EI}{l_s^2} & \dfrac{4EI}{l_s} \end{bmatrix}$$

In the above equations the fact is taken into account that the point in which the spring is joined to the mass of the hammer is in the distance a from the gravity centre.

As it is very difficult to define the real character and intensity of damping of the vibrations, the matrix of damping has a simplified form: the vibrations are damped by a viscous damper acting only in the y direction. The damping coefficient C_{11} is calculated from the simplified formula

$$C_{11} = \frac{2m\Delta}{T} \approx \frac{2m\Delta}{2\pi}\,\alpha_1 \qquad (2)$$

where α_1 is the basic natural frequency of free undamped vibrations of the hammer, and Δ is the assumed logarithmic decrement of damping. Because of this simplification, during all the numerical simulations, much attention should be paid to the influence of this decrement on the motion of the system.

Equations of motion of the rotor, written in the coordinate system connected with the axis of its rotation, are as follows:

$$\begin{bmatrix} B_r & 0 \\ 0 & B_e \end{bmatrix} \begin{Bmatrix} \ddot{\phi}_r \\ \ddot{\phi}_e \end{Bmatrix} + \begin{bmatrix} C_\phi & -C_\phi \\ -C_\phi & C_\phi \end{bmatrix} \begin{Bmatrix} \dot{\phi}_r \\ \dot{\phi}_e \end{Bmatrix} + \begin{bmatrix} K_\phi & -K_\phi \\ -K_\phi & K_\phi \end{bmatrix} \begin{Bmatrix} \phi_r \\ \phi_e \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_e(\dot{\phi}_e) \end{Bmatrix} \tag{3}$$

where

$$K_\phi = \frac{GI_0}{l_\phi}, \quad C_\phi = \frac{2B_r \Delta_\phi}{2\pi} \sqrt{\frac{B_r}{K_\phi}}$$

The coefficient Δ_ϕ is the assumed logarithmic decrement of damping of the torsional vibrations. Because of small (less than a few per cent) changes in the rotational velocity of the engine, the driving moment M_e is taken from the static characteristic of the engine.

Equations of impacts

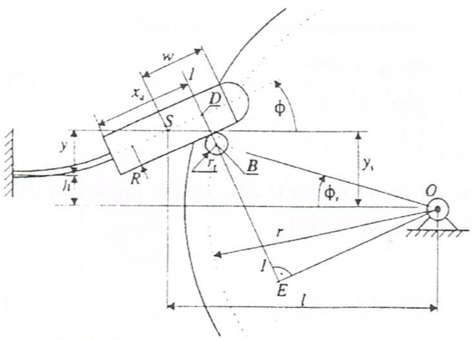

Fig. 2. Geometry of generator - impact of I kind

As the geometry of the generator is rather sophisticated, two various kinds of impacts occur during its operation:

I - the fender collides with the cylindrical part of the hammer, and the line of impact is perpendicular to the hammer axis (Figure 2),

II - the fender collides with the spherical part of the hammer, and the line of impact goes through the centre of the fender and the centre of the basis of the spherical part of the hammer (Figure 3).

As can be seen in Figure 2, the impact of the first kind occurs when the following condition is fulfilled:

$$l < w\cos\phi + (\mathrm{sgn}(y_D - y_B))(r_f + R)\sin\phi + r\cos\phi_r \tag{4}$$

The impact of the second kind occurs when

$$w\cos\phi + (\mathrm{sgn}(y_D - y_B))(r_f + R)\sin\phi + r\cos\phi_r \le l < w\cos\phi + r_f + R + r\cos\phi_r \tag{5}$$

Finally, when

$$w\cos\phi + r_f + R + r\cos\phi_r < l \tag{6}$$

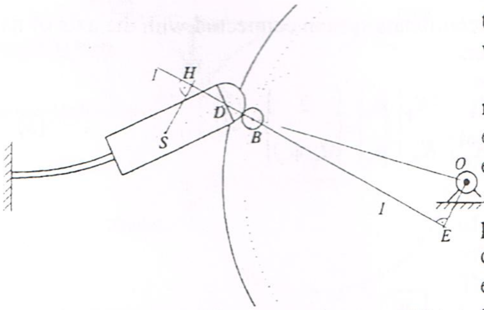

Fig. 3. Geometry of generator - impact of II kind

the hammer and the fender miss each other without impact.

During the numerical simulation of the free motion of both the hammer and the rotor, after each step of the Runge-Kutta procedure, the equation of the line of the potential impact (line *1-1* in Figure 4) and the coordinates of the potential points of collision F_h and F_r are calculated. The change of the sign of the expression (y_{Fh} - y_{Fr}) means that during the last time step the hammer and the fender started to overlap each other (the pass from Figure 4b to Figure 4c). As such a situation is not possible, the procedure comes back one time step (Figure 4b), and then goes ahead with a smaller step of time. The position of the system presented in Figure 4b, for which the distance between points F_h and F_r is smaller than the assumed one (in the described simulations: 0.001R), is treated as the time and the place in which the impact occurs.

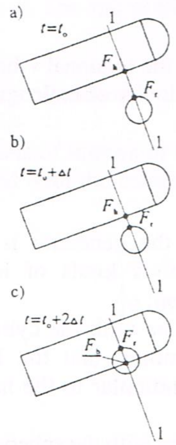

Fig. 4. Finding the point of the impact

The equations describing the impact, following the Newton's law, are as follows:

- for the hammer:

$$m(v'_{sn} - v_{sn}) = S$$
$$B(\dot{\phi}' - \dot{\phi}) = Sl_z \qquad (7)$$

(l_z=l_{SD} for the impact of the I kind or (l_z=l_{SH} for the impact of the II kind - Figures 2 and 3 respectively)

- for the rotor:

$$B_r(\dot{\phi}_r' - \dot{\phi}_r) = -Sl_{EO} \qquad (8)$$

- equation of the velocities of the collision points of the hammer and the fender:

$$v'_{Fhn} - v'_{Fwn} = -k(v_{Fhn} - v_{Fwn}) \qquad (9)$$

The set of equations (7), (8) and (9) allows one to calculate the velocities of the collision points F_h and F_r, and, after a few simple transformations, this set enables one to determine the initial conditions for both the hammer and the rotor motion after the impact.

Data of the system

The next part of this paper presents the results of the numerical simulation of the motion of the system, whose basic data are as follows:

- the hammer:

B=12.14×10^{-6} kgm^2, m=89.83×10^{-3} kg, R=0.01 m, l_s=0.03 m, E=0.206×10^{12} Nm^{-2}, I=63.6×10^{-12} m^4

- the rotor:

B_r=0.01 kgm², B_e=0.015 kgm², r=0.01m, r_T=0.0025m, Δ_ϕ=ln(2), k_s=0.25×10³ Nm, l=0.097 m, h=0.05 m.

Principle of operation

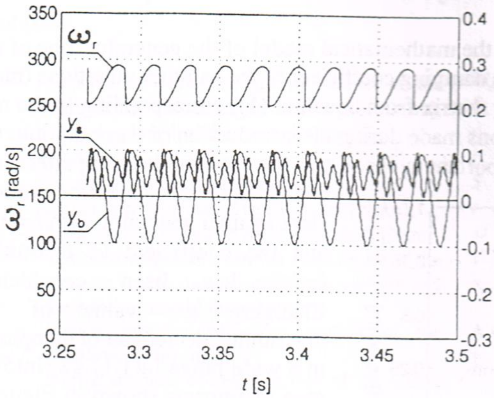

Fig. 5. Time diagram - stable motion of the generator

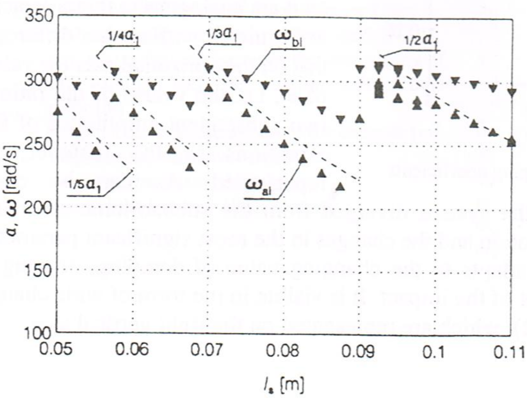

Fig. 6. Map of impacts for various lengths of the spring

A time diagram representing the stable motion of the hammer and the rotor (in this example the length of the cantilever beam l_s=0.08 m, the log decrement Δ=ln(1.5) and the coefficient of restitution k=0.9) is shown in Figure 5. The time of the motion is represented on the horizontal axis; y_s- the vertical coordinate of the centre of the hammer mass (thick line) and y_B - the vertical coordinate of the centre of the fender (thin line) are plotted on the right vertical axis; the angular velocity of the rotor ω_r is shown on the left vertical axis. As can be seen, during each rotation of the rotor, its fender collides with the hammer causing the hammer vibrations and step variations of the angular velocity of the rotor.

Figure 6 presents a map of impacts for many generators which differ in the length of the cantilever beam l_s. The values of the angular velocity of the rotor before (ω_{bi}) and after (ω_{ai}) impacts and, additionally, the curves showing the values of the fractions of the basic frequency of free vibrations of the hammer: $1/2\alpha_1$, $1/3\alpha_1$, $1/4\alpha_1$ and $1/5\alpha_1$ have been shown on this map.

In Figure 6, a close relation between the variations of both the velocities ω_{ai} and ω_{bi} and the function of α_1 can be easily observed. For instance: when l_s=0.07 m, the rotor velocity ω_{bi} (close to its average angular velocity ω_a) is equal to approximately 1/3 of the value of the basic frequency of free vibrations of the hammer α_1. It means that the impact forcing of these vibrations has a subharmonic frequency. Under such conditions the generation of the hammer vibrations occurs at minimal values of the impulse S (see equations 7 and 8). As a consequence, the values of ω_{bi} and ω_{ai} are high, while the difference between them - minimal. An increase in the cantilever beam length causes of course a decrease in the value of $1/3\alpha_1$. Despite it, however, the hammer "wants" the impact forcing of its free vibrations to have a subharmonic character: the impacts are stronger and stronger, which, as a consequence, causes the diminishing of both ω_{bi} and ω_{ai}. This way of affecting the angular velocity by the hammer has been called the self synchronisation of the system. When l_s exceeds the value 0.0875 m, the conditions for easy generation of the hammer vibrations with the next subharmonic frequency $1/2\alpha_1$, arise and the situation repeats.

As can be seen, a choice of the length of the cantilever beam makes it possible to control both the average velocity of the rotor and the intensity of the impacts (the difference ω_{bi}-ω_{ai}).

Influence of the damping coefficient on the dynamical behaviour of the system

During the determination of parameters of the mathematical model of the generator, one of the biggest difficulties is to define a value of the damping coefficient of the hammer vibrations (more strictly - values of the terms of the damping matrix from equation (1)) corresponding to the real value. In the face of significant simplifications made during the introduction of damping into the equations of motion of the hammer, it is important to check how the value of damping affects the character of the generator motion. Here again a generator with l_s=0.08 and the coefficient of restitution k=0.9 has been considered. Changing the value of the logarithmic decrement of damping Δ in a wide range $\ln(1.1) \leq \Delta \leq \ln(5)$, a map of impacts shown in Figure 7 has been prepared. The values seen on it are analogous to those depicted in Figure 6, with such a difference that on the horizontal axis the values of e^{Δ}, i.e. the values of the ratio of two subsequent amplitudes of free vibrations of the hammer, are represented. As can be easily observed, the dynamic behaviour of the system diverged from the subharmonic resonance maintains the regular character of the motion and the changes in the most significant parameters ω_{bi} and ω_{ai} are slight. The generator adapts to the changing value of damping, altering its geometrical configuration at the moment of the impact. It is visible in the form of wide changes in the parameters ϕ_r and y_s (cf. Figure 2), which are represented on the right vertical axis.

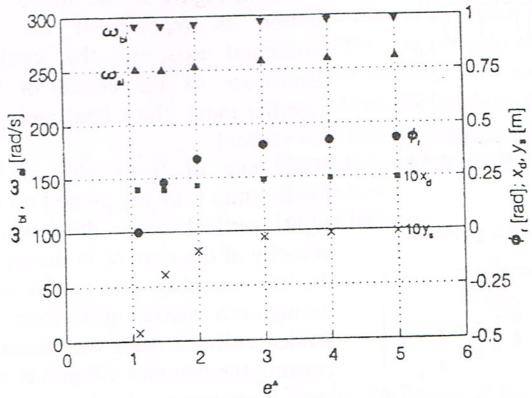

Fig. 7. Map of impacts for various damping coefficients

Conclusions

The object of the numerical investigations presented here is a mechanical impact force generator. During the investigations it was found that a proper choice of the value of the basic frequency of free vibrations of the hammer it was possible to obtain desired changes both in the average rotational velocity of the generator rotor, and in the intensity of impacts, which is expressed as a difference between the value of the angular velocity of the generator rotor before and after the impact. In the system under investigation, the phenomenon of self synchronisation, which consists in the fact that the generator hammer forces such an angular velocity of the rotor at which the motion of both the hammer and the rotor is regular, has been observed.

The values of one of most difficult to determine coefficients, i.e. the coefficient of damping, do not exert a significant influence on the character of the motion of the rotor. The system reacts to the changes in its value by, first of all, a change in the configuration at the moment of the impact. This last feature of the system lets us think that the results obtained by means of numerical experiments are not far from those which could be obtained in laboratory conditions.

References

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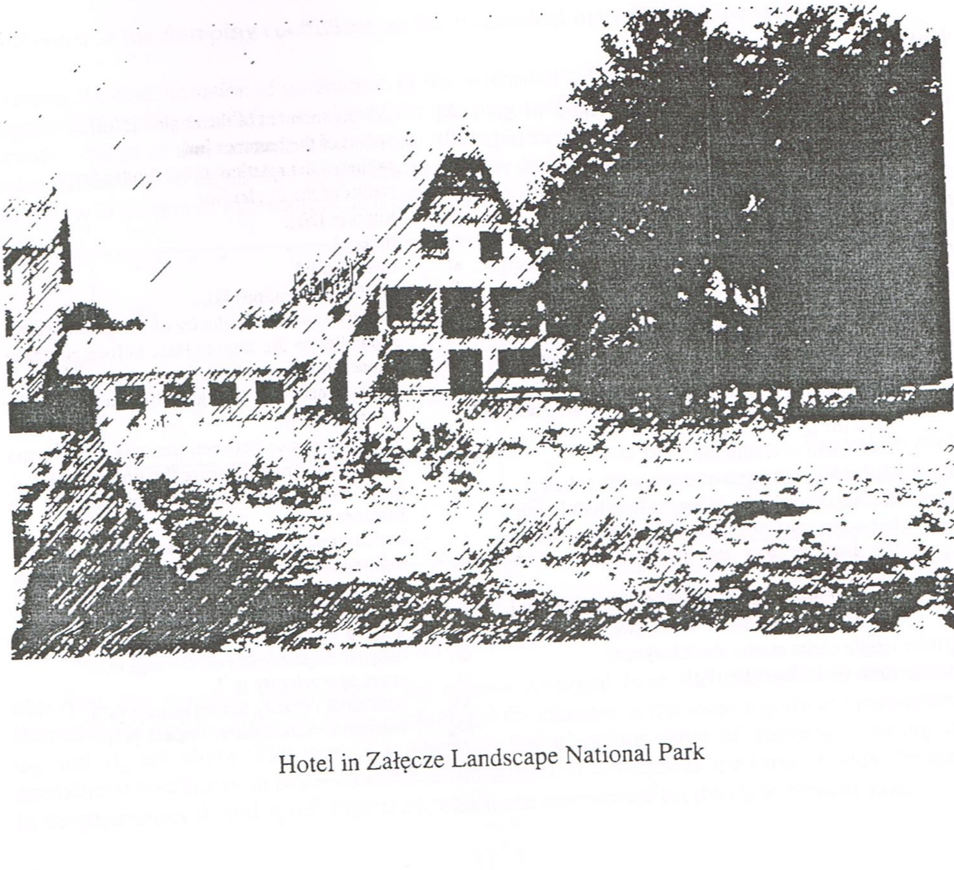
Notation

- a - distance between the gravity centre of the hammer and the end of the cantilever beam [m],
- B - inertial moment of the hammer [kgm^2],
- B_e - inertial moment of the engine rotor [kgm^2],
- B_r - inertial moment of the rotor [kgm^2],
- C_{11} - damping coefficient of the hammer [Ns/m],
- C_ϕ - damping coefficient of the elastic shaft [Nsm],
- h - vertical distance between the mass centre of the hammer in the static equilibrium position and the rotation axis of the rotor [m],
- I - inertial moment of cross section of the cantilever beam [m^4],
- I_o - inertial moment of cross section of the elastic shaft [m^4],
- k - coefficient of restitution,
- K_{ij} - stiffness coefficients of the cantilever beam,
- K_ϕ - stiffness coefficient of the elastic shaft [Nm],
- l - horizontal distance between the mass centre of the hammer and the rotation axis of the rotor [m],
- l_h - length of the cylindrical part of the hammer [m],
- l_s - length of the cantilever beam [m],
- l_ϕ - length of the elastic shaft [m],
- m - mass of the hammer [kg],
- M_e - driving moment of the engine [Nm],
- R - radius of the hammer [m],
- r - radius of the rotation of the fender [m],
- r_f - radius of the fender [m],
- S - impulse [Ns],
- t - time [s],
- Δt - time step [s],
- T - period of vibrations [s],
- v_{sn}, v'_{sn} - component of the velocity of the gravity centre S parallel to the impact line, before and after impact [m/s],
- y - displacement of the gravity centre of the hammer S [m],
- y_s - vertical distance between the axis of rotor and the gravity centre S during the impact [m],
- α_1 - basic natural frequency of vibrations of the hammer [s^{-1}],
- Δ - logharytmic decrement of damping (hammer),
- Δ_ϕ - logharytmic decrement of damping (elastic shaft),
- ϕ - angular displacement of the hammer,
- ϕ_r - angular displacement of the rotor,
- ϕ_e - angular displacement of the engine,
- ω - rotational velocity [s^{-1}],
- ω_{bi} - rotational velocity before impact [s^{-1}],
- ω_{ai} - rotational velocity after impact [s^{-1}].



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