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RESONANT PROPERTIES OF A ROTOR SUPPORTED ON JOURNAL BEARINGS WITH NON-CIRCULAR CONTOUR

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Abstract

In the paper resonant properties of a rigid rotor supported in three-lobe journal bearings are discussed. Equations of motion have been derived and then analyzed numerically. Corresponding relationships and results have been included in appropriate graphs .

1. Introduction

Owing to particular dynamic properties (better stability) multi-lobe journal bearings enjoy a special interest among many researchers. Tondl (1965) studied experimentally three types of multi-lobe bearings elliptical, three- and four-lobe) and made a catalogue of their properties. Analysis of dynamics of rotors supported on non-circular journal bearings attracts numerous problems, already on the level of determining the pressure of the lubricant medium. Even in the case of the short bearing we do non know the exact solution to this problem. This is why approximate methods are used. These methods consist in replacing non-circular contours with combination of circular ones according to given criteria. One of the methods, the geometrical approximation, was presented by Kaniewski and Stasiak (1973).

In the worldwide literature the papers concerning dynamics of rotors supported on journal bearings with non-circular contours scarcely appear. Among those more interesting one can pay attention to theoretical studies by Flack and Allaire (1982) who additionally determined static characteristics of a three-lobe bearing through experiment. The same authors (1984) investigated also regions of parameters securing stable working of the rotor. One of the recent papers due to Müller-Karger, Barret and Flack (1997) focuses on determination of stiffness and damping coefficients of the lubricant.

The main intention of the author of this paper is to present resonant properties of a rotor with bearing having three-lobe contour and undergoing a kinematic excitation.

2. Pressure distribution

The rigid rotor supported on short journal bearings with pericycloidal contour with three oil gaps is considered. The system is shown in Fig. 1.

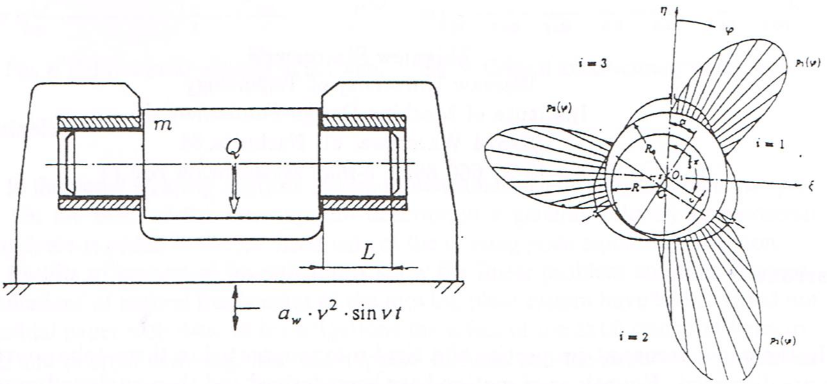

Fig. 1. Model of the system.

For such a model we apply then the geometrical approximation of the oil gap (see Kaniewski, Stasiak). In each segment of the approximated contour of the bearing bushing we find the magnitude of oil gap to be:

$$\begin{aligned} h_1(\varphi) &= R_0 - R + e\cos(\varphi - \alpha) + mK_n[1 - \cos(\frac{\pi}{3} - \varphi)] \\ h_2(\varphi) &= R_0 - R + e\cos(\varphi - \alpha) + mK_n[1 - \cos(\pi - \varphi)] \\ h_3(\varphi) &= R_0 - R + e\cos(\varphi - \alpha) + mK_n[1 - \cos(\frac{5\pi}{3} - \varphi)] \end{aligned} \tag{1}$$

where R_0 is the radius of a circle inscribed into pericycloid, R the radius of the journal, m the eccentricity of the pericycloid, φ the angle measured along the circumference, e the distance between the journal center and the center of the bushing, α the angle describing position of the line connecting these centers, K_n is the approximation coefficient. Assuming that:

$$\varepsilon = R_0 - R\,, \quad \lambda^* = \frac{m}{R_0 - R}\,, \quad \beta = \frac{e}{R_0 - R} \tag{2}$$

where ε is the absolute clearance, λ^* the relative eccentricity of the pericycloid, β – relative eccentricity of the journal. The expressions for the oil gap in non-dimensional version are:

$$\begin{aligned} H_1(\varphi) &= 1 + \beta\cos(\varphi - \alpha) + \lambda^* K_n[1 - \cos(\frac{\pi}{3} - \varphi)] \\ H_2(\varphi) &= 1 + \beta\cos(\varphi - \alpha) + \lambda^* K_n[1 - \cos(\pi - \varphi)] \\ H_3(\varphi) &= 1 + \beta\cos(\varphi - \alpha) + \lambda^* K_n[1 - \cos(\frac{5\pi}{3} - \varphi)] \end{aligned} \tag{3}$$

The equation describing pressure distribution in the oil film in short bearings with all components of the journal velocity taken into account (i.e. rotary motion, cricumferential and radial motion of the journal) has the following form:

$$\frac{\partial}{\partial z}\left(h_i^3 \frac{\partial p}{\partial z}\right) = 6\mu\omega\frac{\partial h_i}{\partial \varphi} + 12\mu\frac{\partial h_i}{\partial t} \quad , \qquad i = 1,2,3 \tag{4}$$

where μ is the absolute viscosity of the lubricant, p is the oil pressure, z - coordinate measured along the length of the bearing, ω - rotation speed. Assuming the boundary conditions:

$$z = 0 \Longrightarrow \frac{\partial p}{\partial z} = 0 \quad , \qquad z = \pm\frac{L}{2} \Longrightarrow p = 0 \tag{5}$$

where L denotes the length of the journal we obtain the following relationship for the oil pressure in each segment of the pericycloid:

$$\begin{aligned} p_i(\varphi, z) \;=\; & \frac{\mu(3L^2 - 12z^2)}{4\{\varepsilon + e\cos(\varphi-\alpha) + mK_n[1-\cos(B_i - \varphi)]\}^3}\Big\{(\omega - 2\dot{\alpha})e\sin(\varphi-\alpha) + \\ & + \omega m K_n \sin(B_i - \varphi) + 2\mu\dot{e}\cos(\varphi-\alpha)\Big\} \text{ and } B_1 = \frac{\pi}{3}, B_2 = \pi, B_3 = \frac{5\pi}{3} \end{aligned} \tag{6}$$

where $\dot{\alpha}$ is the journal velocity component corresponding to circumferential motion, $\dot{e}$ the component corresponding to radial motion of the journal (squeeze effect). By integrating (6) with respect to z, and then taking into account (2) we obtain the following relationship for the oil pressure:

$$p_i(\varphi) = \frac{\mu L^3}{2\varepsilon^2}\frac{(\omega - 2\dot{\alpha})\sin(\varphi-\alpha) + \omega\lambda^* K_n \sin(B_i - \varphi) - 2\dot{\beta}\cos(\varphi - \alpha)}{\{1 + \beta\cos(\varphi-\alpha) + \lambda^* K_n[1 - \cos(B_i - \varphi)]\}^3} \tag{7}$$

Exemplary pressure distribution in particular sections and corresponding to the wedge effect are shown in Fig. 2.

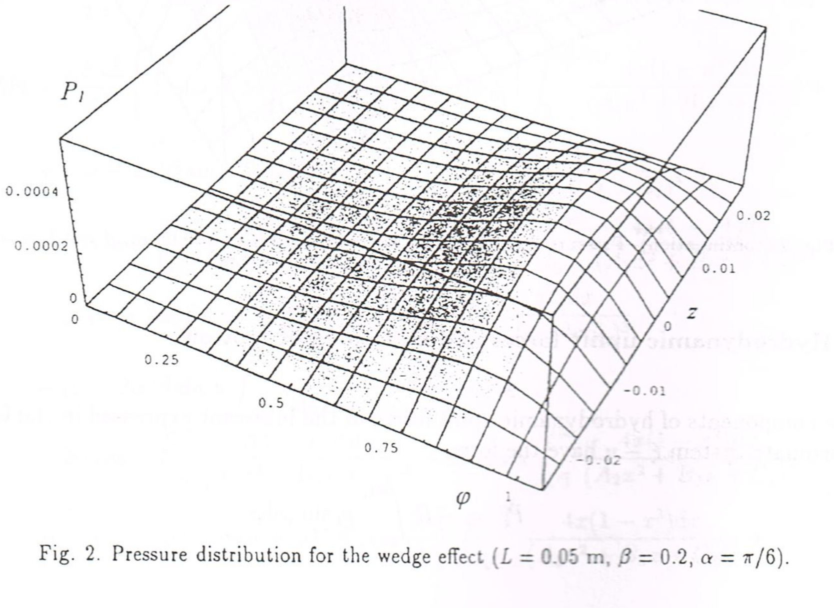

Fig. 2. Pressure distribution for the wedge effect ($L = 0.05$ m, $\beta = 0.2$, $\alpha = \pi/6$).

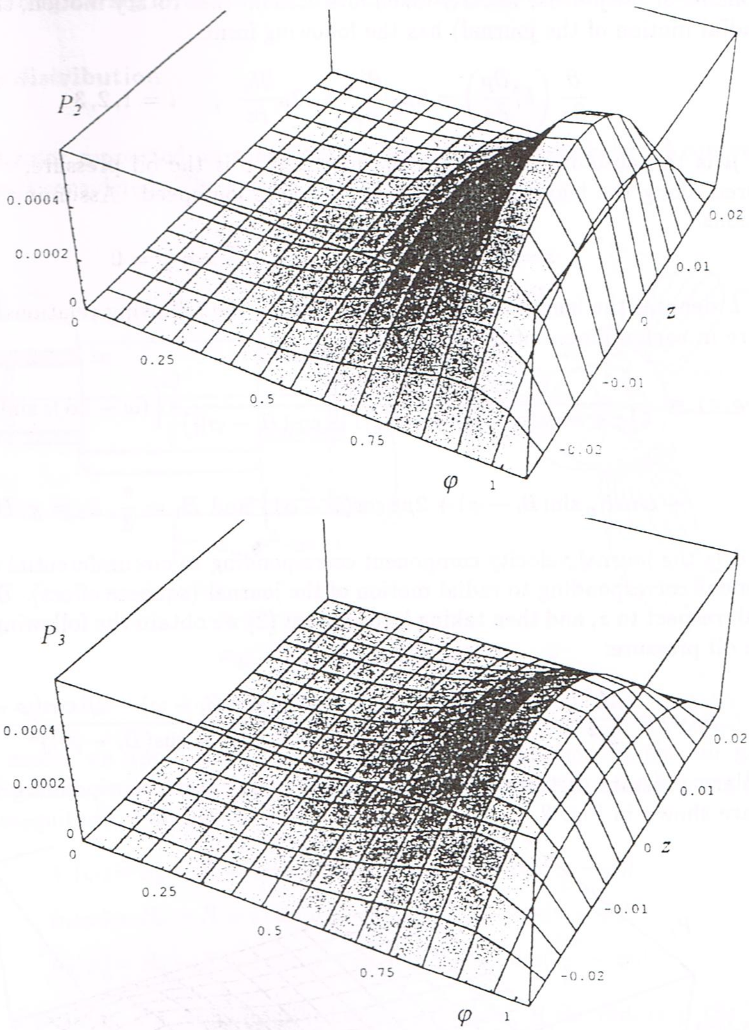

Fig. 2 (continuation). Pressure distribution for the wedge effect ($L = 0.05$ m, $\beta = 0.2$, $\alpha = \pi/6$).

3. Hydrodynamic uplift forces and equations of motion

The components of hydrodynamic uplift forces of the lubricant expressed in the Cartesian coordinate system $\xi - \eta$ have the form:

$$P_{\xi_i} = -R \int_{\varphi_i}^{\varphi_{k_i}} p_i \sin\varphi \mathrm{d}\varphi \tag{8}$$

$$P_{\eta_i} = -R\int_{\varphi_i}^{\varphi_{k_i}} p_i \cos\varphi \mathrm{d}\varphi$$

where φ_i and φ_{k_i} are the boundaries within which the oil film has the positive pressure.

By denoting the mass of the rotor by M, the external loading by Q, the excitation frequency by ν, the excitation amplitude a_ω we write down the equations of motion in the following form:

$$\begin{aligned}
M\ddot{\xi} = \frac{\mu R L^3}{2\varepsilon^2}\Bigg\{ & \Big[(\omega - 2\dot{\alpha})\beta\sin\alpha - \omega\lambda^* K_3 \sin\frac{\pi}{3}\Big]\int_0^{\frac{\sqrt{3}}{3}} \frac{4x(1-x^2)\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + \\
& + \Big[-(\omega - 2\dot{\alpha})\beta\cos\alpha + \omega\lambda^* K_3\cos\frac{\pi}{3}\Big]\int_0^{\frac{\sqrt{3}}{3}} \frac{8x^2\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + \\
& + 2\dot{\beta}\cos\alpha\int_0^{\sqrt{3}} \frac{4x(1-x^2)\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + 2\dot{\beta}\sin\alpha\int_0^{\sqrt{3}} \frac{8x^2\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + \\
& + (\omega - 2\dot{\alpha})\beta\sin\alpha\int_{\sqrt{3}}^{\infty} \frac{4x(1-x^2)\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + \\
& + [-(\omega - 2\dot{\alpha})\beta\cos\alpha - \omega\lambda^* K_3]\int_{\sqrt{3}}^{\infty} \frac{8x^2\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + \\
& + 2\dot{\beta}\sin\alpha\int_{\sqrt{3}}^{\infty} \frac{8x^2\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + 2\dot{\beta}\cos\alpha\int_{\sqrt{3}}^{\infty} \frac{4x(1-x^2)\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + \\
& + \Big[(\omega - 2\dot{\alpha})\beta\sin\alpha - \omega\lambda^* K_3\sin\frac{5\pi}{3}\Big]\int_{-\sqrt{3}}^{-\frac{\sqrt{3}}{3}} \frac{4x(1-x^2)\mathrm{d}x}{(A_3x^2+B_3x+C_3)^3} + \\
& + \Big[-(\omega - 2\dot{\alpha})\beta\cos\alpha + \omega\lambda^* K_3\cos\frac{5\pi}{3}\Big]\int_{-\sqrt{3}}^{-\frac{\sqrt{3}}{3}} \frac{8x^2\mathrm{d}x}{(A_3x^2+B_3x+C_3)^3} + \\
& + 2\dot{\beta}\sin\alpha\int_{-\sqrt{3}}^{0} \frac{8x^2\mathrm{d}x}{(A_3x^2+B_3x+C_3)^3} + 2\dot{\beta}\cos\alpha\int_{-\sqrt{3}}^{0} \frac{4x(1-x^2)\mathrm{d}x}{(A_3x^2+B_3x+C_3)^3}\Bigg\}
\end{aligned} \tag{9}$$

$$\begin{aligned}
M\ddot{\eta} = \frac{\mu R L^3}{2\varepsilon^2}\Bigg\{ & \Big[-(\omega - 2\dot{\alpha})\beta\cos\alpha + \omega\lambda^* K_3\cos\frac{\pi}{3}\Big]\int_0^{\frac{\sqrt{3}}{3}} \frac{4x(1-x^2)\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + \\
& + \Big[(\omega - 2\dot{\alpha})\beta\sin\alpha - \omega\lambda^* K_3\sin\frac{\pi}{3}\Big]\int_0^{\frac{\sqrt{3}}{3}} \frac{2(1-x^2)^2\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + \\
& + 2\dot{\beta}\cos\alpha\int_0^{\sqrt{3}} \frac{2(1-x^2)^2\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + 2\dot{\beta}\sin\alpha\int_0^{\sqrt{3}} \frac{4x(1-x^2)\mathrm{d}x}{(A_1x^2+B_1x+C_1)^3} + \\
& + [-(\omega - 2\dot{\alpha})\beta\cos\alpha - \omega\lambda^* K_3]\int_{\sqrt{3}}^{\infty} \frac{4x(1-x^2)\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + \\
& + (\omega - 2\dot{\alpha})\beta\sin\alpha\int_{\sqrt{3}}^{\infty} \frac{2(1-x^2)^2\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + \\
& + 2\dot{\beta}\cos\alpha\int_{\sqrt{3}}^{\infty} \frac{2(1-x^2)^2\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + 2\dot{\beta}\sin\alpha\int_{\sqrt{3}}^{\infty} \frac{4x(1-x^2)\mathrm{d}x}{(A_2x^2+B_2x+C_2)^3} + \\
& + \Big[-(\omega - 2\dot{\alpha})\beta\cos\alpha + \omega\lambda^* K_3\cos\frac{5\pi}{3}\Big]\int_{-\sqrt{3}}^{-\frac{\sqrt{3}}{3}} \frac{4x(1-x^2)\mathrm{d}x}{(A_3x^2+B_3x+C_3)^3} +
\end{aligned} \tag{10}$$

$$
\begin{aligned}
&+\left[(\omega - 2\dot{\alpha})\beta \sin\alpha - \omega\lambda^* K_3 \sin\frac{5\pi}{3}\right] \int_{-\sqrt{3}}^{-\frac{\sqrt{3}}{3}} \frac{2(1-x^2)^2 \mathrm{d}x}{(A_3x^2 + B_3x + C_3)^3} + \\
&+ 2\dot{\beta}\cos\alpha \int_{-\sqrt{3}}^{0} \frac{2(1-x^2)^2 \mathrm{d}x}{(A_3x^2 + B_3x + C_3)^3} + 2\dot{\beta}\sin\alpha \int_{-\sqrt{3}}^{0} \frac{4x(1-x^2)\mathrm{d}x}{(A_3x^2 + B_3x + C_3)^3} \Bigg\} + \\
&- Q + a_w \nu^2 \sin \nu t
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= 1 - \beta\cos\alpha + \frac{3}{2}\lambda^* K_n \\
B_1 &= 2\beta\sin\alpha - \sqrt{3}\lambda^* K_n \\
C_1 &= 1 + \beta\cos\alpha + \frac{1}{2}\lambda^* K_n \\
A_2 &= 1 + \frac{1}{2}\beta(\cos\alpha - \sqrt{3}\sin\alpha) + \frac{3}{2}\lambda^* K_n \\
B_2 &= -\beta(\cos\alpha + \sin\alpha) - \sqrt{3}\lambda^* K_n \\
C_2 &= 1 - \frac{1}{2}\beta(\cos\alpha - \sqrt{3}\sin\alpha) + \frac{1}{2}\lambda^* K_n \\
A_3 &= 1 - \frac{1}{2}\beta(\cos\alpha - \sqrt{3}\sin\alpha) + \frac{3}{2}\lambda^* K_n \\
B_3 &= -\beta(\sin\alpha - \sqrt{3}\cos\alpha) - \sqrt{3}\lambda^* K_n \\
C_3 &= 1 - \frac{1}{2}\beta(\cos\alpha + \sqrt{3}\sin\alpha) + \frac{1}{2}\lambda^* K_n
\end{aligned}
\tag{11}
$$

$$
\beta = \frac{\sqrt{\xi^2 + \eta^2}}{\varepsilon}, \quad \dot{\beta} = \frac{\xi\dot{\xi} + \eta\dot{\eta}}{\beta\varepsilon^2}, \quad \dot{\alpha} \doteq \frac{\xi\dot{\eta} - \eta\dot{\xi}}{\beta^2\varepsilon^2}, \quad \cos\alpha = \frac{-\xi}{\beta\varepsilon}, \quad \sin\alpha = \frac{-\eta}{\beta\varepsilon}
\tag{12}
$$

4. Numerical simulation

For equations of motion (9) and (10) numerical simulation has been employed. The results corresponding for different excitation amplitudes and different clearances are shown in Fig. 3. The results are compared with those corresponding to cylindrical bearings.

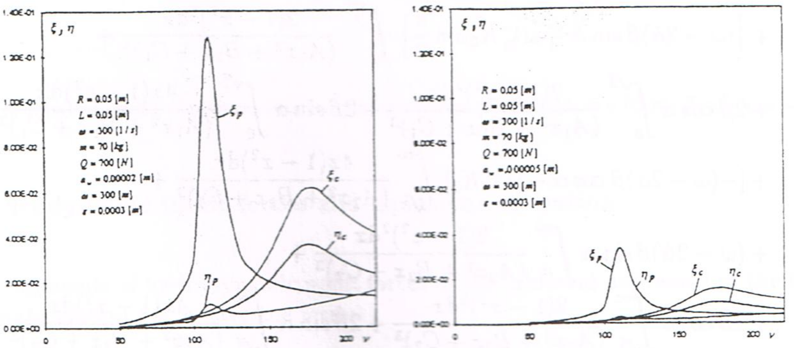

Fig. 3. Resonant characteristics (ξ_p, η_p – displacements in pericycloidal bearing, ξ_c, η_c – in cylindrical bearing).

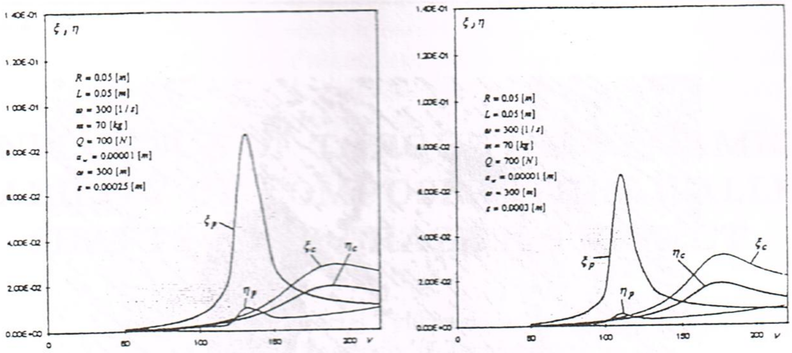

Fig. 3 (continuation). Resonant characteristics (ξ_p, η_p – displacements in pericycloidal bearing, ξ_c, η_c – in cylindrical bearing).

5. Concluding remarks

1. Three-lobe journal bearing exhibits much more stronger damping properties then bearings with cylindrical bushing.
2. The amplitude of resonant vibration is very small in the direction of applied kinematic excitation, and relatively high in perpendicular direction.
3. Three-lobe bearings undergo resonant vibration at lower frequencies. The superresonant vibration has much lower amplitudes when compared to cylindrical bearings.

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