A PROCESS OF SPUR GEARS WITH INVOLUTE TOOTH FORM MESHING

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Abstract

Process of involute tooth form gears meshing mainly depends on their kinematic accuracy. It can be presented by means of kinematic deviation function. Owing to elaborated model of tested and ideal gear mating, a possibility to determine this function by numerical evaluations emerged. This model enables to test connections between the gear manufacturing deviation, and its working features expressed by kinematic deviation function.

1. Introduction

For a description of gears that have involute tooth form, we often use a 'band' model. This model is adequate for geometrically ideal tooth forms meshing (not including geometrical inaccuracy of manufacturing). It assumes, that involute tooth form gears mating is analogical to the process that takes place when two cylinders connected by means of winded, inextensible thread rotate. Diameters of these cylinders refer to the base diameters of analyzed gears. It is obvious, that - under these assumptions - gear ratio (the gears rotational speed ratio) is constant. It depends only on cylinders radii ratio. Such a description of involute tooth form mating is a far reaching simplification. While actual gears mate, this gear ratio changes all the time because of the geometrical inaccuracy of mating tooth forms. Phenomena taking place during involute tooth forms meshing are a subject of research works pursued in Department of Technical Metrology, Technical University of Poznań. To describe them properly, a mathematical model of involute tooth form gears mating was elaborated. It includes geometrical inaccuracies that can be found in actual gears

1,3]. Research conducted with a help of this model made it possible to describe the gears meshing process, when one was a gear having a actual geometry, and the other was an ideal one. A function featuring tooth form meshing process of mating gears is called a kinematic deviation. It is defined as a difference between an actual and nominal rotation angle of mating gears and can be characterised by means of a following relationship:

$$\theta = \phi * i(r) - \phi * i(t)$$

where: $i(t) = z(1)/z(2)$ - nominal gears ratio

$i(r)$ - actual gears ratio

If we assume that one of the mating gears has an ideal geometry, kinematic deviation function will concern only one gear (the one with actual geometry).

2. Mathematical model of mating tooth forms meshing

On the stage of formulating the involute tooth form spur gears mating, it was stablished which of the parameters have significant influence on the process of meshing. Next, they were presented by means of independent geometrical deviations, what finally became a base for model assumptions of mating tooth forms.

When a model for involute tooth forms meshing was created it was assumed, that a gear is an ideal rigid solid. Thus, load influence on meshing process was neglected. Basing on this assumption and geometrical relationships of involute tooth form description, a model gear was defined in the following way:

- it is a set of involutes derived from a base circle with a radius of $r(b)\pm \delta(b)$,
- involutes are located on the circle circumference in the distance referring to circular pitch values $p(t)\pm f(ptr)$,
- distance between rotation axis of the model gear and its geometrical axis is equal to e.

Geometrical accuracy of a gear was thus described using three independent deviations: circular pitch deviation, base radius deviation and axis of rotation eccentricity deviation. It is obvious that these deviations have different values in each planes (sections) alongside flank pitch line. This

description comes down to a following form:

$$A = [a(1), a(2),...,a(m)]$$

where:

$$a_j = \begin{bmatrix} a(1j), b(1j), c(1j) \\ .... \\ a(nj), b(nj), c(nj) \end{bmatrix}$$

$j = 1,2,...,m$
n - number of included sections
a, b, c, -independent deviations
m - number of teeth in tested gear.
Model of involute tooth form meshing was formulated in the following way:

- relationships between particular geometrical deviations, independent on each other and kinematic deviation were presented in the form of mathematical equations
- ideal gear mating with a gear that has given values of manufacturing deviations was presented by means of a group of equations, forming altogether an algorithm.

3. Process of individual involute tooth forms meshing

For determining kinematic deviation function for analysed gear, we must enter input data to algorithm of evaluation. They are gear construction parameters and its accuracy expressed in the form of matrix [A(m)].
In the process of individual teeth couple meshing, three types of meshing can be distinguished: meshing before line of action, meshing on the line of action and meshing after line of action. Meshing before and after line of action was called a stage of edge mating.
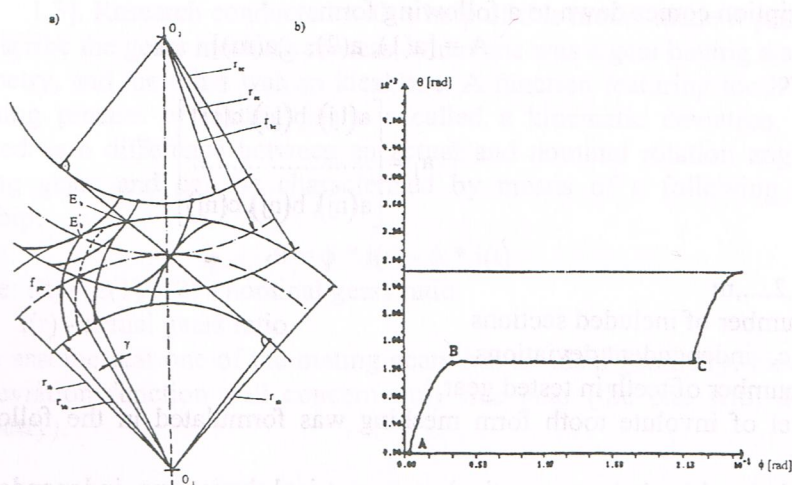
Fig.1.: a) Teeth couple meshing, where one gear toth form has a positive value of circular pitch deviation
b) Teeth couple meshing, when modulus   value m=1, circular pitch deviation f(ptr)=20 μm   AB - stage of edge mating   BC - stage of mating on the line of action
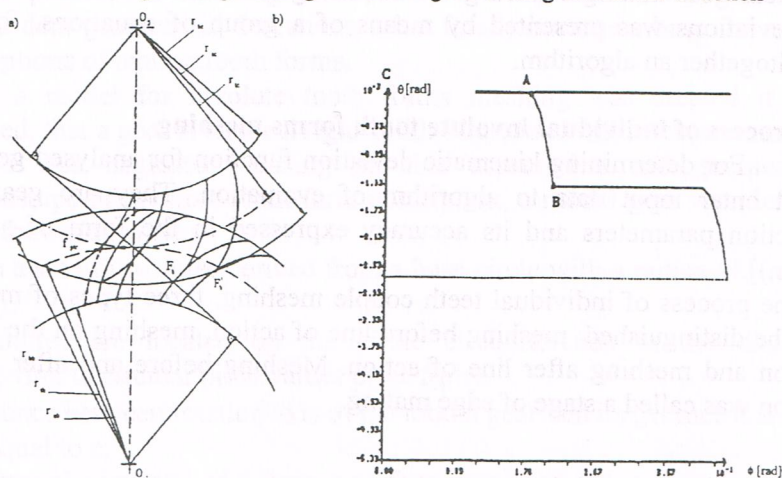

Fig.2.: a) Teeth couple meshing,   where one gear tooth form has a negative value of circular pitch deviation
b) Teeth couple meshing, when modulus   m-1, circular pitch deviation f(ptr)=-20μm
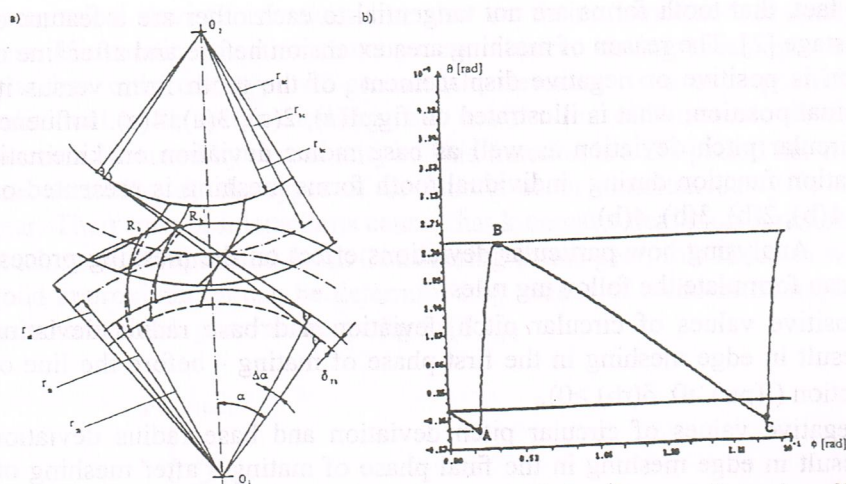CA - stage of mating on the line of action   AB - stage of edge mating

Fig.3.: a) Teeth couple meshing, where one gear tooth form has a positive value of base
radius deviation
b) Teeth couple meshing, when modulus value m=1, base radius deviation δ(rb)=20μm
AB - stage of edge mating   BC - stage of mating on the line of action

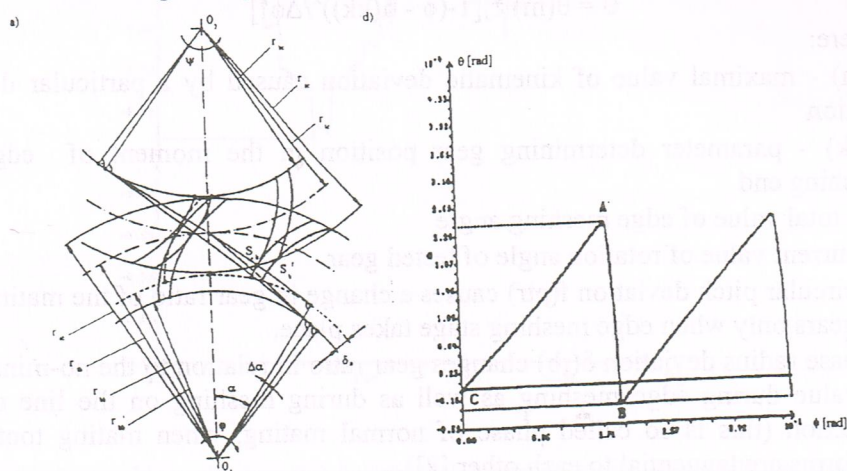
Fig.4.: a) Teeth couple meshing, where one gear tooth form has a negative value of base
radius deviation
b) Teeth couple meshing, when modulus value m=1, base radius deviation δ(rb)=-20μm
CA - stage of mating on the line of action   AB - stage of edge mating

The fact, that tooth forms are not tangential to each other are a feature of this stage [2]. The reason of meshing area extension before and after line of action is positive or negative displacement of the tooth form versus its nominal position, what is illustrated on fig.:1(a), 2(a), 3(a), 4(a). Influence of circular pitch deviation as well as base radius deviation on kinematic deviation function during individual tooth forms meshing is presented on fig.: 1(b), 2(b), 3(b), 4(b).

Analysing how particular deviations effect on the meshing process one can formulate the following rules:

- positive values of circular pitch deviation and base radius deviation result in edge meshing in the first phase of mating - before the line of action (f(ptr) >0, δ(rb) >0),
- negative values of circular pitch deviation and base radius deviation result in edge meshing in the final phase of mating - after meshing on the line of action (f(ptr) <0, δ(rb) <0),
- function characterizing edge meshing caused by f(ptr) or δ(rb) deviation can be shown in the following form:

$$\theta = \theta(m) * [1 - (\phi - \phi(kk))^2 / \Delta\phi^2]$$

where:

$\theta(m)$ - maximal value of kinematic deviation caused by a particular deviation

$\phi(kk)$ - parameter determining gear position in the moment of edge meshing end

$\Delta\phi$ - total value of edge meshing angle

ϕ - current value of rotation angle of tested gear

- circular pitch deviation f(ptr) causes a change in gear ratio of the mating gears only when edge meshing stage takes place,
- base radius deviation δ(rb) changes gear ratio in relation to the no-minal value during edge meshing as well as during meshing on the line of action (this is so called phase of normal mating, when mating tooth forms are tangential to each other [2]).

4. Kinematic deviation

Gear kinematic accuracy can be explicitly featured by kinematic deviation function. It pictures tested and ideal gear meshing process beginning with 0 and ending at 2Π rad. This function depends not only on

geometrical deviations values but also on their distribution in analysed gear. Kinematic deviation that one can observe during meshing when a gear has only one type of geometrical deviation is a superposition of individual tooth forms meshing. Yet another run of kinematic deviation function is caused by eccentricity e. This function is a 2Π period har-monic one. Actually, all the types of deviations occur simultaneously in every gear. Their mutual interactions cause, that kinematic deviation function has much more complicated form. Considering actual gear, this function with a good approximation can be determined by means of elabo-rated model of tested and check gear meshing - fig.5. [1,4].
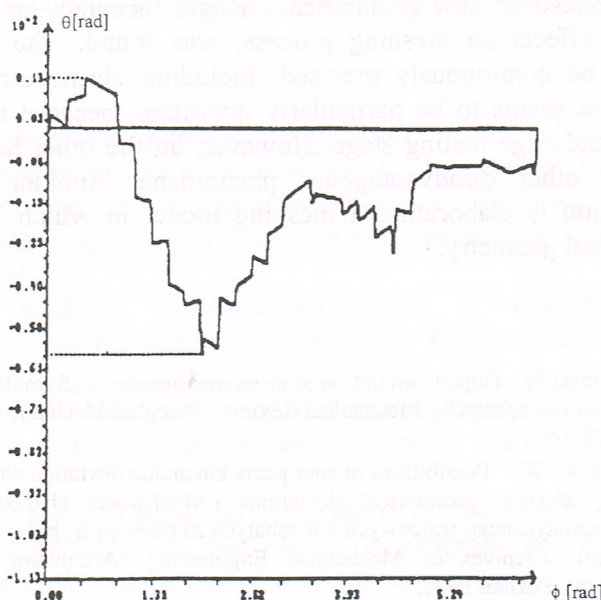

Fig.5. Gear kinematic deviation function determined by means of digital simulation

5. Conclusions

In the process of involute tooth forms meshing, edge mating stage is of a great importance. In this stage significant values of angular acceleration takes place. Especially when edge mating stage turns into mating on the line of action, this phenomenon is intensified. During edge mating, the kinematic deviation function is a second order fun-ction.During mating on the line of action it is a first order function, and its run depends on the type of deviations that can be found. Kinematic deviation function caused by gear axis of rotation eccentricity is a 2Π period harmonic function.Its meaning to a gear functioning increases, as the gear rotational speed grows. Owing to a formulated model of tested and check gear, an answer to the question: how geometrical changes (accuracy of ane of the mating gears) effects on meshing process, was found. The elaborated model should be continuously precised. Including elastic strain in the meshing process seems to be particularly important, because it probably shortens a bit the edge mating stage. However, on the other hand it may produce some other disadvantageous phenomena. Another important research direction is elaborating a meshing model in which both gears would have actual geometry.

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