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GLOBAL AND LOCAL INELASTIC BUCKLING OF THIN-WALLED ORTHOTROPIC COLUMNS BY ELASTIC ASYMPTOTIC SOLUTIONS

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Abstract

The problem of global and local stability in the elasto-plastic range for thin-walled columns is examined on the basis of the incremental theory of plasticity using Hill's Yield Criterion. Columns of closed and open cross-sections built from rectangular orthotropic plates are subjected to the axial compression. A solution of elastic buckling for a thin-walled orthotropic columns based on Koiter's asymptotic method is employed to investigate the elasto-plastic buckling mode of the column and to determine its buckling load. The study is based on the numerical method of the transition matrix. The results of numerical calculations are presented in diagrams.

Notation

E_x, E_y, $E_{\theta 45}$ - Young's moduli in principal (x,y) and 45 degrees directions of orthotropy,
ν_{xy}, ν_{yx} - Poissons' ratios in principal (x,y) directions,
G_{xy} - shear modulus,
E_x^p, E_y^p, $E_{\theta 45}^p$ - hardening moduli in principal (x,y) and 45 degrees directions of orthotropy,
G_{xy}^p - hardening modulus in shear,
σ_{10}, σ_{20}, $\sigma_{\theta 0}$ - proportional limits in principal (x,y) and 45 degrees directions of orthotropy,
τ_{120} - proportional limit in pure shear,
$\bar{\sigma}$ - effective stress,
σ_x, σ_y - normal stresses,

τ_{xy} - shear stress,
σ - uniaxial stress,
σ_{cr} - buckling stress,
h - thickness of a component plate,
l - length of a column,
m - number of axial half-waves,
M_x , M_y, M_{xy} - bending moment resultants for a component plate,
$\overline{\mathbf{N}}$ - force field,
N_x , N_y, N_{xy} - in plane stress resultants of a component plate,
$\overline{\mathbf{U}}$ - displacement field,
u, v,w, - displacement components of middle surface of a component plate,
ε - uniaxial strain,
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$ - membrane strains in a component plate,
$\kappa_x, \kappa_y, \kappa_{xy}$ - bending strains in a component plate,
λ - load parameter,
E_x^*, E_y^* ,G^*, ν_{yx}^* - instantaneous conventional parameters of orthotropy.

1. INTRODUCTION

The aim of this work is to analyse the stability loss of thin-walled orthotropic beam-columns in the range of stresses lying between the proportional limit and the yield limit of a material.

In last years the behaviour of thin-walled elements for which the buckling stress and the yield limit are of the same range is of special importance in the design of machines and many industrial structures.

In numerous studies (e.g. [6], [16], [17]) dealing with elasto-plastic stability of thin-walled structures the isotropy of material in the elastic and inelastic range has been usually assumed. Meanwhile it is well known that many structural materials traditionally considered as isotropic exhibit some degree of anisotropy due to working processes. Also the increased interest in composite materials by the industry demands a better understanding of the strengths of these materials anisotropic by design.

In the present paper the problem of stability in the elasto-plastic range of thin-walled columns is examined using the method elaborated for the analysis of stability of thin-walled orthotropic beam-columns [10], [11]. The relationships between the stresses and strains for a component elasto-plastic plate are derived on the basis of the incremental theory of plasticity [4]. On the other hand the same relationships are written for an orthotropic elastic plate. Comparing the appropriate coefficients in both relations the instantaneous conventional parameters of „elastic orthotropy" can be found out. So the problem of inelastic stability of columns can be investigated in the same way as the problem of stability of the elastic orthotropic columns.

It should be noted that in one of previous papers [12] the authors applied this method to solve the problem of inelastic stability of isotropic columns made from the material that uniaxial stress-strain curve was described analytically by Needlemann-Tvergaard relation or Ramsberg-Osgood formula.

2. FORMULATION OF A PROBLEM

The long thin-walled prismatic columns of length l and composed of plane, rectangular plate segments interconnected along longitudinal edges are considered.
The columns of open and closed cross-sections (Fig.1), simply supported at the ends, subject to the uniform compression.

It is assumed that a component plate is orthotropic in such a way that principal axes of orthotropy (x,y) are parallel to its edges.

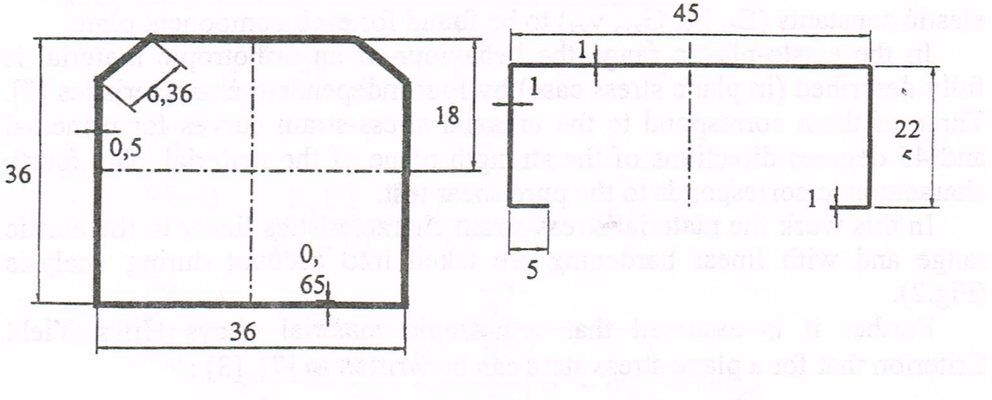
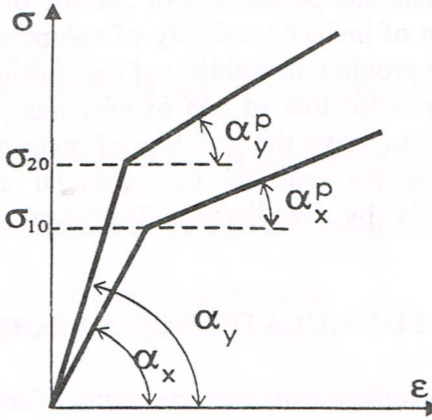


Fig.1. Considered cross-sections (dimensions in mm)

3. CONSTITUTIVE RELATIONS IN THE ELASTO-PLASTIC RANGE



$$E_x = tg\alpha_x\,; \qquad E_y = tg\alpha_y\,;$$
$$E_x^p = tg\alpha_x^p \qquad E_y^p = tg\alpha_y^p$$

Fig. 2 Material stress-strain characteristics

As it is well known for orthotropic materials there are four independent elastic constants (E_x, E_y, G_{xy}, ν_{yx}) to be found for each component plate.

In the elasto-plastic range the behaviour of an orthotropic material is fully described (in plane stress case) by four independent characteristics [7]. Three of them correspond to the uniaxial stress-strain curves for principal and 45 degrees directions of the strength plane of the material. The fourth characteristic corresponds to the pure shear test.

In this work the material stress-strain characteristics linear in the elastic range and with linear hardening are taken into account during analysis (Fig.2).

Further it is assumed that orthotropic material obeys Hill's Yield Criterion that for a plane stress state can be written as [7], [8] :

$$\bar{\sigma}^2 = \bar{a}_1\sigma_x^2 + \bar{a}_2\sigma_y^2 - \bar{a}_{12}\sigma_x\sigma_y + 3\bar{a}_3\tau_{xy}^2 \tag{1}$$

In this expression the parameters $\bar{a}_1 \div \bar{a}_3$ are called anisotropic parameters which depend on the material constants in particular directions.
The initial parameters $\bar{a}_{10} \div \bar{a}_{30}$ depend only on the proportional limits.

$$\bar{a}_{10} = \bar{\sigma}_0^2 / \sigma_{10}^2 \qquad \bar{a}_{20} = \bar{\sigma}_0^2 / \sigma_{20}^2, \qquad \bar{a}_{30} = \bar{\sigma}_0^2 / (3\tau_{120}^2), \qquad \bar{a}_{330} = \bar{\sigma}_0^2 / \sigma_{\theta 0}^2,$$

when $\theta = 45^0$,: (2)

$$\bar{a}_{120} = \bar{a}_{10} + \bar{a}_{20} + 3\bar{a}_{30} - 4\bar{a}_{330}.$$

For stress-strain hardening material the uniaxial elasto-plastic stresses vary with increasing plastic deformations and therefore the anisotropic parameters should also vary since they are functions of current stresses. The anisotropic parameters $\bar{a}_1 \div \bar{a}_3$ depend on constants of the material characteristic and also on the current stress (for more details see [14]):

$$\bar{a}_1 = \frac{\bar{\sigma}^2}{\sigma_x^2} = \frac{\bar{\sigma}^2}{\dfrac{E_x E_x^p}{E_x - E_x^p}\dfrac{E - E^p}{E\,E^p}\left(\bar{\sigma}^2 - \sigma_0^2\right) + \sigma_{10}^2};$$

.

$$\bar{a}_2 = \frac{\bar{\sigma}^2}{\sigma_y^2} = \frac{\bar{\sigma}^2}{\dfrac{E_y E_y^p}{E_y - E_y^p}\dfrac{E - E^p}{E\,E^p}\left(\bar{\sigma}^2 - \sigma_0^2\right) + \sigma_{20}^2};$$

$$\bar{a}_{33} = \frac{\bar{\sigma}^2}{\sigma_{\theta=45}^2} = \frac{\bar{\sigma}^2}{\dfrac{E_\theta E_\theta^p}{E_\theta - E_\theta^p}\dfrac{E - E^p}{E\,E^p}\left(\bar{\sigma}^2 - \sigma_0^2\right) + \sigma_{\theta 0}^2}; \qquad (3)$$

$$\bar{a}_3 = \frac{\bar{\sigma}^2}{3\tau_{xy}^2} = \frac{\bar{\sigma}^2}{3\dfrac{G_{xy} G_{xy}^p}{G_{xy} - G_{xy}^p}\dfrac{E - E^p}{E\,E^p}\left(\bar{\sigma}^2 - \sigma_0^2\right) + 3\tau_{120}^2}.$$

$\bar{a}_{12} = \bar{a}_1 + \bar{a}_2 + 3\bar{a}_3 - 4\bar{a}_{33}$ for $\theta = 45^o$.

where: E, E^p, σ_0 - Young's modulus, hardening modulus and proportional limit for a characteristic of reference.

When the characteristic in the longitudinal direction (direction of loading) is chosen as a characteristic of reference than $a_1=a_{10}=1$.

For small elasto-plastic strains it is assumed that they don't depend on the history of deformation [4]. Therefore the relations between infinitesimal increments of membrane sectional forces and strains according to the incremental theory of plasticity take a form (4), where S_{ij} (i,j=x,y) denotes deviatoric stress for orthotropic material and Λ is a scalar positively defined [14].

$$\begin{aligned}
\delta N_x &= \frac{E_x h}{(1-\eta\nu^2)}[\delta\varepsilon_x + \nu\eta\delta\varepsilon_y - \Lambda(S_{xx} + \nu\eta S_{yy})],\\
\delta N_y &= \frac{E_y h}{(1-\eta\nu^2)}[\delta\varepsilon_y + \nu\delta\varepsilon_x - \Lambda(S_{yy} + \nu S_{xx})],\\
\delta N_{xy} &= Gh(\delta\gamma_{xy} - \Lambda S_{xy});
\end{aligned} \tag{4}$$

where:

$$\eta = \frac{E_y}{E_x}; \qquad \nu = \nu_{yx},$$

$$S_{xx} = \frac{1}{3}(2\bar{a}_1\sigma_x - \bar{a}_{12}\sigma_y),$$

$$S_{yy} = \frac{1}{3}(2\bar{a}_2\sigma_y - \bar{a}_{12}\sigma_x),$$

$$S_{xy} = 2\bar{a}_3\tau_{xy}.$$

On the other hand the variations of sectional forces can be written as follows:

$$\begin{aligned}
\delta N_x &= h[A_{11}\delta\varepsilon_x + A_{12}\delta\varepsilon_y],\\
\delta N_y &= h[A_{12}\delta\varepsilon_x + A_{22}\delta\varepsilon_y],\\
\delta N_{xy} &= hA_{33}\delta\gamma_{xy}.
\end{aligned} \tag{5}$$

When the uniaxial state of loading is considered ($\sigma_x = \sigma$, $\sigma_y = \tau_{xy} = 0$; $\bar{\sigma} = \bar{a}_1 \sigma$), the parameters $A_{11} \div A_{33}$ and Λ can be easily determined as functions of elastic constants of orthotropy and current parameters of anisotropy $\bar{a}_1 \div \bar{a}_3$ (see [14]):

$$\Lambda = \frac{3}{\sigma A_0}\left[(2\bar{a}_1 - \nu\eta\bar{a}_{12})\delta\varepsilon_x + (2\nu\eta\bar{a}_1 - \eta\bar{a}_{12})\delta\varepsilon_y\right]$$

$$A_{11}/C = 1 - \frac{(2\bar{a}_1 - \nu\eta\bar{a}_{12})^2}{A_0}; \quad A_{12}/C = \nu\eta - \eta\frac{(2\bar{a}_1 - \nu\eta\bar{a}_{12})(2\nu\bar{a}_1 - \bar{a}_{12})}{A_0};$$

$$A_{22}/C = \eta - \eta^2\frac{(2\nu\bar{a}_1 - \bar{a}_{12})^2}{A_0}; \quad A_{33}/C = \frac{G\,(1-\nu^2\eta)}{E_x}; \tag{6}$$

$$C = \frac{E_x}{\left(1 - \eta\nu^2\right)},$$

where: $$A_0 = 4(1-\nu^2\eta)\frac{E^p\dfrac{E}{E_x}}{E - E^p}\bar{a}_1 + 4\bar{a}_1^2 - 4\nu\eta\bar{a}_1\bar{a}_{12} + \eta\bar{a}_{12}^2. \tag{7}$$

4. BASIC EQUATIONS FOR ORTHOTROPIC ELASTIC COLUMNS

Let's now consider basic relationships for an elastic orthotropic plate. For a component plate exact geometrical relationships between strains and displacements are taken into account in aim to consider both out-of-plane and in-plane bending of a plate [10], [11]:

$$\varepsilon_x = u_{,x} + 0.5(u_{,x}^2 + v_{,x}^2 + w_{,x}^2);$$
$$\varepsilon_y = v_{,y} + 0.5(u_{,y}^2 + v_{,y}^2 + w_{,y}^2); \tag{8}$$
$$\gamma_{xy} = u_{,y} + v_{,x} + u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y};$$
$$\kappa_x = -w_{,xx}; \qquad \kappa_y = -w_{,yy}; \qquad \kappa_{xy} = -w_{,xy};$$

Physical relationships for a component plate treated as an orthotropic with principal axes of orthotropy parallel to its edges are formulated in the following way:

$$N_x = h(K_{11}\varepsilon_x + K_{12}\varepsilon_y);$$
$$N_y = h(K_{12}\varepsilon_x + K_{22}\varepsilon_y); \tag{9}$$
$$N_{xy} = hK_{33}\gamma_{xy}.$$

where:

$$K_{11} = E_x / (1 - \nu_{xy}\nu_{yx}); \qquad K_{22} = E_y / (1 - \nu_{xy}\nu_{yx});$$
$$K_{12} = \nu_{yx}K_{11} = \nu_{xy}K_{22}; \qquad K_{33} = G.$$

and

$$M_x = \frac{h^3}{12(1-\nu_{xy}\nu_{yx})}\left(E_x\kappa_x + \nu_{yx}E_y\kappa_y\right)$$
$$M_y = \frac{h^3}{12(1-\nu_{xy}\nu_{yx})}\left(\nu_{yx}E_x\kappa_x + E_y\kappa_y\right) \tag{10}$$
$$M_{xy} = \frac{G\,h^3}{12}\kappa_{xy}.$$

The dependence in Young's moduli and Poisson's ratios in (9) and (10) is as follows:

$$\eta = E_x / E_y = \nu_{xy} / \nu_{yx}. \tag{11}$$

The differential equilibrium equations resulting from the Principle of Virtual Work for the component plate can be written in a form:

$$\int_S \left[N_{x,x} + N_{xy,y} + \left(N_x u_{,x}\right)_{,x} + \left(N_y u_{,y}\right)_{,y} + \left(N_{xy} u_{,x}\right)_{,y} + \left(N_{xy} u_{,y}\right)_{,x} \right] \delta u dS = 0$$

$$\int_S \left[N_{xy,x} + N_{y,y} + \left(N_x v_{,x}\right)_{,x} + \left(N_y v_{,y}\right)_{,y} + \left(N_{xy} v_{,x}\right)_{,y} + \left(N_{xy} v_{,y}\right)_{,x} \right] \delta v dS = 0$$

$$\int_S \left[M_{x,xx} + M_{y,yy} + 2M_{xy,xy} + \left(N_x w_{,x}\right)_{,x} + \left(N_y w_{,y}\right)_{,y} + \left(N_{xy} w_{,x}\right)_{,y} + \left(N_{xy} w_{,y}\right)_{,x} \right] \delta w dS = 0 \qquad (12)$$

The solution of these equations for each plate should satisfy kinematic and static conditions at the junctions of adjacent plates (that are given in [13]) and following boundary conditions at the ends of the column (corresponding to the free support):

$$\begin{aligned} &N_x(x = 0; l, y) = N_x^0, \qquad && v(x = 0; l, y) = 0, \\ &w(x = 0; l, y) = 0, && M_{,xx}(x = 0; l, y) = 0. \end{aligned} \qquad (13)$$

5. SOLUTION OF THE PROBLEM

The method of solution is based on the fact that the relationships between stresses and strains in the elasto-plastic and elastic range for the orthotropic plate have the identical form (5), (9):

Elasto-plastic range	*Elastic range*
$N_x = h*[A_{11}*\varepsilon_x + A_{12}*\varepsilon_y]$,	$N_x = h*(K_{11}*\varepsilon_x + K_{12}*\varepsilon_y)$;
$N_y = h*[A_{12}*\varepsilon_x + A_{22}*\varepsilon_y]$,	$N_y = h*(K_{12}*\varepsilon_x + K_{22}*\varepsilon_y)$; (14)
$N_{xy} = h*A_{33}*\gamma_{xy}$.	$N_{xy} = h*K_{33}*\gamma_{xy}$.

So, the coefficients A_{11} -A_{33} can be replaced by coefficients K_{11} -K_{33}. In other words equating K_{11} =A_{11}, etc. the instantaneous conventional parameters of orthotropy E_x^*, E_y^*, G^*, ν_{yx}^* can be found out as functions of parameters describing the elasto-plastic behaviour of a material. Therefore the method for the elastic orthotropic columns can be applied to calculate, for the assumed number of half-waves *m*, the buckling load value of a given column in the elasto-plastic range.

Now, we have the elastic problem that is solved by the asymptotic Koiter's method [9]. Displacement $\overline{\mathbf{U}}$ and sectional force $\overline{\mathbf{N}}$ fields are expanded in power series in the buckling mode amplitude ζ, (ζ is the amplitude of buckling mode divided by the thickness of the first component plate).

$$\begin{aligned} \overline{\mathbf{U}} &= \lambda * \overline{\mathbf{U}}^{(0)} + \zeta * \overline{\mathbf{U}}^{(1)} + \\ \overline{\mathbf{N}} &= \lambda * \overline{\mathbf{N}}^{(0)} + \zeta * \overline{\mathbf{N}}^{(1)} + \end{aligned} \tag{15}$$

where: $\overline{\mathbf{U}}^{(0)}$, $\overline{\mathbf{N}}^{(0)}$ - prebuckling fields,
$\overline{\mathbf{U}}^{(1)}$, $\overline{\mathbf{N}}^{(1)}$ - buckling modes fields.

By substituting the expansion (15) into equations of equilibrium (12), junction conditions and boundary conditions, the boundary value problems of zero and first order can be obtained. The zero approximation describes the prebuckling state while the first order approximation that is the linear problem of stability is reduced to a system of homogenous differential equations with respect to displacements. This system with appropriate junction conditions is solved by modified transition matrices method [20]. The numerical integration of the equilibrium equations in a transverse direction with respect to the orthogonalization method by Godunov [1] is used. Finally the relation between state vectors on two longitudinal edges is obtained. In the solution and in the computer program Byskov-Hutchinson [2] asymptotic expansion is employed.

The solution of the first order approximation enables to determine the buckling loads of global and local value and the buckling modes. It should be mentioned that this approach enables to find out all buckling modes and their combinations (e.g. „mixed modes" [3], [5]) and to include the shear lag phenomenon and the effect of cross-sectional distortions.

The problem is solved in a numerical way. For a given geometrical parameters, material constants and for the assumed number of half-waves the elastic buckling stress for the considered orthotropic structure is calculated. If this value is greater than the proportional limit in the longitudinal direction σ_{10}, the value of a stress σ is found out for a considered value of uniaxial strain ε and next corresponding to it conventional „parameters of orthotropy" are also found. Applying these parameters in the elastic solution for elastic orthotropic plates a new value of the buckling stress is calculated. Further a method of secants is used to obtain the value of buckling elasto-plastic stress with accuracy 0.05%.

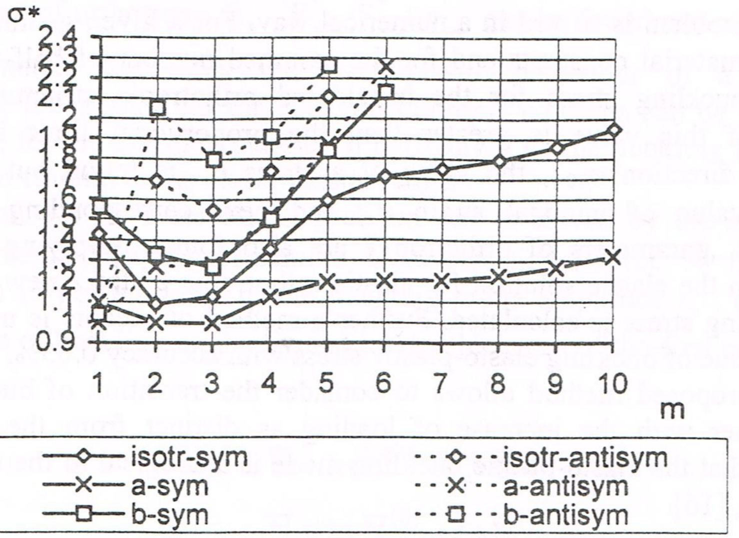
The proposed method allows to consider the transition of buckling mode together with the increase of loading as distinct from the usual assumption that the elasto-plastic buckling mode is analogical to the elastic one [6], [14], [16].

6. RESULTS OF NUMERICAL CALCULATIONS

Thin walled prismatic columns of open and closed cross-sections shown in Fig.1 are considered. The dimensions are chosen in such a way that the buckling occurs in the elasto-plastic range.

The results of calculations are presented in diagrams showing the dimensionless buckling stress $\sigma^* = \sigma_{cr} / \sigma_{10}$ as a function of axial half waves m.

Because rather long columns are considered (in case of open cross-section l= 200 mm and for columns of closed cross-section l=441 mm) the global buckling occurs for $m=1$ and local buckling for $m>1$.



isotr - $\mathbf{E_y = E_x}$; **a** - $\mathbf{E_y = 0{,}5E_x}$; $\nu_{yx}=0{,}15$; **b** - $\mathbf{E_y = 2E_x}$; $\nu_{xy}=0{,}15$;

Fig. 3a. Influence of values of elastic modulus E_y on the dimensionless buckling stress
(other material constants unchanged)

The material properties are taken from the works of Owen and Figueiros[18], [19] and in the case of the isotropic material are as follows:

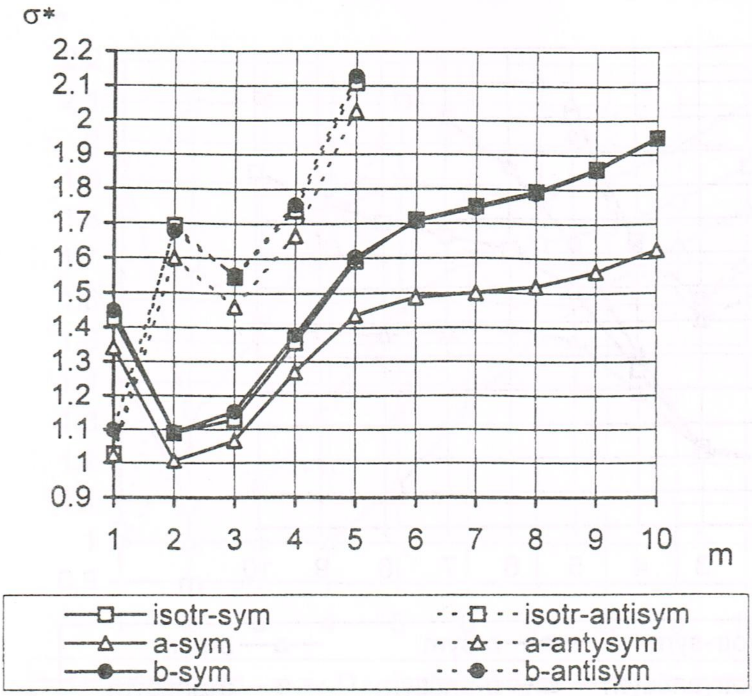
$\mathbf{E_x = E_\theta = E_y = 30000\ MPa}$; $\mathbf{G = 10000\ MPa}$; $\nu_{xy}=\nu_{yx}=0{,}3$;

$\mathbf{E_x^p = E_y^p = E_\theta^p = 3000\ MPa}$; $\mathbf{G^p = 1000\ MPa}$;

$\mathbf{\sigma_{10} = \sigma_{20} = \sigma_{\theta 0} = 30\ MPa}$; $\mathbf{\tau_{12o} = 17{,}32\ MPa}$;

During the numerical analysis the influence of changes of elastic moduli and hardening moduli in principal directions of orthotropy (while the others parameters remain unchanged) on the buckling stress values and also on the buckling mode is investigated.

For **open cross-sections** the results obtained under the assumption of symmetry conditions along the cross-section symmetry axis are shown by full lines and the results obtained under the assumption of antisymmetry conditions along the cross-section symmetry axis by dotted lines.

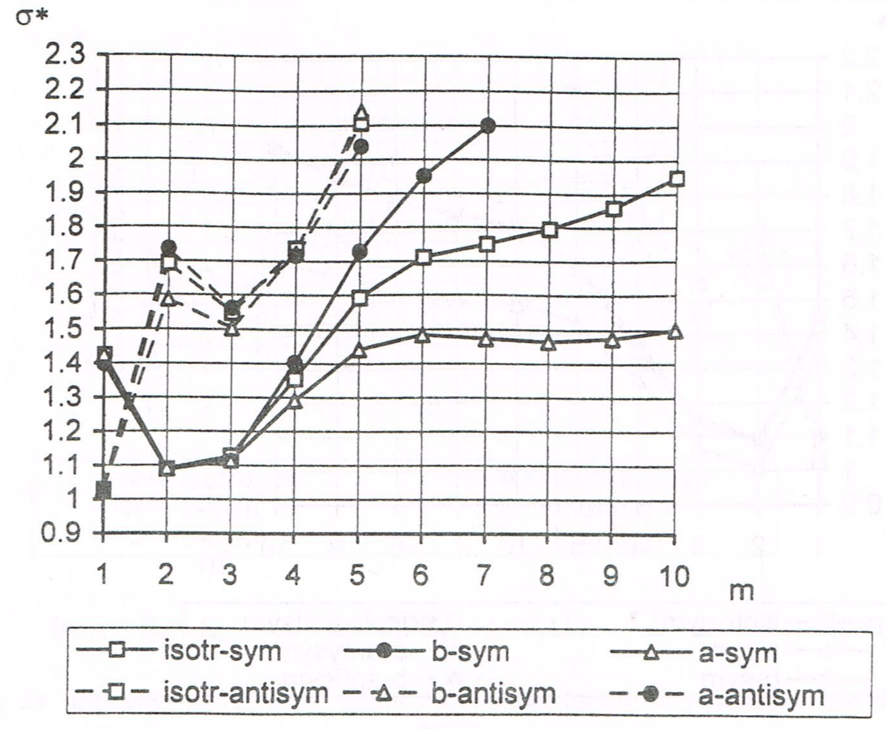


isotr - $E_x = E_y$; **a - $E_x = 0{,}5E_y$;** $\nu_{xy}=0{,}15$; **b - $E_x = 2E_y$;** $\nu_{yx}=0{,}3$;

Fig. 3 b. Influence of values of elastic modulus E_x on the dimensionless buckling stress
(other material constants unchanged)

Change of the elastic modulus E_y (in the direction perpendicular to the direction of loading) does not cause the change in global buckling stress value that is of antisymmetrical mode (Fig.3a). The significant influence is observed on the values of local buckling stress (symmetrical mode) - it can be noted that number of half-waves m corresponding to the minimum also changes (m=2 in case „a" and m=3 in case „b").

It is rather surprising that the influence of values of elastic modulus E_x (in the direction of loading) is less significant (Fig.3b).

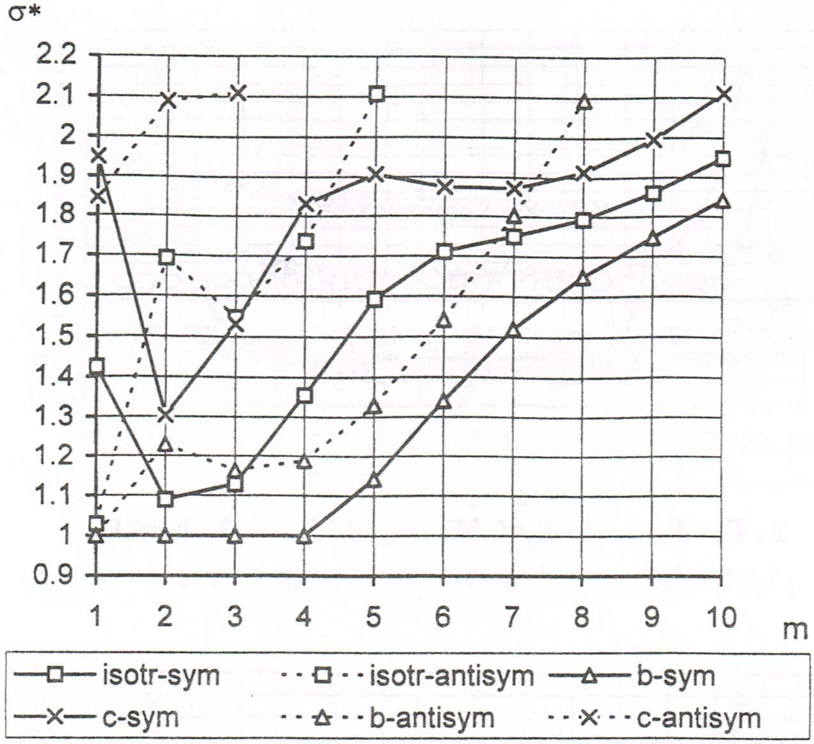


isotr - $E_y^p = E_x^p$; **a** - $E_y^p = 2E_x^p$; **b** - $E_y^p = 0{,}5E_x^p$;

Fig. 4a. Dimensionless buckling stress versus number of axial half-waves for different values of hardening moduli E_y^p (other material constants unchanged)

Analysing the influence of changes of hardening moduli that describe material characteristics slopes in the inelastic range the opposite phenomenon is observed.

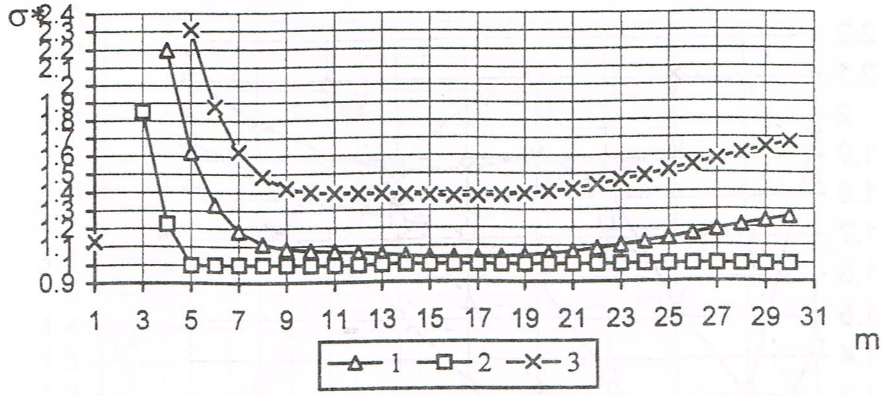
Change of the values of hardening modulus E_y^p (in the direction perpendicular to the direction of loading) practically does not affect the values of global and local stress (Fig.4a), while the change of values of plastic modulus E_x^p (in the direction of loading) has the significant influence on values of global and local buckling stresses (Fig.4b).



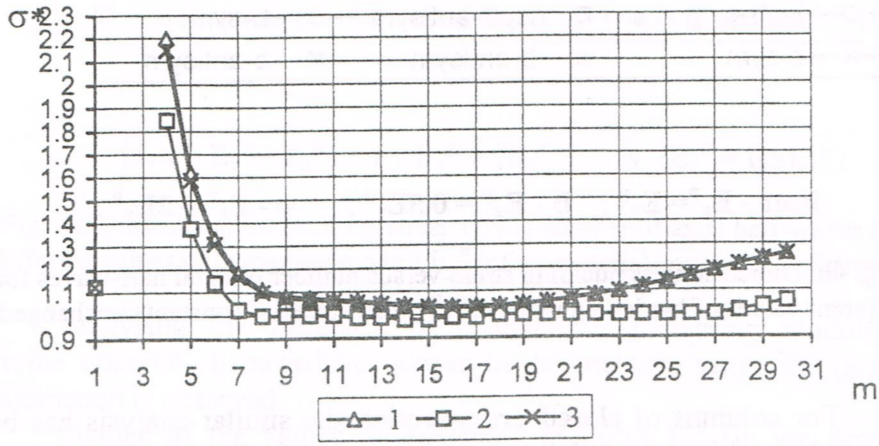
isotr - $E_x^p = E_y^p$; **b** - $E_x^p = 0{,}5E_y^p$; **c** - $E_x^p = 2E_y^p$;

Fig. 4b. Dimensionless buckling stress versus number of axial half-waves for different values of hardening moduli E_x^p (other material constants unchanged)

For columns of **closed cross-section** the similar analysis has been conducted. In this case only the results corresponding to the symmetry conditions along the cross-section symmetry axis are shown because the values of global flexural-torsional buckling stress (antisymmetrical) are much greater than flexural ones what is caused by a large torsional rigidity of a closed section.

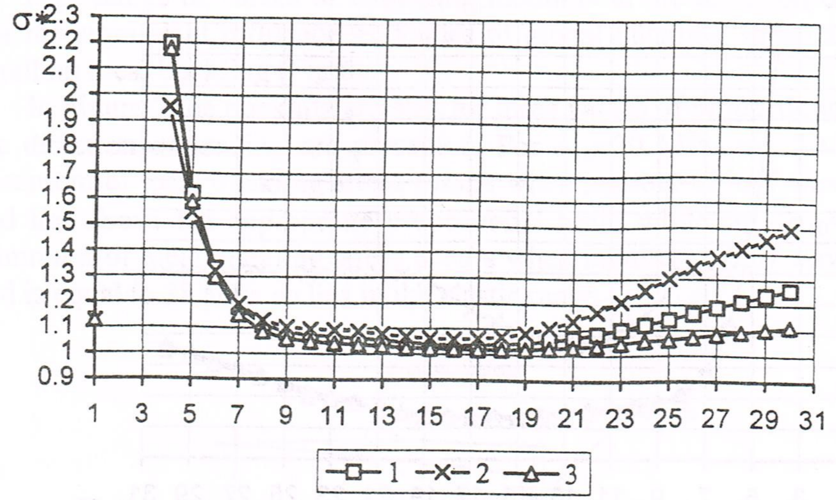


1 - $E_y = E_x$; **2 - $E_y=0,5E_x$**; $\nu_{yx}=0,15$; **3 - $E_y=2E_x$**; $\nu_{xy}=0,15$;

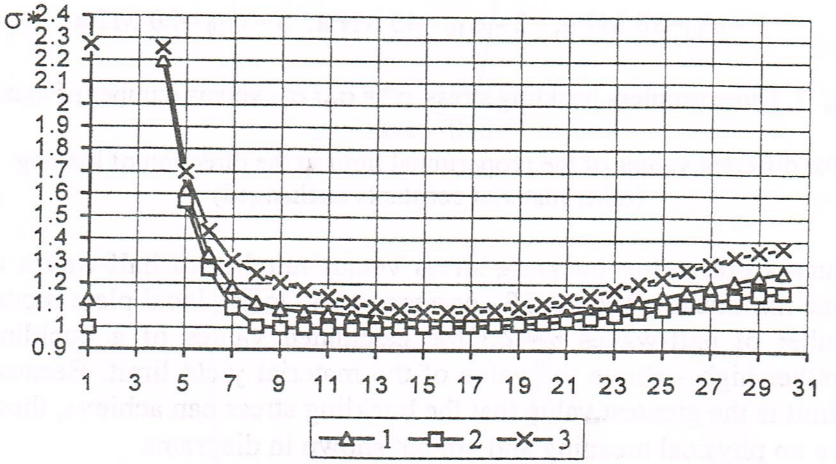


1- $E_x = E_y$ **2 - $E_x = 0,5E_y$** ; $\nu_{xy}=0,15$; **3 - $E_x = 2E_y$**; $\nu_{yx}=0,3$;

Fig. 5. Influence of values of elastic modulus E_y and E_x on the dimensionless buckling stress (other material constants unchanged)

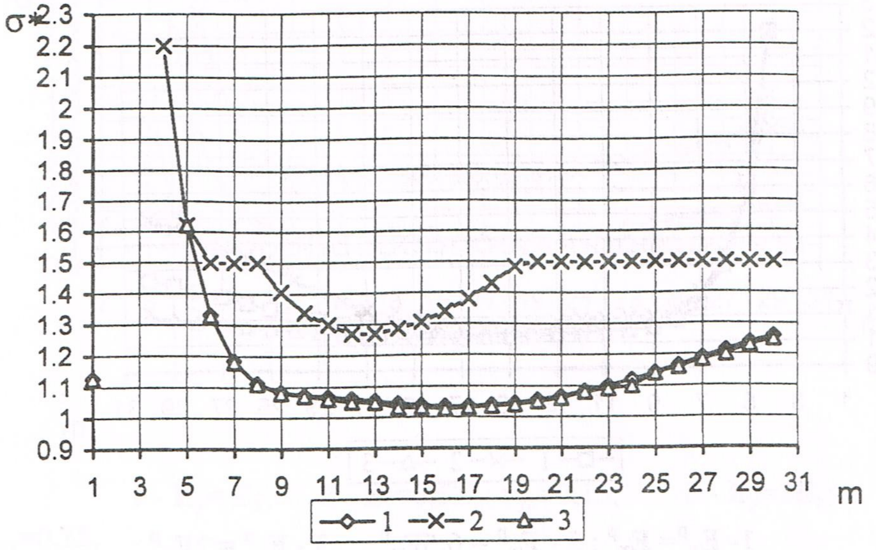


1- $E_y^p = E_x^p$; 2 - $E_y^p = 0{,}5E_x^p$;   3 - $E_y^p = 2E_x^p$;



1 - $E_x^p = E_y^p$;   2 - $E_x^p = 0{,}5E_y^p$;   3 - $E_x^p = 2E_y^p$;

Fig. 6. Dimensionless buckling stress versus number of axial half-waves for different values of hardening moduli E_x^p and E_y^p (other material constants unchanged)



1 - σ_{10}=30 MPa; 2 - σ_{10} =45 MPa; 3 - σ_{10} =20 MPa;

Fig. 7. Dimensionless buckling stress $\sigma^{*}=\sigma_{cr}/\sigma_{20}$ versus number of axial half-waves
for different values of the proportional limit in the direction of loading
(other material constants unchanged)

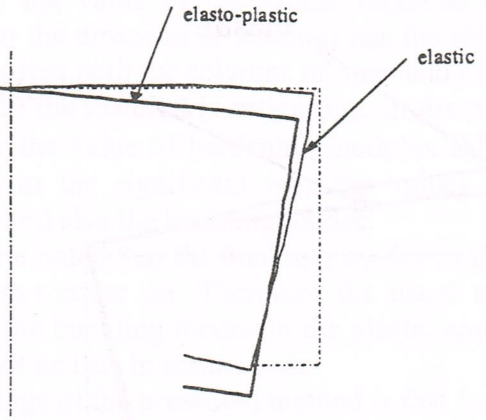
The curves presenting buckling stress versus number of half-waves *m* has only one minimum when *m>10* - corresponding to the local plate mode. When number of half-waves *m=2,3* the calculated values of a buckling stress lie rather high - above the value of the material yield limit. Because the yield limit is the greatest value that the buckling stress can achieve, these results have no physical meaning and are not shown in diagrams.

The different values of elastic moduli does not affect the global buckling stress but the change of elastic modulus E_y in the direction perpendicular to the direction of loading has a significant influence on the values of local buckling stress and number of half-waves m corresponding to the minimum (Fig.5).

The change of values of hardening modulus in the direction of loading has the significant influence on values of global buckling stress and rather small on local buckling (Fig.6).

In Figure 7 the buckling stresses for three values of proportional limit in the direction of loading are presented. For σ_{10}=20 MPa and 30 MPa the minimum of local buckling stress occurs when number of half waves m=17 and lies above the appropriate proportional limit, while for σ_{10}=45 MPa minimum of local buckling stress occurs when number of half waves m=12 and is equal to 38 MPa so lies in the elastic range.

a)



b)

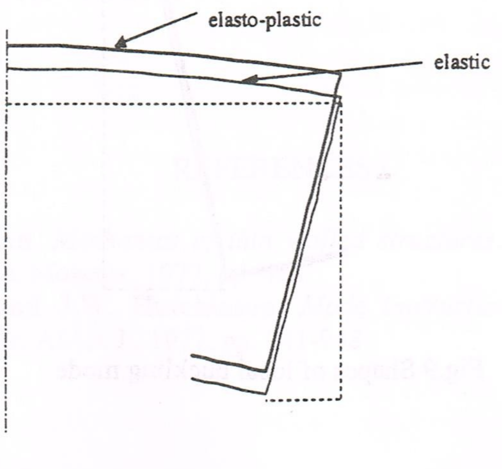


Fig.8. Shapes of global buckling modes

In Fig.8 the shapes of global buckling modes (the material constants as for results shown in Fig.4b curve a, m=1) are presented when antisymmetry (Fig.8a) or symmetry conditions (Fig.8b) are imposed along the cross-section symmetry axis. In the case of a column with open cross-section the minimum value of global buckling stress corresponds to the antisymmetry conditions and in the elasto-plastic range the buckling mode is pure flexural-torsional, while in the elastic range the corresponding mode is flexural-distorsional one ("mixed mode" [3], [5]). When the symmetry conditions are imposed the buckling modes in the elastic and elasto-plastic range are flexural ones.

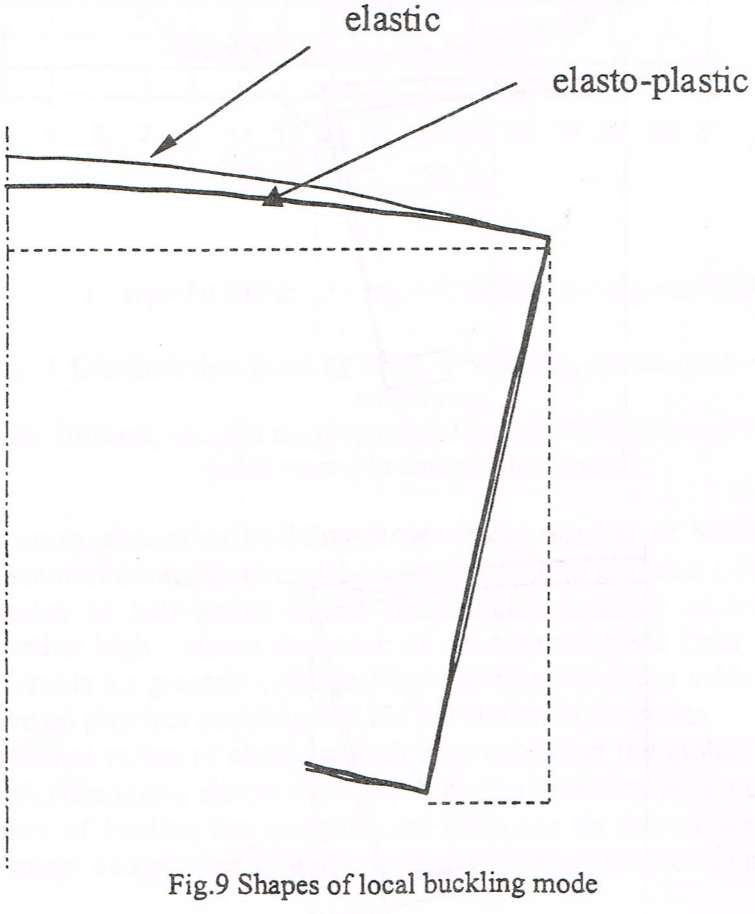


Fig.9 Shapes of local buckling mode

The shapes of local buckling modes that corresponds to the case "a" shown in Fig.4b (number of half-waves m=2) are presented in Fig.9. It can be seen that both modes (elastic and elasto-plastic) correspond to the "mixed" modes – local buckling plate mode with distortional one.

FINAL REMARKS

The results of numerical analysis show that:

a) in the case when the material is orthotropic in the elastic range ($E_x \neq E_y$, $E_x^p = E_y^p$) only the value of the elastic modulus E_y (in the direction perpendicular to the direction of loading) has the strong influence on the local buckling stress both for columns of open and closed cross-section,
b) in the case when the material is orthotropic in the plastic range ($E_x = E_y$, $E_x^p \neq E_y^p$) only the value of hardening modulus E_x^p (in the direction of loading) affect in the significant way the values of local and global buckling stress and also the buckling modes.

It should be noted that the buckling modes in the elastic and plastic range do not always cover up. Therefore the usual assumption, made in many works, that the buckling modes in the elastic and elasto-plastic range are identical can not be true in some cases.

The advantage of the presented method is that it allows to find out all possible buckling modes (global and local, symmetrical and antisymmetrical) and also to investigate "mixed modes" occurring in columns of open cross-sections.

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