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STABILITY OF AXIALLY MOVING PAPER WEB

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Abstract

Dynamic analysis of axially moving paper web at under- and supercritical transport speeds is presented in this paper. A new mathematical model derived from moving orthotropic plate has been used in dynamic investigations. Numerical calculations have been carried out for kraft paper and filter paper. To compare results a membrane model of both paper webs has been investigated as well.

In undercritical region of transport speed natural frequencies of the web decrease during the axial velocity increase. For both papers the critical speeds have been identified at which the fundamental natural frequency vanishes indicating divergence instability of the system. The change of the web orthotropy factor in the published range of experimental results does not influence the critical speed value. The rolls support stiffness influences in the most rate the critical speed of paper web. At supercritical transport speeds between divergence and flutter instability areas the second stability region may appear. The width and position of the second stable region depend on the slenderness ratio of the web and the rolls support stiffness.

Introduction

In the paper production line there is the rewinding-and-slicing machine as the basic auxiliary device. Usually this device is bounded with the paper machine. The operating speed of the rewinding-and-slicing machine has to be higher than the value of analogical parameter of the paper machine, to prevent the paper machine capacity decrease. It is well known that the operating speed of contemporary paper machines exceeds significantly the value 1000 m/min. Hence, the operating speed of contemporary rewinders exceeds 2500 m/min. In many cases one can observe wrinkling and breaking of the paper web under so high transport speeds. Thus, characterization of vibration and dynamic stability of the moving web is requisite for analysis and optimal design of such technological devices as the rewinding-and-slicing machines.

A lot of earlier works in dynamics field focused on dynamic investigations of string-like, beam-like and plate-like axially moving isotropic systems [1],[2]. Recent works analyse equilibrium displacement, stress distribution [3] , the wrinkling phenomenon [4] and stability of axially moving isotropic plate [5]. On the other hand a lot of earlier works in paper physics field show that paper belongs to materials behave elastically as orthotropic continua. For example, Jones [6] carried out experimental investigations of the in-plane elastic modules of different kinds of papers and compared them with theoretical dependencies. On the base of received results one can say that in linear region of the tensile stress-strain dependence all investigated papers may be treat in plane as orthotropy materials. Uesaka [7] investigated biaxial tensile behaviour of paper in experimental way. Tests concerned wide scope of tensile stresses until the breaking stress. Investigations results for three different kinds of papers show that the Poisson ratio increases during the tensile stress increase. Seo [8] shows an accurate method for determination of the in-plane shear modulus of machine-made paper. The in-plane shear modulus and Poisson ratio were received from measurements of the Young modulus and the shear modulus conducted at various orientation angles. Also a simply criterion has been established for the optimum angular region, where the measurements should be performed to achieve the best accuracy.

The aim of this paper is to analyse dynamic behaviour of two different kinds of paper webs in undercritical and supercritical regions of transport speed. To derive the equations of motion, non-linear thin-walled orthotropic plate theory has been modified to include the inertial forces resulting from the moving web [9]. In this paper there are results of numerical investigations, which show the linear problem solution. The effect of axial transport speed, orthotropy properties of the web and the rolls support system on transverse and torsional natural frequencies and stability of the web motion are presented.

1. Mathematical model of the moving web system

Long elastic moving web of the length l is considered. System coordinates and geometry are shown in Figure 1. In theoretical investigations the considered web is composed of plane rectangular plate segments of principal axes of orthotropy parallel to their edges. These moving plates are interconnected along the longitudinal edges. The orthotropic materials of the moving web obey Hook's law. Because of significant transport velocity elasto-plastic deformation and reologic phenomena have not been taken into account.

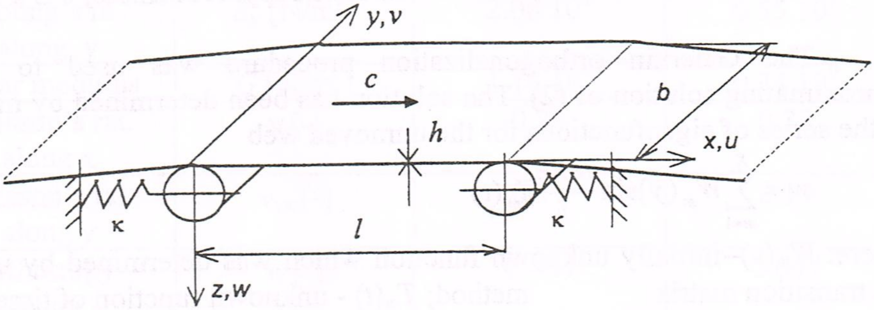

Fig.1. Axially moving web

The web moves with constant velocity c. One assumes that the web is tensioned only in its longitudinal direction. The relationships among stresses and strains in the web are formulated in the following way:

$$N_x = \frac{Eh}{1-\eta\nu^2}(\varepsilon_x + \eta\nu\,\varepsilon_y); \quad M_x = -D\,(w_{,xx} + \eta\nu\,w_{,yy});$$
$$M_y = -\eta\,D\,(w_{,yy} + \nu\,w_{,xx}); \quad M_{xy} = -D_1 w_{,xy} \tag{1}$$

where a comma denotes partial differentiation; $\eta = E_y / E$ – orthotropy factor; E, E_y - Young's modulus along x and y axes, respectively; G– modulus of non-dilatational strain; ν - Poisson's ratio; ε_x, ε_y - strain tensor components; N_x-stress resultant along the x direction;

M_x, M_y, M_{xy} - bending moment resultants for the web;

$$D = \frac{E\,h^3}{12(1-\eta\nu^2)}$$ - flexural stiffness of the web;

$D_1 = \dfrac{G\,h^3}{6}$ -torsional stiffness of the web.

The linear differential equilibrium equations of the web motion resulting from the Hamilton's principle can be written as follows (derivation details [9])

$$\rho h(w_{,tt} + 2cw_{,xt} + \kappa c^2 w_{,xx}) - N_x w_{,xx} + Dw_{,xxxx} + 2(\nu\eta D + D_1)w_{,xxyy} + \eta D w_{,yyyy} = 0 \quad (2)$$

where: ρ - mass density of the paper;

κ - dimensionless coefficient of the rolls support stiffness, $0 \le \kappa \le 1$.

The Galerkin orthogonalization procedure was used to find approximating solution of (2). The solution has been determined by means of the series of eigenfunctions for the unmoved web

$$w = \sum_{m=1}^{K} W_m(y) \sin\frac{m\pi x}{l} T_m(t) \quad \ldots.. \quad (3)$$

where: $W_m(y)$–initially unknown function which was determined by using the transition matrix method; $T_m(t)$ - unknown function of time. On the base of (2) and (3) one can determine eigenvalues and eigenvectors and investigate stability of the system [9].

2. Numerical results and discussion

Numerical investigations have been carried out for two different kinds of paper: kraft and filter paper. Parameters values are given in Table 1.

Fig.2 and Fig.3 show the plots of the lowest transverse natural frequency versus the axial transport speed for the kraft paper and the filter paper, respectively. These plots are shown for three various values of the rolls support stiffness. For the value $\kappa = 0$, the rolls are free to displace relative to each other under the tension variation. For $\kappa = 1$, two rolls are rigidly fixed with respect to each other, eliminating the web tension changing with the axial transport speed. For the values $0 < \kappa < 1$, the rolls

Table 1. Numerical data [6]

Parameters	Symbols	Kraft paper	Filter paper
length	l [m]	1.194	1.194
width	b [m]	0.597	0.597
thickness	h [mm]	0.3	0.3
mass density	ρ [kg/m^3] ([g/m^2])	133.33 (40)	25.0 (7.5)
axial stress	N_x [N/m]	55.0	55.0
Young's m. along x	E [N/m^2]	4.48 10^9	0.85 10^9
Young's m. along y	E_y [N/m^2]	2.08 10^9	0.55 10^9
shear modulus	G [N/m^2]	1.14 10^9	0.24 10^9
Poisson's rat. along x	ν [-]	0.26	0.27
Poisson's rat. along y	ν_{yx} [-]	0.11	0.19
orthotropy factor	η [-]	0.464	0.647

support system has finite stiffness and the axial tension decreases with transport speed. Results in Fig.2 and 3 show for the constant axial tension of the web that the lowest eigenfrequency decreases with the axial velocity increase at a rate dependent on the rolls support stiffness. For all investigated values of $\kappa > 0$ a velocity exists, at which the fundamental natural frequency vanishes, indicating divergence instability of the system. The increase of the rolls support stiffness diminishes the critical axial speed of the web. Numerical results are compared with the membrane model ($D = 0$) of the kraft paper (Fig.2) and the filter paper (Fig.3). The critical transport speed determination by the membrane model is much better in the case of the filter paper. For the rigid rolls support $\kappa = 1$ this model overestimates the critical speed value and the error is less than 14%.

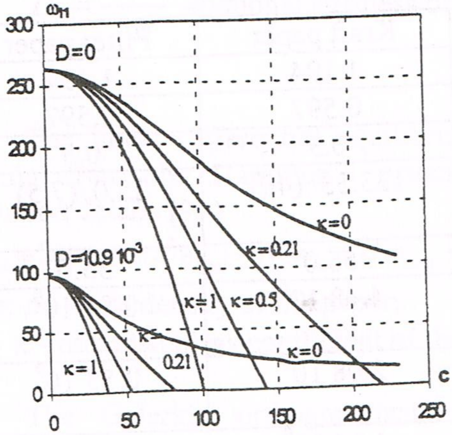

Fig.2. The lowest transverse eigenvalues (kraft paper)

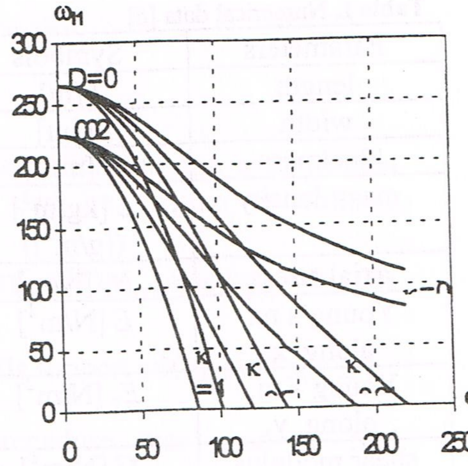

Fig.3. The lowest inverse eigenvalues (filter paper)

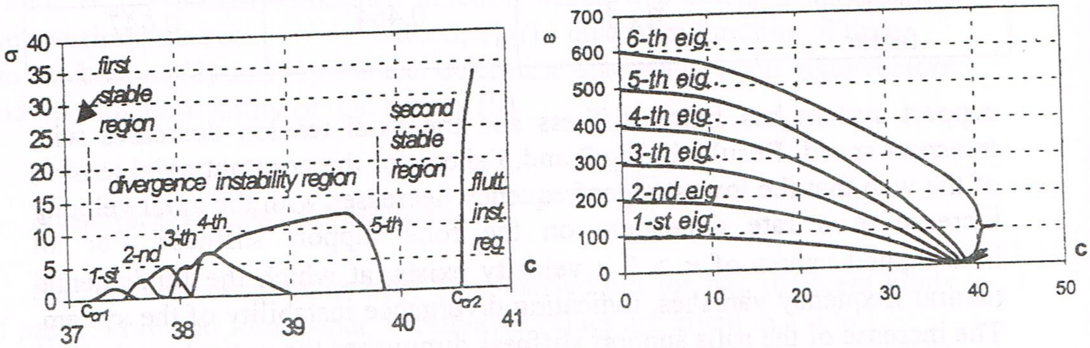

Fig.4. The real part (σ) and imaginary part (ω) of the lowest transverse eigenvalues (kraft

Dynamic behaviour of the paper web was investigated above the critical transport speed as well. Let σ and ω denote the real part and the imaginary part of eigenvalues, respectively. The non-zero value of σ indicates instability of the system and ω is natural frequency of the web. To show dynamic behaviour in supercritical area the first six complex eigenvalues are presented in Fig.4a,b for the kraft paper and in Fig.5a,b for the filter paper. In supercritical transport speeds at first the web experiences divergent instability (the fundamental mode with non-zero σ and zero ω) and next flutter instability (non-zero σ and non-zero ω). The second critical transport speed is denoted by c_{cr2}. Between these two instability regions there is a second stability region where σ = 0. The width and position of the second stable region are dependent on the rolls support stiffness and are practically independent on the orthotropy factor of the web.

The plot of the critical transport speeds versus the rolls support stiffness and the orthotropy factor for the kraft paper is shown in Fig.6 and for the filter paper in Fig.7. In the filter paper case the divergence instability region is very narrow. Bold line in Fig.7 represents this region. Both plots show that the width of the second stable region increases when the rolls support stiffness diminishes. For the specified stiffness the first critical speed c_{cr1} and the second critical speed c_{cr2} have the same values for any orthotropy factor from the range $0.464 \leq \eta \leq 1$ for the kraft paper and from the range $0.647 \leq \eta \leq 1$ for the filter paper, respectively. The change of the web orthotropy factor corresponds to the published experimental results of this factor for various kinds of paper.

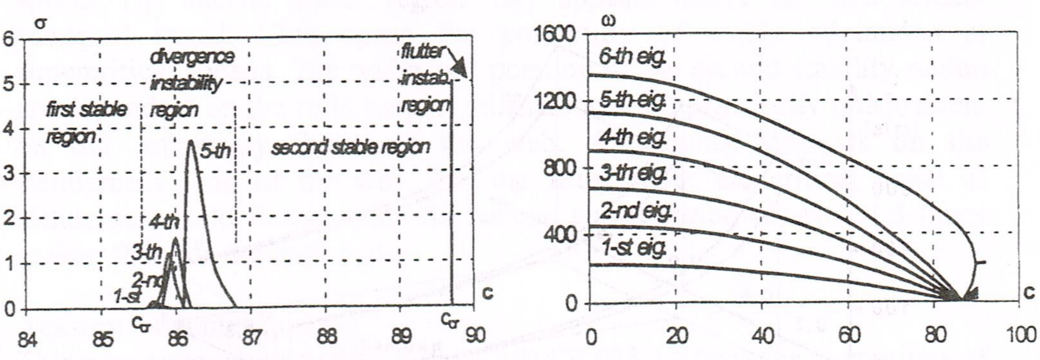

Fig.5. The real part (σ) and imaginary part (ω) of eigenvalues (filter paper)

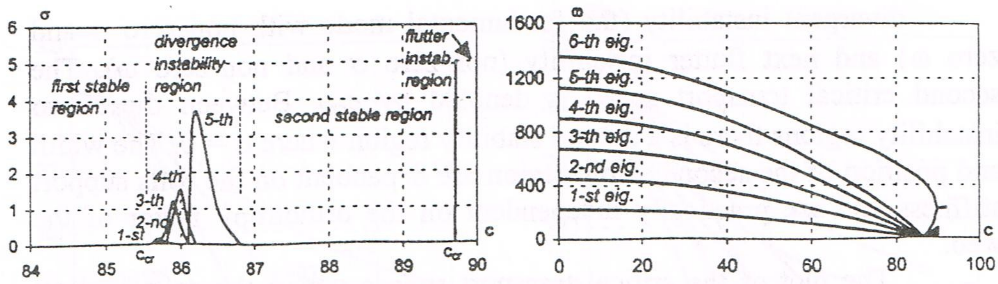

Fig.5. The real part (σ) and imaginary part (ω) of eigenvalues (filter paper)

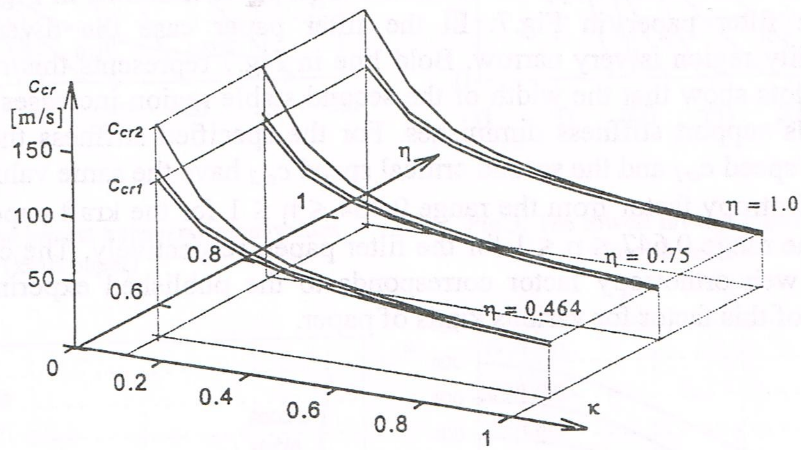

Fig.5a. Critical transport speeds of the kraft paper for various orthotropy factors

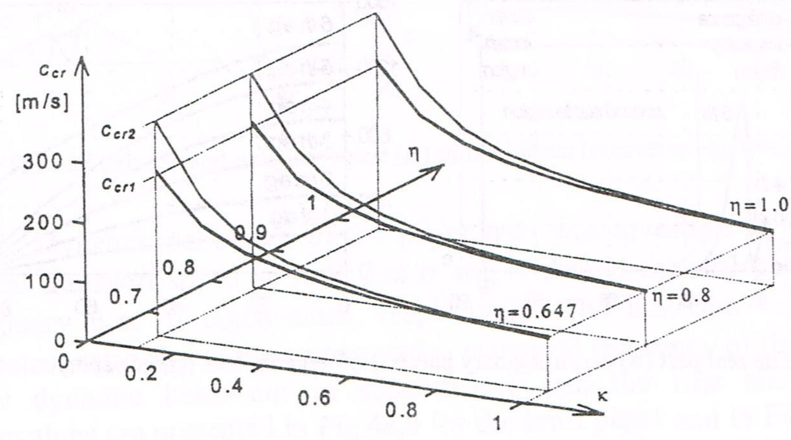

Fig.5b. Critical transport speeds of the filter paper for various orthotropy factors

Appearance of the second stability region depends on the slenderness ratio of the web. For the investigated kraft paper this region may exist for webs with the slenderness ratio greater than $\xi = l / b = 0.92$. For the investigated filter paper the critical slenderness ratio is equal $\xi = 1.35$.

3. Conclusions

In the paper dynamic behaviour of axially moving paper web is presented. The equilibrium equation of motion is derived from the Hamilton's principle on the base of the thin plate model of axially moving web. Results of published experimental investigations have been used as the data in numerical calculations. Numerical investigations have been carried out for kraft paper and filter paper.

In subcritical region of transport speed for the constant axial tension of the web the lowest transverse natural frequency decreases during the axial velocity increase. At first critical transport speed the fundamental eigenfrequency vanishes indicating divergence instability. The rolls support stiffness influences in the most rate the first critical speed value. The critical speed determination by the membrane model of the web has a sense only in the filter paper case.

In supercritical region of transport speed at first the web experiences divergent instability and next flutter instability above the second critical speed. The second stable region may appears above the first critical transport speed. This opens the possibility of stable operations at supercritical speeds. The width and position of the second stability region are dependent on the rolls support stiffness and are practically independent on the orthotropy factor of the web. Appearance depends on the slenderness ratio of the web. For the filter paper the critical value of slenderness ratio above which the second stable region exists is 1.5 times greater than for the kraft paper.

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