THE INFLUENCE OF THE CLEARANCE ON THE MOTION OF IMPACT OSCILLATOR

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Abstract

The numerical analysis of the influence of the clearance on the motion's character of impact oscillator is general purpose of this paper. The basic criterion of distinguishing between the various kinds of motion was the number of impacts of the masses per one period of the exciting force. The regions of existence of the motion with exactly two impacts are included in the regions of the results of the theoretical analysis or are very close to them. The presented investigations confirm the basic properties of systems with impacts.

Introduction

The phenomenon of vibrations with impacts, i.e. of the so called impact oscillators [1], occurs in many branches of technology. In numerous cases, e.g. in impact machines, vibration dampers or any type of shakers, this phenomenon plays a very useful role. On the other hand, however, its occurrence is very often undesirable, as it causes e.g. additional dynamic loads, as well as faulty operation of machines and devices.

The importance of problems of dynamics of mechanical systems is manifested by the fact that theoretical and experimental investigations of them have been carried out in various scientific centres for many years. One of such centres is the Division of Dynamics, Technical University of Lodz, which together with the Institute of Termomechanics

in Prague and Strathclyde University in Glasgow, deals with analysis of relations between parameters of such systems and the character of their motion [2-6] and the occurrence of the phenomenon of intermittence [2].

Within the present investigations, a detailed analysis of a mechanical system motion, in which impacts forced by an external force occur has been conducted. The knowledge of the dynamics of such a vibratory-impact system will let one employ this system in a better way, avoid faulty operation of machines and provide data which can be used in calculations and design of such constructions in the future.
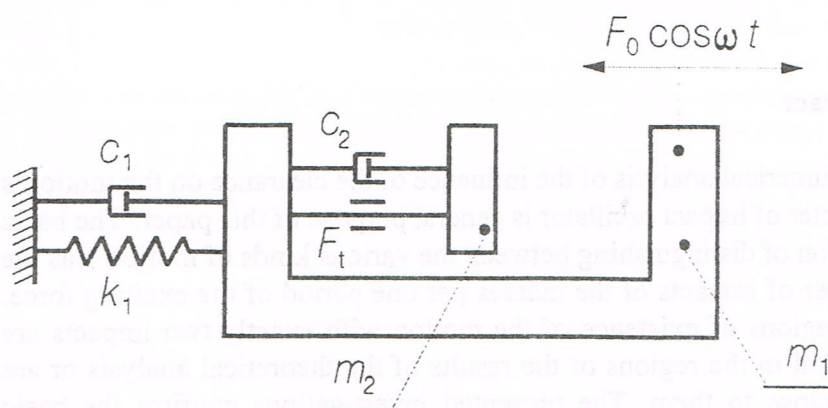

Figure 1. Model of the system.

In this paper we consider a simple physical system shown in Fig. 1. The mass m_1 is connected to a vibrator giving the sinusoidal force $F_0 \cos \omega t$ through the spring-damper system with the stiffness coefficient k_1 and the damping coefficient c_1. The second mass m_2 is placed on the mass m_1 and its movement is limited by two stops. The motion of the mass m_2 on the mass m_1 is influenced by the friction force F_t.

The main purpose of this paper is numerical analysis of the influence of the clearance on the motion's character of impact oscillator and to show it graphically in the form of regions of existence of different kinds of impact motion for our system.

Owing to the fact that the main parameters of the system have been taken from the real experimental system, we could investigate the basic properties of the system with impacts, compare the results of the theoretical analysis [6] with the results of the computer simulation and conclude whether this simulation is correct or not.

Dimensionless equations of motion

The considered model can be described by the following dimensionless equations:

$$X_1^{||} + b_1 X_1^{|} + X_1 + \lambda - b_1 \delta (X_1^{|} - X_2^{|}) = \cos \eta \, \iota$$
$$X_2^{||} - (\lambda / \mu) - (\delta / \mu) b_1 (X_1^{|} - X_2^{|}) = 0 \tag{1}$$

where: c_1, c_2 - coefficients of viscous damping, r - static clearance, F_t - friction force, F_0 - amplitude of excitation force, k_1 -spring stiffness, m_1, m_2 - masses of bodies, R -coefficient of restitution, $x_{st} = F_t / k_1$ - static displacement, ω - angular frequency of the excitation force, $\Omega_1 = (k_1/m_1)^{1/2}$ -own frequency of the system, $b_1 = c_1/\Omega_1$, $b_2 = c_2/\Omega_1$, $\bar{r} = r/x_{st}$ -clearance, $X^{|} = dX/d\tau$, $X^{||} = d^2X/d\tau^2$, $\eta = \omega/\Omega_1$, $\lambda = F_t/F_0$, $\tau = \Omega_1 t$ -time transformation, $\mu = m_2/m_1$, $\delta = b_2/b_1$, $f_T = F_t/m_2 g$ and g is gravitational acceleration. The equations describing the conditions of impact are included in e.g. [3].

To describe the dry friction force was considered a linear model given by the relation

$$f_T = \lambda_0 \, \text{sgn} (X_1^{|} - X_2^{|}) \tag{2}$$

where λ_0 is a friction coefficient depending on the surface in contact [7].

Numerical results

This chapter demonstrates how the results of a theoretical analysis of the fundamental impact motion [6] can be not only explained in detail by analogue computer simulation of the system motion but also completed by an analysis of more complex motions, whose theoretical analysis is very difficulty or impossible.

The motion of the mathematical system with impacts depends on the combination of the following parameters: μ, R, b_1, δ, λ, $\bar{r}$ and η.

For the system from Fig. 1 with the parameters $\mu = 0.693$, $R = 0.6$, $b_1 = \delta = 0$, $\lambda = 0$ we established the regions of existence of different kinds of impact motion depending on the frequency $\eta = \omega/\Omega_1$

of the existing force and on the relative clearance $\bar{r} = r\, k_1/F_0$, (Fig. 2). The boundaries of the regions were determined by changing the parameters ω, r in very small steps during sufficiently long intervals, so
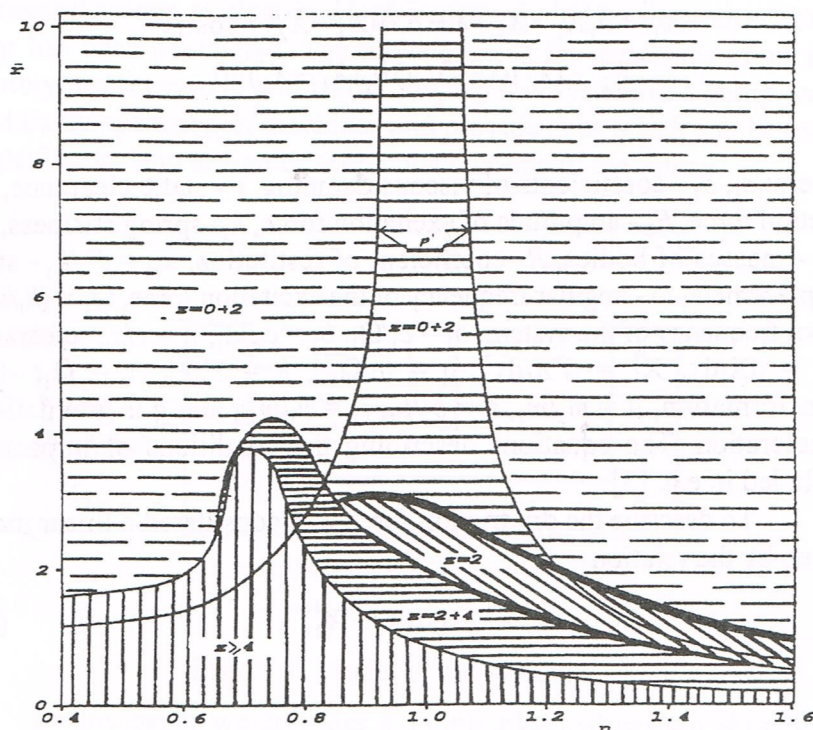

Figure 2. Regions of existence of different kinds of the impact motion
for our system for $b_1 = \delta = \lambda = 0$.

that the boundaries may be considered the boundaries of existence of a certain stationary motion. The kind of the motion is indicated by hatching in the region. In the regions marked by dashed lines, impact motion need not arise. In the darkened regions, a quasi-periodic motion exists.

The basic criterion of distinguishing between the various kinds of motion was the number z, i.e. the number of impacts of the masses per one period of the exciting force τ. The regions of existence of the motion with exactly two impacts ($z = 2$ - see the region with hatched sloping lines) are included in the regions of earlier results of the

attaining properties of an impact-beat motion ($z = 0$ to 2, Fig. 5b). The impact-beat motion is characterised by periodic changes of the amplitudes of the motion of the system masses within certain limits. The regions with such behaviour border with the regions which are characterised by exactly two impacts per one period of the motion ($z = 2$,
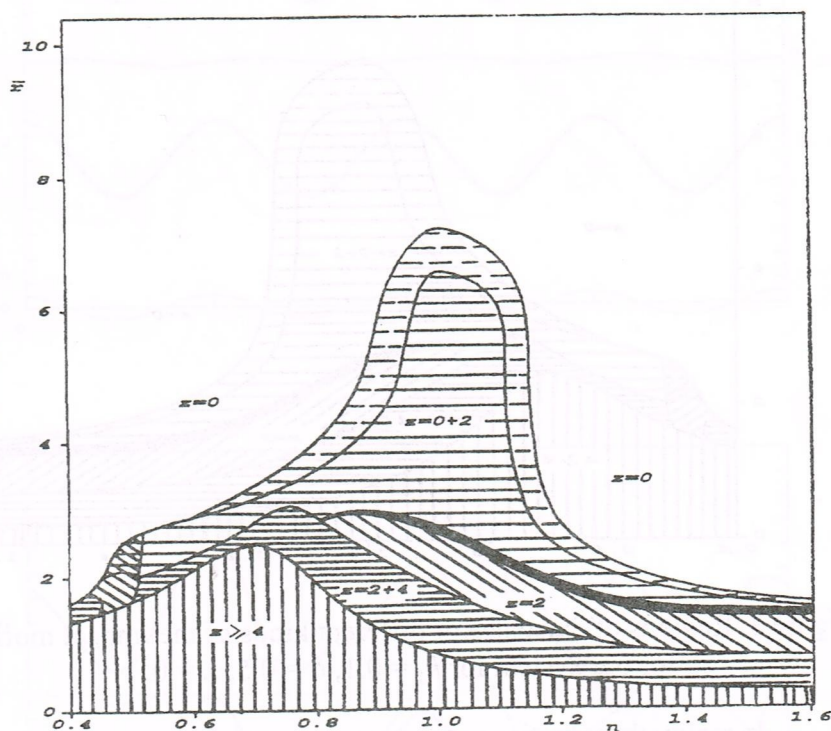

Figure 4. Regions of existence of different kinds of the impact motion for our system for $b_1 = 0.1$, $\delta = 0.5$, $\lambda = 0.02$.

Fig. 6a). Next, the fundamental two-impact motion becomes asymmetric (Fig. 6b), and with a further change in r, this motion becomes periodic, but with the period two, i.e. the period of the motion repeats after two periods of the exciting force (Fig. 7a). A wide range is occupied by the motion in which $z = 2 \div 4$ (Fig. 7b), it borders with a four-impact and multi-impact motion ($z \geq 4$, Fig. 8a, 8b). This is an impact motion with more than two impacts occurring during the period τ of the exciting force.

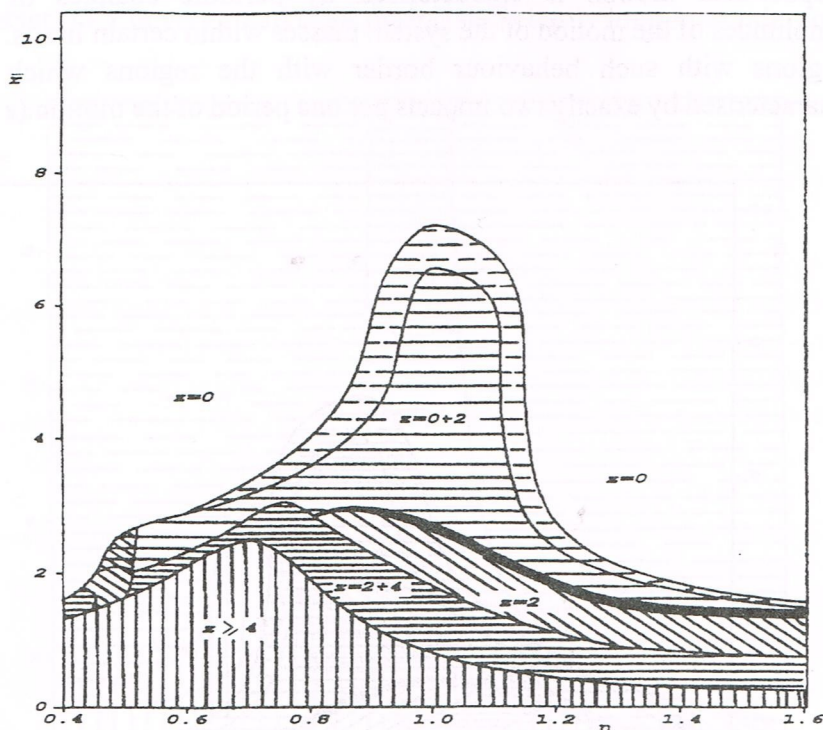theoretical analysis or are very close to them [6].

Figure 3. Regions of existence of different kinds of the impact motion for our system for $b_1 = 0.1$, $\delta = 0.5$, $\lambda = 0$.

In reality the vibrations of the masses m_1 and m_2 are subjected to damping proportional to the velocity of the motion, which enlarges the region of existence of the two-impact motion (see Fig. 3 showing the influence of the linear damping $b_1 = 0.1$, $\delta = 0.5$ on the displacement of the boundaries of existence of the impact motions) and permits to use impact dampers in a larger frequency interval of the exciting force. In the cross-hatched regions two different motions can exist. Which of them will arise depends on the initial conditions of the motion, external impulse, direction of the change of the parameters ω, r, etc.

In the region with a large value of r, the motion without any impacts occurs ($z = 0$, Fig. 5a). When this value is decreased, the motion of this system becomes completely free with a tendency towards
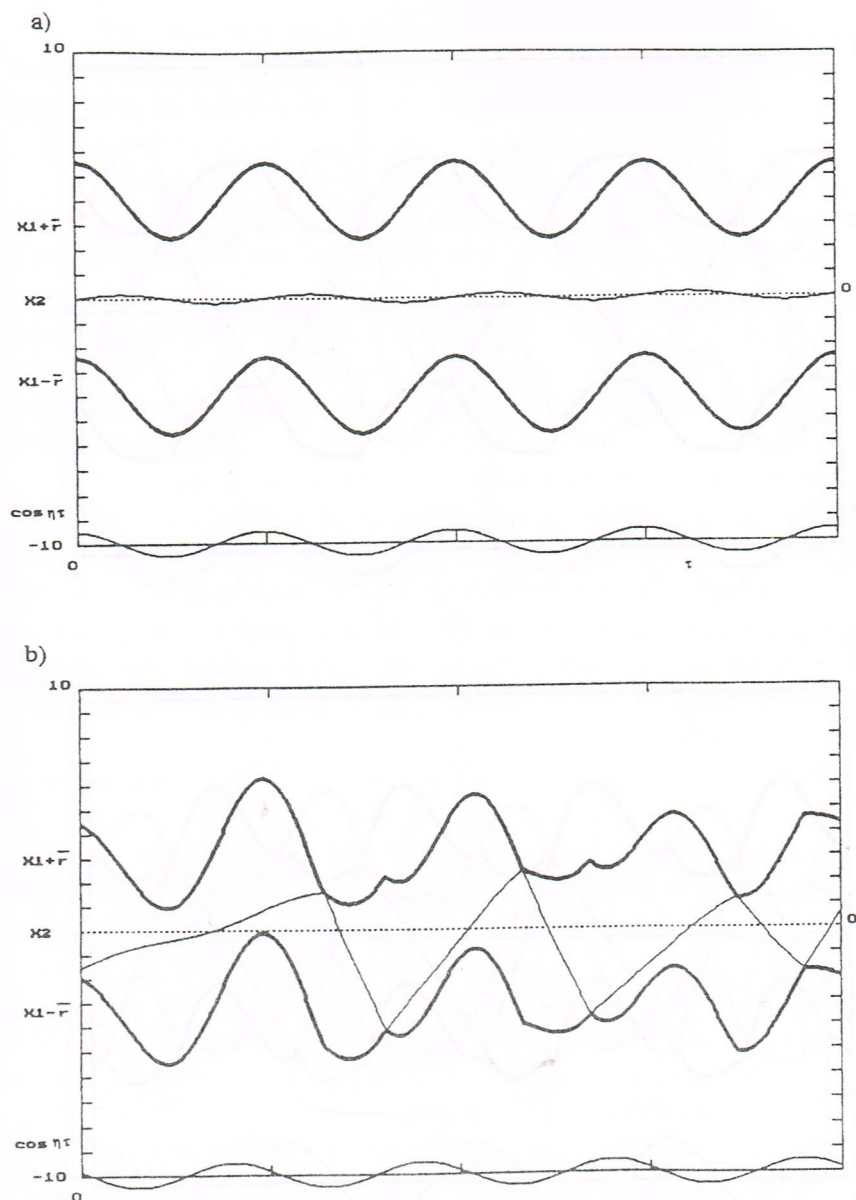
a)

b)

Figure 5. Examples of various kinds of the motion of the system with impacts for parameters: $R = 0.6$, $b_1 = 0.1$, $\delta = 0.5$, $\lambda = 0$; a) $\eta = 0.6$, $\bar{r} = 4$; b) $\eta = 0.9$, $\bar{r} = 3.2$.
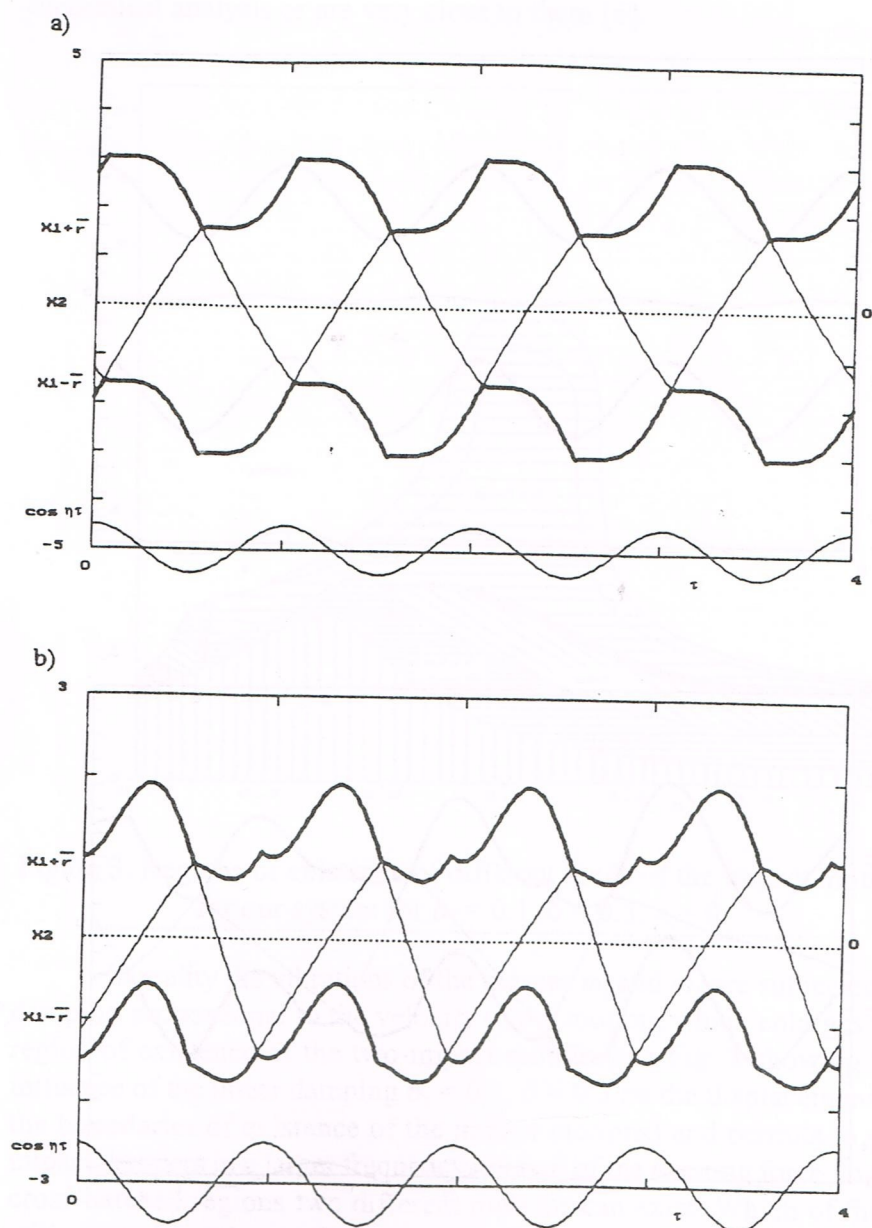
a)

b)

Figure 6. Examples of various kinds of the motion of the system with impacts for parameters: $R = 0.6$, $b_1 = 0.1$, $\delta = 0.5$, $\lambda = 0$; a) $\eta = 1.05$, $\overline{r} = 2.3$; b) $\eta = 1.15$, $\overline{r} = 1.23$.

a)

b)

Figure 7. Examples of various kinds of the motion of the system with impacts for parameters: $R = 0.6$, $b_1 = 0.1$, $\delta = 0.5$, $\lambda = 0$; a) $\eta = 1.15$, $\overline{r} = 1.2$; b) $\eta = 1.15$, $\overline{r} = 1$.
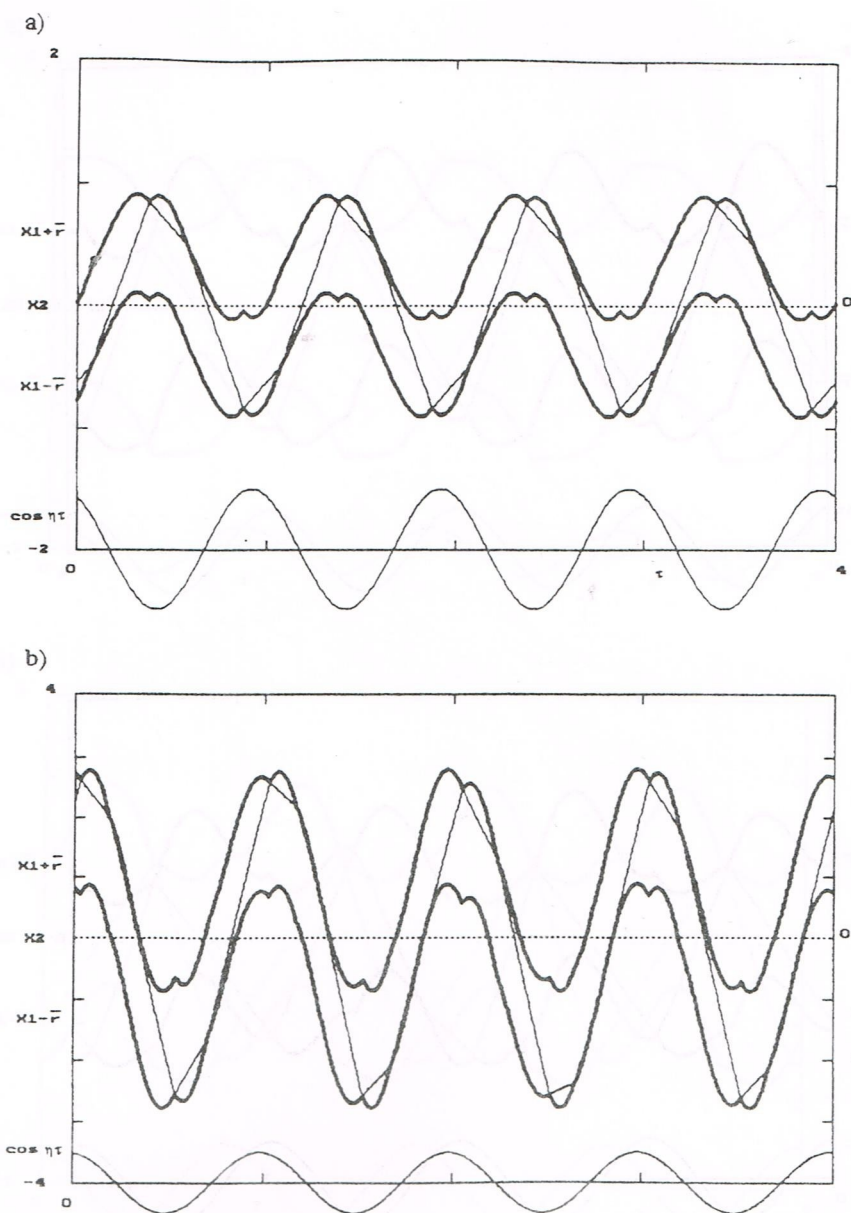
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Figure 8. Examples of various kinds of the motion of the system with impacts for parameters: $R = 0.6$, $b_1 = 0.1$, $\delta = 0.5$, $\lambda = 0$; a) $\eta = 1.2$, $\overline{r} = 0.4$; b) $\eta = 0.52$, $\overline{r} = 0.95$.

Fig. 4 depicts the regions of existence of different kinds of the motion, taking into account a friction force ($\lambda = 0.02$, the remaining parameters have been introduced without any changes). It describes the behaviour of the system from Fig.1.

Comparing these two last diagrams, one can see an influence of the friction force manifested by a slight enlargement of the two-impact region situated on the left ($z = 2$) at the expense of a decrease in the beat-motion region ($z = 2 \div 4$). It is advantageous, taking into consideration the fact that it is possible to use this system as an impact damper. Moreover, attention should be also paid to another property - a system with a friction force achieves more quickly the boundary of an impactless motion.

Comparing the results, we find that the impact motion of individual systems is of the same character, with the fundamental, beat and multi-impact motions existing in all of these systems. The average number z of impacts per one period of the exciting force grows with the decreasing static clearance $\bar{r}$. With the aid of Figs. 2, 3 and Fig. 4 and on the basis of existing studies, we can generalise some of the properties ofthe impact motion.

Conclusions

A simple system with impacts has been analysed in the present investigation. By means of the computer simulation three systems with impacts have been investigated with regards to the parameters obtained from the real impact system. Each of these systems can operate as an impact damper of vibrations. The basic criterion of distinguishing between the various kinds of motion was the number of impacts of the masses per one period of the exciting force. The average number of impacts per period of the exciting force grows with the decreasing static clearance. The regions of existence of the motion with exactly two impacts are included in the regions obtained in [6] as the results of the theoretical analysis or are very close to them. The presented numerical analysis confirms the fundamental properties of systems with impacts.

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