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STABILITY OF A SHAFT ROTATING WITH FLUCTUATING ANGULAR VELOCITY

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Abstract

In this paper the technique of the dynamic stability analysis proposed for the conventional laminated structures is extended to the structures rotating with the time-dependent angular velocity. The rotating angle-ply symmetrically laminated circular cylindrical shell is treated as a beam-like structure. The shaft is subjected to a constant torque. The velocity stochastic component is assumed in the form of the wide - band Gaussian processes modelled as a Wiener process. The fluctuating component of angular velocity implies a stochastic parametric excitation of shaft motion. The structure buckles dynamically when the axial parametric excitation becomes so large that the structure does not oscillate about the unperturbed state, and a new increasing mode of oscillations occurs. The uniform stochastic stability criteria involving a damping coefficient, a rotation speed and geometrical and material parameters are derived using Liapunov's direct method. Formulas determining dynamic stability regions are written explicitly.

Introduction

The dynamic stability of isotropic elastic shafts has been studied for several years (cf. Bishop [3], Tylikowski [13]). The increased use of advanced composite materials in various applications has caused a great research effort in the structural dynamic and acoustic analysis of composite materials. Latest works in this area have been shown that the dynamic stability regions are highly sensitive to structural parameters. The dynamics of laminated composiyes has been object of considerable attention over the past quarter of the century. The first analysis of the stability of simply supported laminated shafts rotating with a constant speed is due to the present author [12]. Using the perturbation technique Kammer and Schlack [6] investigated the effect of time-dependent angular velocity on the vibration of a rotating beam. Natsiavas [8] analysed nonlinear dynamic response of circular rings rotating with spin speed which involves small fluctuations from

a constant average value. The uniform stability of laminated shaft modelled as composite shells rotating with a constant angular velocity under the combined axisymmetric loading was investigated by Tylikowski [16]. Recently, composite materials find an increased range of applications for high-performance rotating shafts (e.g., see Napershin and Klimov [9], and Bauchau [2]). Thin-walled standard angle-ply laminated tubes meet relatively easy the requirements of torsional strength and stiffness but are more flexible to bending and have specific elastic and damping properties which depend on the system geometry, physical properties of plies, and on the laminate arrangement. Such systems are also sensitive to a lateral buckling. The Shape Memory Alloy (SMA) hybrid composites are a class of materials capable of changing both their stiffnesses through the application of in-plane loads and their elastic properties. The stiffness modification occurs as a result of the thermally induced martensite phase transformation of the SMA fibers which are embedded in standard laminated composite structures. The Young's modulus of the nitinol (Nickel-titanium alloys), which is an example of such a material increases 3 to 4 times when the temperature changes from that below M_f (i.e., in the martensite phase) to that above A_f (i.e., in the austenite phase). The damping of vibrations in the SMA due to internal friction exhibits also an important characteristics. The low-temperature martensic phase is characterised by a large damping coefficient while the high-temperature austenic phase shows a low damping coefficient. The decrease ratio is approximately equal to 1 : 10. Comprehensive studies of eigen-frequencies and eigen-functions of the SMA hybrid adaptive panels with uniformly and piecewise distributed actuation have been presented in papers [1], and [10].
One of the first important studies applying the direct Liapunov technique to was a rotating shaft stability analysis [15] in which the closed form analytical criteria were derived. The dynamic stability criterion of rotating composite shafts subjected to a plain axial force was reduced to an effective algebraic inequality [12].

This work investigates a dynamic stability of thin-walled shafts rotating with angular velocity which involves fluctuations from a constant average value. The time-dependent spin speed variations introduce new terms to dynamic equations and lead to the parametric excitation. In order to analyse behaviour of solutions of dynamic equations we introduce a measure of distance, $\|\cdot\|$, of the solution of dynamics equations with nontrivial initial conditions from the trivial solution. Using the appropriate energy-like Liapunov functional, the sufficient stability conditions for the uniform stability of the shaft equilibrium are derived. In our dynamics study the rotating angle-ply symmetrically laminated circular cylindrical shell will be treated as a beam-like structure. The reduction is justified by a symmetric plies arrangement and negligible circumferrential stresses in the shaft (see [2]). Despite the fact that in case of viscoelastic orthotropic plies the resulting constitutive equation is of higher order, the simple Voigt-Kelvin model is assumed. The stochastic parametric excitation is assumed to be a wide-band Gaussian process. Thus, it can be written in terms of the Wiener process and dynamics of shaft have to be understood in the stochastic sense. The dynamic equations are rewritten in as Itô differential equations in a suitable Hilbert space. The shaft is subjected to the torque destabilizing the rectilinear shape of the shaft, and implies a noncirculatory problem [18]. The increase of constant angular velocity component leads to buckling while the increase of time-dependent component results in the growing parametric vibration. The direct Liapunov method is used to analyse the uniform stochastic stability of the equilibrium state. Since the dynamic equations are strongly nonlinear, special attention is paid to a positive-definiteness of the appropriate energy-like Liapunov functional. Analysing the local positive-definiteness and the supermartingale property leads to sufficient stability conditions, expressed in terms of the rotation speed, the damping coefficients, the bending stiffness and the torque characteristics.

Constitutive equation

Let us consider a geometrically perfect ylindrical shell of radius R and total thickness h. Due to the symmetry assumption equations relating inplane moments and force resultants with the strain state components decouple and we can write the equation of symmetrically laminated tube in the form+ (as the coupling stiffness matrix B is equal to zero)

$$\begin{bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{bmatrix} \tag{1}$$

where ϵ_{ij} and A_{ij} denote strains and inplane laminate stiffness matrix, respectively.
If the shaft consists of a large number of orthotropic layers the A_{16} and A_{26} are negligible and the matrix equation (2) decouples. As the circumferential force N_{22} is much smaller than the the axial one, we can omit N_{22} in the second equation of (2) and calculate the reduced Young modulus of the beam-like cylindrical shell

$$E_0 = \bar{Q}_{11} - \bar{Q}_{12}^2/\bar{Q}_{22}, \tag{2}$$

where the transformed in-plane stiffnesses $\bar{Q}_{ij}$ are expressed by the orthotropic lamina invariants U_i in the following way $\bar{Q}_{11} = U_1 + U_2\cos 2\theta + U_3\cos 4\theta$, $\bar{Q}_{22} = U_1 - U_2\cos 2\theta + U_3\cos 4\theta$, $\bar{Q}_{12} = U_4 - U_3\cos 4\theta$. The invariants can be calculated using the in-plane sttiffnesses Q_{ij} and the lamination angle θ [5].
Using the engineering constants $E_{11}, E_{22}, G_{12}, \nu_{12}$ we can express the in-plane stiffnesses as follows

$$Q_{11} = E_{11}/(1 - \nu_{12}\nu_{21}), \qquad Q_{22} = Q_{11}E_{22}/E_{11}, \qquad Q_{12} = Q_{22}\nu_{12}, \qquad Q_{66} = G_{12}. \tag{3}$$

Analysing the viscoelastic behaviour of laminate Young's moduli and the stiffnesses are the operators $\bar{Q}^*_{ij} = \bar{Q}^*_{ij}(s)$ and the reduced operator Young modulus and the engineering constants have the following form

$$E^* = \bar{Q}^*_{11} - \bar{Q}^*_{12}/\bar{Q}^*_{22} \tag{4}$$

$$E_{11}^{*} = E_{11}(1+\beta_{11}s), \qquad E_{22}^{*} = E_{22}(1+\beta_{22}s), \qquad G_{12}^{*} = G_{12}(1+\beta_{12}s). \tag{5}$$

where according to [11] the different engineering constants E_{11}, E_{22}, G_{12} have different viscoelastic properties described β_{11}, β_{22}, β_{12}, respectively.

By analogy we can write the reduced in-plane stiffnesses in the form of operators

$$\bar{Q}_{ij}^{*} = \bar{Q}_{ij}^{e} + \bar{Q}_{ij}^{v}s, \tag{6}$$

where $\bar{Q}_{ij}^{e}$ and $\bar{Q}_{ij}^{v}$ are given as follows

$$Q_{11}^{e} = E_{11}, \; Q_{22}^{e} = E_{22}, \; Q_{66}^{e} = G_{12}, \; Q_{11}^{v} = \beta_{11}E_{11}, \; Q_{22}^{v} = \beta_{22}E_{22}, \; Q_{66}^{v} = \beta_{12}G_{12}.$$

Finally the constitutive equation in the first order approach has the form

$$\sigma = E_0(\varepsilon + \beta_1\dot{\varepsilon}), \tag{7}$$

where $E_0 = \bar{Q}_{11}^{e} - (\bar{Q}_{12}^{e})^2/\bar{Q}_{22}^{e}$, $\beta_1 = (\bar{Q}_{11}^{e}\bar{Q}_{22}^{v} + \bar{Q}_{22}^{e}\bar{Q}_{11}^{v} - 2\bar{Q}_{12}^{e}\bar{Q}_{12}^{v})/(\bar{Q}_{11}^{e}\bar{Q}_{22}^{e} - (\bar{Q}_{12}^{e})^2)$.

Equations of motion

The shaft transverse displacements in the movable rotating with the angular velocity ω coordinate system (y, z) are denoted by u, v, respectively. The absolute acceleration is equal to the sum of the acceleration of transportation, the Coriolis acceleration and the relative acceleration, therefore its components are as follows

$$a_y = u_{,TT} - \omega^2 u + 2\omega v_{,T} + \epsilon v \tag{8}$$

$$a_z = v_{,TT} - \omega^2 v - 2\omega u_{,T} - \epsilon u \tag{9}$$

Taking into account the internal and the external damping equations of motion of the center shaft line in the movable coordinates (u, v) have the form

$$\rho A a_u + EJu_{,XXXX} + \alpha EJu_{,TXXXX} + b_e(u_{,T} + \Omega v) + M_s v_{,XXX} = 0 \tag{10}$$

$$\rho A a_z + EJv_{,XXXX} + \alpha EJv_{,TXXXX} + b_e(v_{,T} - \Omega u) + M_s u_{,XXX} = 0 \tag{11}$$

$$X \in (0, \ell)$$

where ρ is the averaged density of the shaft, b_e is the external damping coefficient, and M_s represents the constant torque, which can destabilize the smooth rotation motion.

Introducing the dimensionless coordinates

$$X = Lx, \qquad T = k_t t$$

where

$$k_t = L^2\sqrt{\frac{A\rho}{EJ}}, \qquad \lambda = \alpha/k_t, \qquad \beta = \frac{b_e k_t}{A\rho}, \qquad \omega = \Omega k_t, \qquad \epsilon = \frac{d\omega}{dt}$$

the dynamics equations can be written as

$$u_{,tt} - \omega^2 u + 2\omega v_{,t} + \epsilon v + eu_{,xxxx} + \lambda u_{,txxxx} + \beta(u_{,t} + \omega v) + Lv_{,xxx} = 0 \tag{12}$$

$$v_{,tt} - \omega^2 v - 2\omega u_{,t} - \epsilon u + ev_{,xxxx} + \lambda v_{,txxxx} + \beta(v_{,t} - \omega u) - Lu_{,xxx} = 0 \qquad x \in (0,1) \tag{13}$$

where $\omega = \omega_o + \Delta\omega$, $\epsilon = \frac{d\Delta\omega}{dt}$ and ω_o is constant. The shaft is assumed to be simply supported at both ends. It means that the displacements of the shaft in supporting bearings are small as compared with the displacements of thin-walled flexible shaft.

The shaft is assumed to be simply supported at its ends. As the torque M_s are acting at the shaft ends it is necessary to remind relations between the torque and bending moments. When the torque is pure tangential to the deformed shaft axis in supports the bending moments vanish [18]. It means that the transverse displacements and the bending moments are equal to zero

$$u(0,t) = u(\ell,t) = v(0,t) = v(\ell,t) = 0 \tag{14}$$

$$u_{,xx}(0,t) = u_{,xx}(\ell,t) = v_{,xx}(0,t) = v_{,xx}(\ell,t) = 0 \tag{15}$$

The linear equations (4) and (5) have the trivial solution (equilibrium state) $u = v = 0$.

We assume that the stochastic component of angular velocity $\Delta\omega$ has the mean value equal to zero and the time-dependent wide-band Gaussian part with the intensity ς expressed as the formal time-derivative of the Wiener process $\mathcal{W}$,

$$\omega(t) = \omega_o + \varsigma \frac{d\mathcal{W}}{dt} \tag{16}$$

We also neglect the last terms of accelerations y and a_z in formulae (8) and (9) in comparison with the others [17]. The dynamic equations (23) and (24) can be rewritten in the Itô form

$$\begin{aligned}
du &= u_{,t}dt && (17)\\
du_{,t} &= \left\{\omega_o^2 u - 2\omega_o v_{,t} - eu_{,xxxx} - \lambda u_{,xxxxt} - \beta(u_{,t} + \omega_o v) + Lv_{,xxx}\right\} dt + \\
&\quad + \varsigma(2\omega_o u - 2v_{,t} - \beta v)d\mathcal{W} && (18)\\
dv &= v_{,t}dt && (19)\\
dv_{,t} &= \left\{\omega_o^2 v + 2\omega_o u_{,t} - ev_{,xxxx} - \lambda v_{,xxxxt} - \beta(v_{,t} - \omega_o u) - Lu_{,xxx}\right\} dt + \\
&\quad + \varsigma(2\omega_o v + 2u_{,t} - \beta u)d\mathcal{W} && (20)
\end{aligned}$$

As we would like to interpret the the white noise process as a limit case of Gaussian wide-band differentiable processes the stochastic differential equations have to be modified [ ] adding the Wong-Stratonovich correction terms

$$\begin{aligned}
du &= u_{,t}dt && (21)\\
du_{,t} &= \left\{\omega_o^2 u - 2\omega_o v_{,t} - eu_{,xxxx} - \lambda u_{,xxxxt} - \beta(u_{,t} + \omega_o v) + Lv_{,xxx} + \right.\\
&\quad \left. -\varsigma^2(2\omega_o v + 2u_{,t} + \beta u)\right\} dt + \varsigma(2\omega_o u - 2v_{,t} - \beta v)d\mathcal{W} && (22)\\
dv &= v_{,t}dt && (23)\\
dv_{,t} &= \left\{\omega_o^2 v + 2\omega_o u_{,t} - ev_{,xxxx} - \lambda v_{,xxxxt} - \beta(v_{,t} - \omega_o u) - Lu_{,xxx} + \right.\\
&\quad \left. + \varsigma^2(2\omega_o u - 2v_{,t} - \beta v)\right\} dt + \varsigma(2\omega_o v + 2u_{,t} - \beta u)d\mathcal{W} && (24)
\end{aligned}$$

Uniform stochastic stability analysis

In order to investigate the stability of trivial solution $u = v = 0$ corresponding to the smooth shaft motion it is necessary to introduce a precise stability definition. The trivial solution is uniformly stochastically stable if the following logic sentence is true

$$\bigwedge_{\epsilon\geq 0}\bigwedge_{\delta\geq 0}\bigvee_{r\geq 0} \|u(.,0), v(.,t)\| \leq r \Rightarrow P(\sup_{t\geq 0} \|u(.,t), v(.,t)\| \geq \epsilon) \leq \delta$$

where $\|u(.,t), v(.,t)\|$ denotes a measure of distance of solutions with nontrivial initial conditions from the trivial one.

We choose the Liapunov functional in the positive energy-like form [16]

$$V = \frac{1}{2}\int_0^\ell \Big\{ \big(u_{,t} + \omega_o v\big)^2 + \big(u_{,t} + \omega_o v + \beta v + \lambda u_{,xxxx}\big)^2 + \big(v_{,t} - \omega_o u\big)^2 + \\ + \big(v_{,t} - \omega_o u + \beta v + \lambda v_{,xxxx}\big)^2 + 2e\big(u_{,xx}^2 + v_{,xx}^2\big)\Big\} dx \tag{25}$$

Therefore

$$\|u, v\| = V^{1/2} \tag{26}$$

In order to calculate the differential dV along the trajectory of Eqs. (21)-(24) we use the appropriate generalized Itô lemma [4]

$$dV = \int_0^\ell \Big\{ \big(u_{,t} + \omega_o v\big)\big(du_{,t} + \omega_o dv\big) + \big(v_{,t} - \omega_o u\big)\big(dv_{,t} - \omega_o du\big) +$$

$$+\big(u_{,t} + \omega_o v + \beta v + \lambda u_{,xxxx}\big)\big(du_{,t} + \omega_o dv + \beta dv + \lambda du_{,xxxx}\big) +$$

$$+\big(v_{,t} - \omega_o u + \beta v + \lambda v_{,xxxx}\big)\big(dv_{,t} - \omega_o du + \beta dv + \lambda dv_{,xxxx}\big) + 2e\big(u_{,xx} du_{,xx} + v_{,xx} dv_{,xx}\big) +$$

$$+ \varsigma^2\left[(2\omega_o u - 2v_{,t} - \beta v)^2 + (2\omega_o v + 2u_{,t} - \beta u)^2\right] dt \Big\} dx \tag{27}$$

Eliminating du, dv, $du_{,t}$, $dv_{,t}$ by means of Eqs. (21-(24) and integrating by parts with respect to x we rewrite the differential of Liapunov functional in the form

$$dV = -\int_0^\ell \mathcal{F} dx dt + \int_0^\ell \mathcal{G} dx dW \tag{28}$$

where

$$\mathcal{F} = \beta\big(u_{,t}^2 + v_{,t}^2\big) + \beta\omega_o^2\big(u^2 + v^2\big) + 2\beta\omega_o\big(vu_{,t} - uv_{,t}\big) + \big(e\beta - \beta\varsigma^2\lambda - \lambda\omega_o^2\big)\big(u_{,xx}^2 + v_{,xx}^2\big) +$$

$$+e\lambda\big(u_{,xxxx}^2+v_{,xxxx}^2\big)+\lambda\big(u_{,xxt}^2+v_{,xxt}^2\big)+2\lambda\big(u_{,xxt}u_{,xx}+v_{,xxt}v_{,xx}\big)-L\beta\big(u_{,xxx}v-v_{,xxx}u\big)+$$
$$+2L\big(u_{,t}v_{,xxx}-v_{,t}u_{,xxx}\big)+2L\omega_o\big(uv_{,xxx}-vu_{,xxx}\big)++L\lambda\big(u_{,xxxx}v_{,xxx}-v_{,xxxx}u_{,xxx}\big) \quad (29)$$

$$\mathcal{G}=\varsigma\Big[(2\omega_o u-2v_{,t}-\beta v)(2u_{,t}+2\omega_o v+\beta u+\lambda u_{,xxxx})+$$
$$+(2\omega_o v+2u_{,t}-\beta u)(2v_{,t}-2\omega_o v+\beta u+\lambda v_{,xxxx})\Big] \quad (30)$$

Integrating Equation (28) from $t=s$ to $\tau_\delta(t)$, where $\tau_\delta(t)$ is the first random time of a trajectory exit from the domain $V^{1/2}=\delta$, conditionally averaging ($\mathcal{E}$) we have

$$\mathcal{E}V(\tau_\delta(t))=V(s)-\mathcal{E}\int_s^{\tau_\delta(t)}\int_0^\ell \mathcal{F}(t)dt \quad (31)$$

Neglecting the first four positive terms of integrand and using the elementary inequalities for arbitrary $\xi,\ \eta,\ \zeta,\ \psi\in(0,1)$

$$\pm ab=\pm\eta ab/\eta\le\frac{1}{2}(a^2\eta^2+b^2/\eta^2)$$

we calculate the lowerbound of the function $\mathcal{F}$.

$$\mathcal{F}\ge\Big[\omega_o^2\beta-\frac{1}{2}\eta^2\beta^3-\omega_o^2\beta-\frac{\omega_o^2\beta}{1-\xi^2}\Big](u^2+v^2)+\Big[e\beta-\lambda\omega_o^2-\varsigma^4+\lambda\beta\varsigma^2\Big](u_{,xx}^2+v_{,xx}^2)+$$
$$-L^2\Big[\frac{1}{\beta\xi^2}+\frac{1}{2\beta\eta^2}+\frac{1}{\beta\zeta^2}+\frac{\lambda}{2e\psi^2}\Big](u_{,xxx}^2+v_{,xxx}^2)+e\lambda\big(1-\frac{\psi^2}{2}\big)(u_{,xxxx}^2+v_{,xxxx}^2) \quad (32)$$

Using the supermartingale property and proceeding similarly to the proof of Chebyshev's inequality we find that the trivial solution of Eqs. (21)-(24) is uniformly stochastically stable if the functional $\mathcal{F}$ is positive-definite. It is equivalent to the following algebraic inequality

$$\Big\{\Big[e\lambda\big(1-\frac{\psi^2}{2}\big)\frac{\pi^2}{\ell^2}-L^2\Big(\frac{1}{\beta\xi^2}+\frac{1}{2\beta\eta^2}+\frac{1}{\beta\zeta^2}+\frac{\lambda}{2e\psi^2}\Big)\Big]\frac{\pi^2}{\ell^2}e\beta-\lambda\omega_o^2+\varsigma^2(\lambda\beta-\varsigma^2)\Big\}\frac{\pi^4}{\ell^4}+$$
$$+\omega_o^2\beta-\frac{1}{2}\eta^2\beta^3-\omega_o^2\beta-\frac{\omega_o^2\beta}{1-\xi^2}\ge 0 \quad (33)$$

The critical angular velocity can be obtained maximizing the left side of the inequality (33) over admissible ξ, η, ζ and ψ As example we find the maximal value of mean angular velocity for $\eta = \zeta = \psi = 1$ It is easy to notice that the suboptimal value is obtained for ξ minimalizing the following expression

$$f(\xi) = L^2 \frac{1}{\beta\xi^2} \frac{\pi^6}{\ell^6} + \frac{\omega_o^2\beta}{1-\xi^2}$$

The minimalizing value of parameter ξ is equal to

$$\xi_m^2 = \frac{\sqrt{\frac{1}{\beta}\frac{\pi^6}{\ell^6}L^2}}{\sqrt{\frac{1}{\beta}\frac{\pi^6}{\ell^6}L^2} + \omega_o\sqrt{\beta}} \tag{34}$$

Therefore the stability domain as function of fluctuating angular velocity characteristics is defined as follows

$$\left\{\left[\frac{1}{2}e\lambda\frac{\pi^2}{\ell^2} - L^2\left(\frac{1}{\beta\xi_m^2} + \frac{3}{2\beta} + \frac{\lambda}{2e}\right)\right]\frac{\pi^2}{\ell^2}e\beta - \lambda\omega_o^2 + \varsigma^2(\lambda\beta - \varsigma^2)\right\}\frac{\pi^4}{\ell^4} - \beta\left(\frac{\beta^2}{2} + \frac{\omega_o^2}{1-\xi_m^2}\right) \geq 0 \tag{35}$$

Taking into account the Brazier ovalization effect [15] we notice that the functional (25) is locally positive definite. Therofore, increasing of ξ enlarges the stability domain in system parameter space but decreases the stability domain in the state space described by the norm $\|.\|$. It means that the region of initial disturbances (initial conditions) described by the norm becomes smaller. The increase of noise intensity ς decreases the admissible rotation speed ω_o.

Conclusions

A technique has been presented for the analysis the dynamic stability of a shaft rotating with nonconstant angular velocity and subjected to a constant torque. The dynamic stability and the stochastic stability problem is reduced to the problem of the positive definiteness of the auxiliary functional. The explicit criterium derived in the paper defines the stability region in the form of algebraic inequality. The boundary of the parametric resonance instability is defined by the geometrical and material properties, the lamination angle, as well as the constant value and intensity of the fluctuating angular velocity. The results indicate that the decrease of the angular velocity intensity increases the admissible constant component of the rotation speed.

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