MECHANICS AND MECHANICAL ENGINEERING
International Journal
Vol. 3 No. 2 1999

NATURAL FREQUENCIES OF A ROTATING VISCOELASTIC SHELL

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Abstract

The paper deals with a simply supported cylindrical shell rotating about its longitudinal axis. The shell is modelled according to the thin shell theory and small deflection assumption. The equations of relative motion are derived considering the acceleration including the Coriolis component. The analysis is focused on the passive damping effect created by viscoelastic material. The complex eigenvalue problem is formulated and solved. The results show the influence of material damping property on the modal equivalent damping parameter for variations in the angular velocity of the shell. The relations between the modal damping parameter and velocity of rotation for different retardation time (loss factor) values are also presented.

1. Introduction

Dynamics of cylindrical shells rotating about their longitudinal axes has been the topic of many investigations and theoretical publications. Thin cylindrical shells are commonly used as the models of cylinders or shafts rotating with a high speed. The dynamic problems due to the high-speed rotation are presented in [1], [4], [5], where critical parameters of the system and the effect of rotational speed on natural frequencies are studied. The analysis of natural frequencies of the rotating isotropic shells including the Coriolis component is presented in [1] among others. The influence of a non-linear material damping on the natural frequencies of the rotating shell is discussed in [2].

In the present paper the analysis is focused on passive damping effects caused by the Kelvin-Voigt viscoelastic material of the shell. The dynamic equations are composed in terms of displacements U, V, W in the axial, circumferential and radial directions, respectively, according to the relative rotating system ξ,η,ζ . The solution of the complex eigenvalue problem formulated assuming simply supported conditions leads to the natural frequencies and modal damping parameters of the shell.

2. Free vibration analysis

The analysis of rotating shell is based on the thin shell theory and the Kirchhoff-Love hypothesis [3].

Let us consider a surface of revolution S, which corresponds to the shell midplane surface with its longitudinal symmetrical axis coincides with axis X of the fixed co-ordinate system X,Y,Z. The shell equations of motion are formulated in the relative co-ordinate system

ξ, η, ζ rotating with the surface S. The co-ordinate system and displacement components U, V, W are shown in Fig. 1.

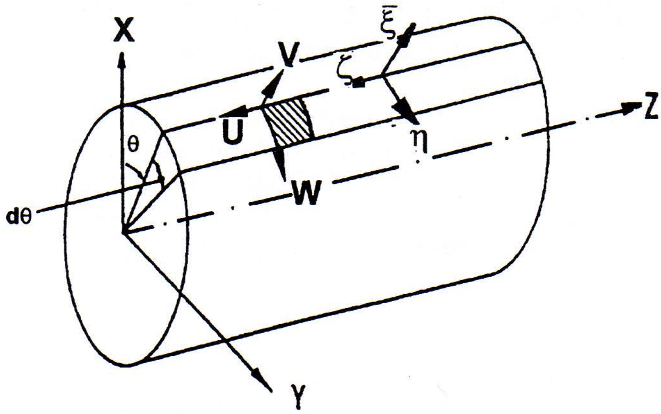

Fig. 1. Geometry of the shell and the co-ordinate systems

The angular velocity of the shell is given by the vector

$$\bar{\Omega} = -\bar{k}\Omega \tag{1}$$

Taking into account the transportation movement one may obtain the following formula of the acceleration vector relative to X, Y, Z system

$$\bar{a} = \bar{a}_0 + \bar{\Omega} \times (\bar{\Omega} \times \bar{r}_0) + \bar{\varepsilon} \times \bar{r}_0 + 2\bar{\Omega} \times \bar{v}_r + \bar{a}_r \tag{2}$$

where:

$\bar{a}_0 = -\bar{j}\Omega^2 R$ – centripetal acceleration of the origin of the ξ, η, ζ reference system,

$\varepsilon = \dfrac{d^2\Omega}{dt^2}$ – angular acceleration of the rotation,

$\bar{r}_0 = \bar{i}V + \bar{j}W + \bar{k}U.$ – position vector of any point of the shell in the rotating co-ordinates,

$\bar{v}_r = \bar{i}\dfrac{\partial V}{\partial t} + \bar{j}\dfrac{\partial W}{\partial t} + \bar{k}\dfrac{\partial U}{\partial t}$ – relative velocity of the shell point,

$\bar{a}_r = \bar{i}\dfrac{\partial^2 V}{\partial t^2} + \bar{j}\dfrac{\partial^2 W}{\partial t^2} + \bar{k}\dfrac{\partial^2 U}{\partial t^2}$ – relative acceleration of the shell point,

For the constant rotational speed ($\varepsilon = 0$) the acceleration components are as follows

$$a_\xi = -\Omega^2 V + 2\Omega\frac{\partial W}{\partial t} \tag{3}$$

$$a_\eta = -\Omega^2 R - \Omega^2 W - 2\Omega\frac{\partial V}{\partial t} \tag{4}$$

$$a_\zeta = \frac{\partial^2 U}{\partial t^2} \tag{5}$$

The complex form of the Young modulus can describe viscoelastic properties of material

$$E^*(\omega_{mn}) = E(1+i\eta) \tag{6}$$

where:

ω_{mn} – natural frequency for *m-n* mode of vibration,

η – loss factor defined as the ratio of the imaginary and real part of the complex modulus.

In case of the Kelvin-Voigt material loss factor η is given by the relation

$$\eta = \omega_{mn}\mu \tag{7}$$

where:

μ – retardation time of viscoelastic material.

Taking into account Eqs. (3÷6) after eliminating the axial and circumferential acceleration components and assuming external viscous damping the equations of motion referred to the rotating system (co-ordinate ξ, η, ζ) my be described as follows:

$$-\frac{\partial^2 U}{\partial x^2} - \frac{1-\nu}{2}\frac{\partial^2 U}{\partial y^2} - \frac{1+\nu}{2}\frac{\partial^2 V}{\partial x \partial y} - \frac{\nu}{R}\frac{\partial W}{\partial x} + 2\frac{\rho h \beta}{A(1+i\eta)}\frac{\partial U}{\partial t} = 0, \tag{8}$$

$$-\frac{1+\nu}{2}\frac{\partial^2 U}{\partial x \partial y} - \frac{\partial^2 V}{\partial y^2} - \frac{1-\nu}{2}\frac{\partial^2 V}{\partial x^2} - \frac{1}{R}\frac{\partial W}{\partial y} - \left(\frac{\Omega^2}{A}V - 2\frac{\Omega}{A}\frac{\partial W}{\partial t}\right)\frac{\rho h}{1+i\eta} + $$
$$+ \frac{2\rho h \beta}{A(1+i\eta)}\frac{\partial V}{\partial t} = 0, \tag{9}$$

$$\frac{\nu}{R}\frac{\partial U}{\partial x} + \frac{1}{R}\frac{\partial V}{\partial y} + \frac{W}{R^2} + \frac{D}{A}\Delta^2 W + \frac{N_x}{A(1+i\eta)}\frac{\partial^2 W}{\partial x^2} + \frac{N_y}{A(1+i\eta)}\frac{\partial^2 W}{\partial y^2} +$$
$$+ \frac{\rho h}{A(1+i\eta)}\left(\frac{\partial^2 W}{\partial t^2} - 2\Omega\frac{\partial V}{\partial t} - \Omega^2 W\right) + 2\frac{\rho h \beta}{A(1+i\eta)}\frac{\partial W}{\partial t} = 0 \tag{10}$$

where:

$A = \frac{Eh}{1-\nu^2}$ – longitudinal stiffness,

$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{Ah^2}{12}$ – flexural stiffness,

$\tilde{N}_y = N_y - R^2\Omega^2\rho h$ – intensity of circumferential forces,

ν – Poisson ratio,

β – external damping coefficient,

ρ – mass density,

Δ – Laplace operator.

The solutions to the equations of motion Eqs (8÷10), which satisfy the simply supported conditions, are assumed in the form:

$$U = \sum_{m,n=1}^{\infty}\left[U_{mn}(t)\cos\beta_n y + \tilde{U}_{mn}(t)\sin\beta_n y\right]\cos\alpha_m x, \tag{11}$$

$$V = \sum_{m,n=1}^{\infty}\left[V_{mn}(t)\sin\beta_n y + \tilde{V}_{mn}(t)\cos\beta_n y\right]\sin\alpha_m x \tag{12}$$

$$W = \sum_{m,n=1}^{\infty}\left[W_{mn}(t)\cos\beta_n y + \tilde{W}_{mn}(t)\sin\beta_n y\right]\sin\alpha_m x. \tag{13}$$

where:

m, n – wave numbers.

Substituting the above set of displacements into equations of motion results in the ordinary differential equation system related to the time dependent functions $U_{mn}, V_{mn}, W_{mn}, \tilde{U}_{mn}, \tilde{V}_{mn}, \tilde{W}_{mn}$

$$2d\frac{dU_{mn}}{dt} + \left(\alpha_m^2 + \frac{1-\nu}{2}\beta_n^2\right)U_{mn} - \frac{1+\nu}{2}\alpha_m\beta_n V_{mn} - \frac{\nu}{R}\alpha_m W_{mn} = 0, \tag{14}$$

$$2d\frac{dV_{mn}}{dt} - \frac{1+\nu}{2}\alpha_m\beta_n U_{mn} + \left(\beta_n^2 + \frac{1-\nu}{2}\alpha_m^2 - c\Omega\right)V_{mn} + \frac{\beta_n}{R}W_{mn} + 2c\frac{d\tilde{W}_{mn}}{dt} = 0, \tag{15}$$

$$\frac{\rho h}{A(1+i\eta)}\frac{d^2W_{mn}}{dt^2} + 2d\frac{dW_{mn}}{dt} - \frac{\nu}{R}\alpha_m U_{mn} + \frac{\beta_n}{R}V_{mn} +$$
$$+\left[\frac{1}{R^2} + \frac{\nu}{A(1+i\eta)}\left(\alpha_m^2 + \beta_n^2\right) - \frac{N_x\alpha_m^2 - \tilde{N}_y\beta_n^2}{A(1+i\eta)} - c\Omega\right]W_{mn} + 2c\frac{d\tilde{V}_{mn}}{dt} = 0, \tag{16}$$

$$2d\frac{d\tilde{U}_{mn}}{dt} + \left(\alpha_m^2 + \frac{1-\nu}{2}\beta_n^2\right)\tilde{U}_{mn} - \frac{1+\nu}{2}\alpha_m\beta_n\tilde{V}_{mn} - \frac{\nu}{R}\alpha_m\tilde{W}_{mn} = 0, \tag{17}$$

$$2d\frac{d\bar{U}_{mn}}{dt}-\frac{1+\nu}{2}\alpha_m\beta_n\bar{U}_{mn}+\left(\beta_n^2+\frac{1-\nu}{2}\alpha_m^2-c\Omega\right)\bar{V}_{mn}+\frac{\beta_n}{R}\bar{W}_{mn}-2c\frac{dW_{mn}}{dt}=0, \tag{18}$$

$$\frac{\rho h}{A}\frac{d^2\bar{W}_{mn}}{dt^2}+2d\frac{d\bar{W}_{mn}}{dt}-\frac{\nu}{R}\alpha_m\bar{U}_{mn}+\frac{\beta_n}{R}\bar{V}_{mn}+$$
$$+\left[\frac{1}{R^2}+\frac{\nu\left(\alpha_m^2+\beta_n^2\right)}{A(1+i\eta)}-\frac{N_x\alpha_m^2-\tilde{N}_y\beta_n^2}{A(1+i\eta)}-c\Omega\right]\bar{W}_{mn}-2c\frac{dV_{mn}}{dt}=0. \tag{19}$$

where:

$a=\dfrac{\rho h}{A}$ – constant depending on material parameters,

$d=\dfrac{2\rho h\beta}{A}=2a\beta$ – external viscous factor,

$c=\dfrac{2\rho h\Omega}{A}=2a\Omega$ – rotational speed factor.

It can be noticed that the equations (14÷19) are coupled because of rotation and the terms representing the elastic forces.

Using symbols L_{ij} to designate the elements of the stiffness matrix, the following differential system of equations is obtained:

$$2d\frac{dU_{mn}}{dt}+L_{11}U_{mn}+L_{12}V_{mn}+L_{13}W_{mn}=0, \tag{20}$$

$$2d\frac{dV_{mn}}{dt}+2c\frac{d\bar{W}_{mn}}{dt}+L_{21}U_{mn}+L_{22}V_{mn}+L_{23}W_{mn}=0, \tag{21}$$

$$\frac{\rho h}{A(1+i\eta)}\frac{d^2W_{mn}}{dt^2}+2d\frac{dW_{mn}}{dt}+2c\frac{d\bar{V}_{mn}}{dt}+L_{13}U_{mn}+L_{23}V_{mn}+L_{33}W_{mn}=0, \tag{22}$$

$$2d\frac{d\bar{U}_{mn}}{dt}+L_{11}\bar{U}_{mn}+L_{12}\bar{V}_{mn}+L_{13}\bar{W}_{mn}=0, \tag{23}$$

$$2d\frac{d\bar{V}_{mn}}{dt}-2c\frac{d\bar{W}_{mn}}{dt}+L_{12}\bar{U}_{mn}+L_{22}\bar{V}_{mn}+L_{23}\bar{W}_{mn}=0, \tag{24}$$

$$\frac{\rho h}{A(1+i\eta)}\frac{d^2\bar{W}_{mn}}{dt^2}+2d\frac{d\bar{W}_{mn}}{dt}-2c\frac{dV_{mn}}{dt}+L_{13}\bar{U}_{mn}+L_{23}\bar{V}_{mn}+L_{33}\bar{W}_{mn}=0. \tag{25}$$

where:

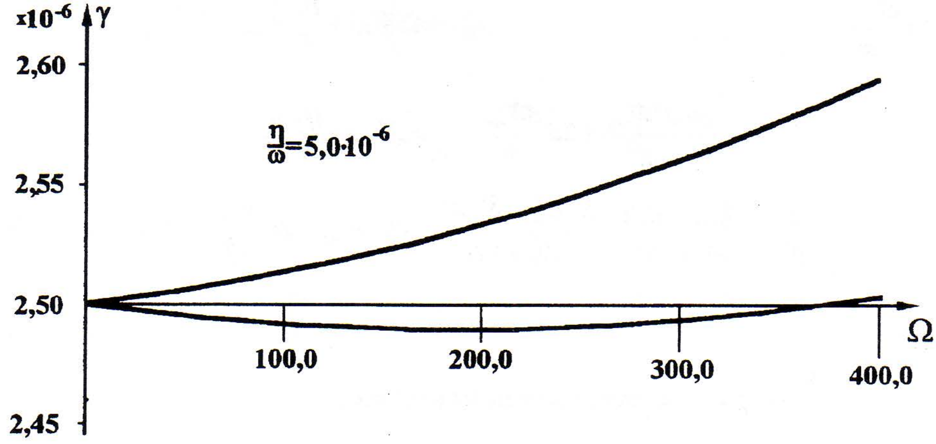

Fig. 2. Influence of the rotational speed Ω on the modal damping parametry γ $\left(\eta/_{\omega} = 5 \bullet 10^{-6}[s]\right)$

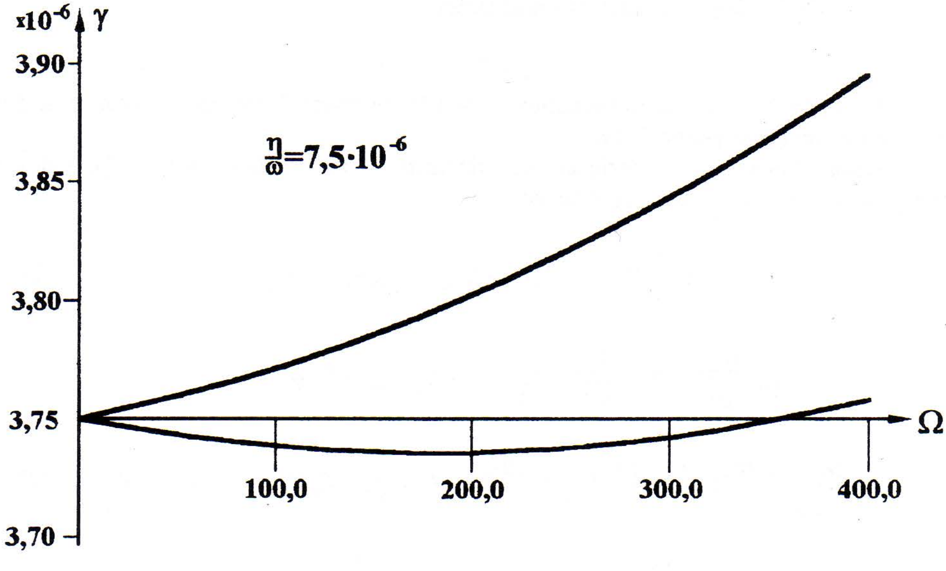

Fig. 3. Influence of the rotational speed Ω on the modal damping parametry γ

$\left(\eta/_{\omega} = 7.5 \bullet 10^{-6}[s]\right)$

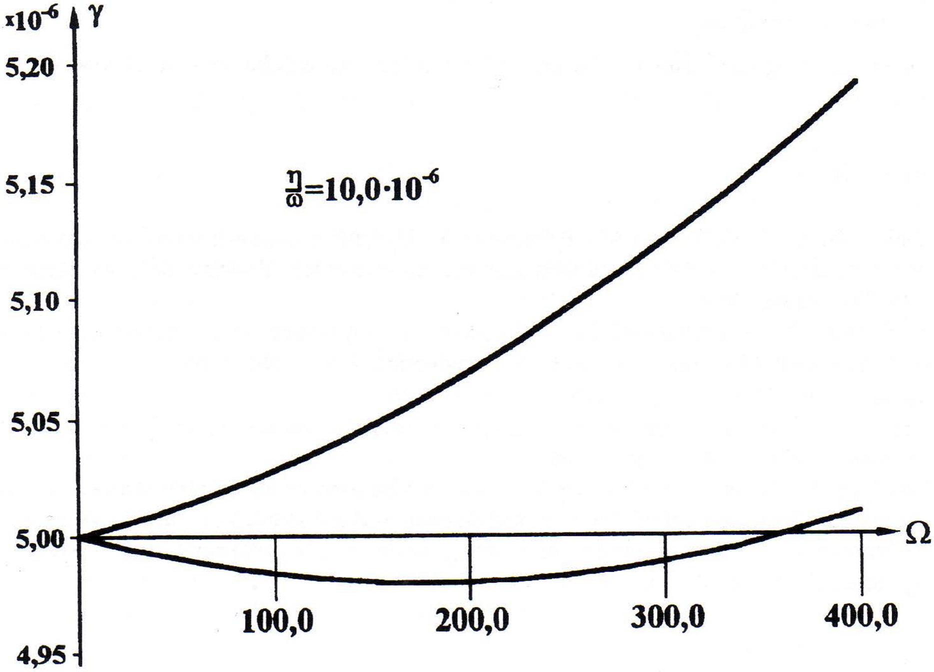

Fig. 4. Influence of the rotational speed Ω on the modal damping parametry γ $\left(\eta/_{\omega} = 10^{-5}[s]\right)$

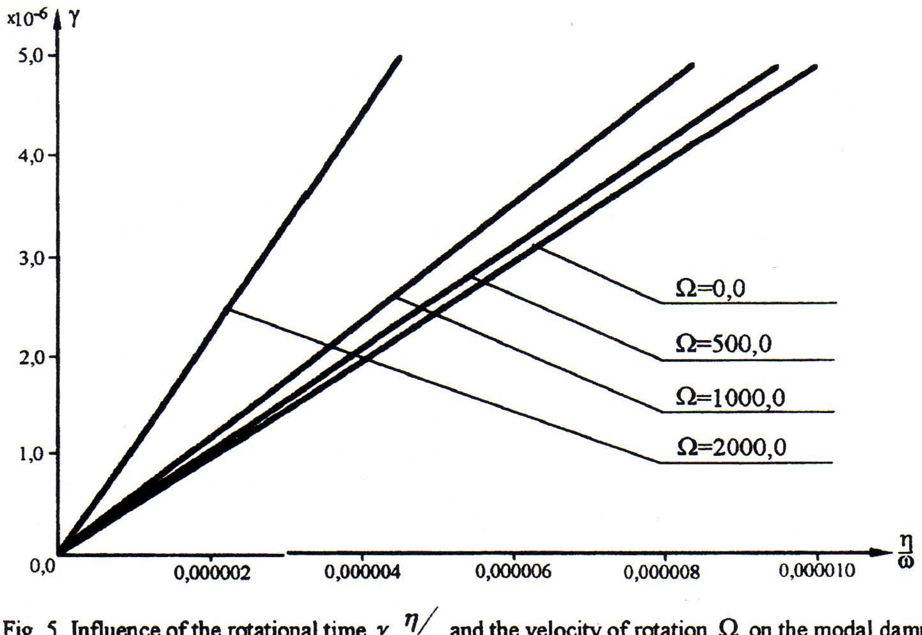

Fig. 5. Influence of the rotational time γ $\eta/_{\omega}$ and the velocity of rotation Ω on the modal damping parameter γ.

for the relatively slow rotation, when the damping parameter values correspond to the lower curve of the above relation.

The modal damping parameter relates almost linearly on the retardation time of viscoelastic material.

5. References

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