MECHANICS AND MECHANICAL ENGINEERING
International Journal

Vol. 3 No. 2 **1999**

FORCE FIT OF A HOLLOW SHAFT IN A HUB OF LARGE RADIUS

Ismail A. Koleif,
M. Helal Kamel
Cairo University, Depart. Of Engineering Math. & Physics,
Faculty of Eng. , Giza, EGYPT

Abstract

In this paper the problem of force fit of a hollow shaft in a hub of large radius is completed under the assumption of plane polar elasticity. Different materials are admitted and radial displacement of the inner and outer radius of the shaft together with that of the hub are allowed. Complete solutions are obtained and are compared with that of the solid shaft. In the case of solid shaft, the results showed that the strain energy is equally shared between the shaft and the hub.

I. Introduction

It is indicated in literature that the problem of a hub force fitted to a shaft which has a great interest in most engineering applications must be handled properly. In spite of the simplicity of the general problem statement, it has been difficult to obtain exact solutions in most particular cases. In 1987, Grunau and Hahn [3] studied the strength behavior of bonded and shrunk hub and shaft joints under continuous quasi-static stress as a function of the choice of adhesive and found that the necessary manufacturing tolerances should be seen as one aspect of the economical production of the parts to be joined. In 1990, Orcan and Gamer [4] studied the shrink fit consisting of elastic hollow shaft and nonlinearly hardening elastic-plastic hub. General expressions foe stresses and displacement are derived in a shrink fit consisting of elastic hollow inclusion and elastic-plastic hub. The thermal assembly of an elastic-plastic hub and a solid shaft is investigated by Mack [5],1993. In 1995, Esebec and Goetz [6] introduced an experimental study on ceramic-metal joints for shafthub connections in gas turbines. In 1997, Gao and Alturi [7] presented an analytical solution for the axisymmetric shrink-fit problem with a thin strain-hardening hub and an elastic solid shaft. In our investigation, the problem is solved for both hollow and solid shafts and also for different materials. In addition the distribution of the strain energy in the shaft and the hub is presented.

2. Formulation of the problem

Consider the plane stress in the cylindrical polar coordinates, the equilibrium condition is

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{1}$$

and Hook's law;

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \upsilon\sigma_\theta\right) = \frac{du}{dr}$$
$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \upsilon\sigma_r\right) = \frac{u}{r} \tag{2}$$

Here we have,

u is the radial displacement,
σ_r and σ_θ are the radial and tangential stresses respectively,
ν is poison ratio,
E is the Young's modulus of elasticity and
ε_r and se are the radial and tangential strains respectively.

The system of equations (1) and (2) reduces to the equation:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0 \tag{3}$$

which has the general solution, $u = Ar + \frac{B}{r}$ where A and B are arbitrary B are constants. It is implied that the solution inside the shaft should be $u_s = A_s r + \frac{B_s}{r}$ while the solution in the hub of large outer radius should be $u_h = \frac{B_h}{r}$. The following schematic diagram for the shaft and the hub before fit.

The following notations are used for the radii.

The outer radius of the shaft before fit is r_o, and after fit is $r_o - \delta_s$,

The inner radius of the shaft before fit is r_i, and after fit is $r_i - \delta_{si}$,

The radius of the hub before fit is R, and after fit is R+ δ_h.

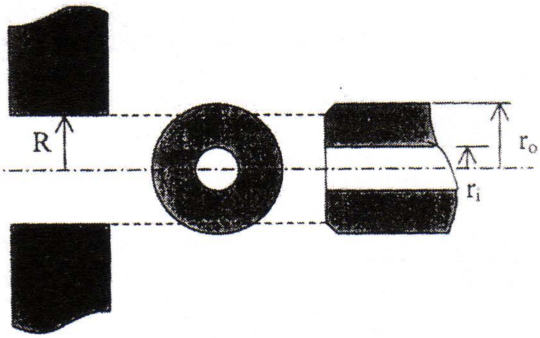

Fig. 1: Schematic diagram for the shaft and the hub before fit.

The strain field for the shaft is,

$$\varepsilon_{rs} = A_s - \frac{B_s}{r^2} \quad \text{and} \quad \varepsilon_{\theta s} = A_s + \frac{B_s}{r^2} \tag{4}$$

and the strain field for the hub is,

$$\varepsilon_{rh} = -\frac{B_h}{r^2} \quad \text{and} \quad \varepsilon_{\theta h} = \frac{B_h}{r} \tag{5}$$

From equations (2) and (4), the stress field for the shaft is,

$$\begin{aligned} \sigma_{rs} &= \frac{E_s}{1-\nu_s^2}\left[A_s(1+\nu_s) - \frac{B_s}{r^2}(1-\nu_s)\right] \\ \sigma_{\theta s} &= \frac{E_s}{1-\nu_s^2}\left[A_s(1+\nu_s) + \frac{B_s}{r^2}(1-\nu_s)\right] \end{aligned} \tag{6}$$

and the stress field for the hub is,

$$\begin{aligned} \sigma_{rh} &= \frac{-E_h}{1-\nu_h^2}\left[\frac{B_h}{r^2}(1-\nu_h)\right] \\ \sigma_{\theta h} &= \frac{E_s}{1-\nu_h^2}\left[\frac{B_h}{r^2}(1-\nu_h)\right] \end{aligned} \tag{7}$$

Applying the following boundary conditions in equations (6) and (7) to determine the three unknowns A_s, B_s and B_h,

$$\begin{aligned} &\text{In the hub} && u_h \big|_{r=R+\delta_h} = \delta_h \\ &\text{In the shaft - hub contact} && \sigma_{rs}. \big|_{r=R+\delta_h} = \sigma_{rh} \big|_{r=R+\delta_h} \\ &\text{In the shaft inner radius} && \sigma_{rs} \big|_{r=r_{is}-\delta_{si}} = 0 \end{aligned} \tag{8}$$

We get,

$$B_h = \delta_h(R+\delta_h)$$
$$A_s = -\frac{E_h}{E_s}\frac{1-\nu_s}{1+\nu_h}\frac{\delta_h}{R}\frac{1}{1-\left(\frac{r_i}{R}\right)^2} \tag{9}$$
$$B_s = \frac{E_h}{E_s}\frac{1+\nu_s}{1+\nu_h}\frac{\delta_h}{R}\frac{r_i^2}{1-\left(\frac{r_i}{R}\right)^2}$$

Using the following condition on both shaft radii,

$$R+\delta_h = r_o - \delta_s$$
$$u_s\Big|_{r=r_{is}-\delta_{is}} = \delta_{is} \tag{10}$$
$$u_s\Big|_{r=R+\delta_s} = \delta_s$$

From equations (9) and (10) and the solution of equation (3) we deduced the general solution of δ_s,δ_{is} and δ_h,

$$\delta_s = (r_o - R)\frac{\phi}{1+\phi},$$
$$\delta_h = (r_o - R)\frac{1}{1+\phi} \quad \text{and} \tag{11}$$
$$\delta_{is} = (r_o - R)\frac{\psi}{1+\phi}$$

where

$$\phi = \frac{E_h}{E_s}\frac{1}{1-\left(\frac{r_i}{R}\right)^2}\left[\frac{1-\nu_s}{1+\nu_h}+\left(\frac{r_i}{R}\right)^2\frac{1+\nu_s}{1+\nu_h}\right] \quad \text{and}$$

$$\psi = \frac{E_h}{E_s}\frac{\frac{r_i}{R}}{1-\left(\frac{r_i}{R}\right)^2}\left[\frac{2}{1+\nu_h}\right]$$

3. Special Case (I) : Same material with a Numerical Example

If the shaft and the hub are of the same material ϕ and ψ take the form,

$$\delta_s = (r_o - R)\frac{\phi}{1+\phi},$$
$$\delta_h = (r_o - R)\frac{1}{1+\phi} \quad \text{and} \tag{12}$$
$$\delta_{is} = (r_o - R)\frac{\psi}{1+\phi}$$

Taking the following numerical data,
r_i= 20 mm, R=50 mm, r_o = 51 mm and ν=0.3 and substituting into (12) to determine δ_s,δ_{is} and δ_h we get,
δ_s = 0.454 mm , ,δ_{is} = 0.4 mm and δ_h = 0.546 mm.

4. Special Case (II) : Solid Shaft

In the case of solid shaft, the displacement will be u = $A_s r$ which gives the following stress fields,

$$\sigma_{rs} = \frac{E_s}{1-\nu_s^2}\left[A_s(1+\nu_s)\right] = \sigma_{\theta s} \tag{13}$$

with the boundary condition,

In the shaft - hub contact $\sigma_{rs}\big|_{r=R+\delta_h} = \sigma_{rh}\big|_{r=R+\delta_h}$ this gives,

$$A_s = \frac{E_h}{E_s}\frac{1+\nu_s}{1+\nu_h}\frac{\delta_h}{R+\delta_h}$$

with the condition $u_s\big|_{r=R+\delta_s} = \delta_s$ we get,

$$\delta_s = \frac{E_h}{E_s}\frac{1+\nu_s}{1+\nu_h}\delta_h \tag{14}$$

For same material we conclude,

$$\delta_s = \delta_h = \delta = \frac{1}{2}(r_o - R) \tag{15}$$

5. Strain Energy Sharing

The strain energy $\Phi(r)$ is defined by, $\Phi(r) = \sigma_r\varepsilon_r + \sigma_\theta\varepsilon_\theta$ so for the shaft and the hub takes the form,

$$\begin{aligned} \Phi_s(r) &= \frac{2E_s}{1+\nu_s}A_s = \frac{2E_h}{1+\nu_h}\frac{\delta_h^2}{R^2} \\ \Phi_h(r) &= \frac{2E_h}{1+\nu_h}\frac{B_h^2}{r^4} \end{aligned} \tag{16}$$

The total strain energy is,

$$\text{In the hub}\quad \Phi_h = 2\pi\int_R^{\infty} r\Phi_h(r)dr = \frac{2\pi E_h}{1+\nu_h}\delta_h^2$$

$$\text{In the shaft}\quad \Phi_s = \pi R^2\left(\frac{2E_h}{1+\nu_h}\frac{\delta_h^2}{R^2}\right) = \frac{2\pi E_h}{1+\nu_h}\delta_h^2 \tag{17}$$

From the result (16), the total energy $\frac{4\pi E_h}{1+\nu_h}\delta_h^2$ is equally shared between shaft and hub.

6. Conclusions

It is proved that a central hole in the shaft reduces the shaft radial contraction at the outer radius and the strain energy is equally shared between the solid shaft and the hub of large radius. Calculation of radial stresses at the contact surface and for the same material depends on the radial displacement and is greater for the hollow shaft than that of the solid shaft, for the presented numerical example, $\sigma_r / E_h = 0.00858$ (for hollow shaft) and $\sigma_r / E_h = 0.007616$ (for solid shaft).

References

[1.] J. E. Shigley, Mechanical Engineering Design, Mc Graw-Hill, Book Company, N.Y.,1977.

[2.] Crandall, Dahl and Lardner, An Introduction to the Mechanics of Solids, Mc Graw-Hill, Book Company, N.Y.,1978.

[3.] G. Arbogast and H.Ortwin, Strength Behavior of Bonded and Shrunk Hub and Shaft Joints Under Continuous Quasi-Static Stress, *Schweissen-Schneiden*, Vol.**39**, No.4, E120-E121, West Ger.,1987.

[4.] Y.Orcan and U. Gamer, Shrink Fit Consisting of Elastic Hollow Shaft and Nonlinearly Hardening Elastic-Plastic Hub, *Acta-Mech.*, Vol.**81**, No.1-2, 97-108,1990.

[5.] W. Mack, Thermal Assembly of an Elastic-Plastic Hub and A Solid Shaft, *A.of Applied Mech.*, Vol.**63**, No.1, 42-50, Austria.,1993.

[6.] V. Esebeck, M. kising and U. Neuhof, Investigation on Ceramic-Metal Joints for Shaft-Hub Connections in Gas Turbines, *Proc.of American Society of Mech. Eng.(ASME)*, **7**, 95-CTP-52, N.Y., U.S.A.,1995.

[7.] G. Lin and N. Atluri, Elasto-Plastic Analytical Solution for the Shrink-Fit Problem with a Thin Strain - Hardening Hub and an Elastic Solid Shaft, *J. of Math.& Mech.of Solids*, Vol.**2**, No.3, 335-342, WI, U.S.A.,1997.