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NEAR-CRITICAL BIFURCATING VIBRATION OF A ROTATING SHAFT WITH PIEZOELECTRIC ELEMENTS

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Abstract

The paper is concerned with problem of active stabilisation of rotating shafts by making use of piezoelectric elements and their effect on non-linear response of such systems, i.e. on bifurcating vibration that appears near the critical threshold. Piezoelectric elements serve in the system as sensors measuring internal bending moment in a given cross-section of the shaft and actuators producing bending moment out of phase with respect to the measured moment and in accordance with a specially devised control strategy. The purpose of incorporating piezoelectric elements is to enlarge domain of stability of rotating shafts; i.e. to increase the critical rotation speed at which dynamic stability is lost and self-excited vibration occurs. Self-excitation observed in shafts, in terms of mathematics, consists in bifurcating of the static equilibrium position into an oscillating state representing in fact either stable or unstable limit cycle. Bifurcation of a static solution into a periodic one is called Hopf's bifurcation. In rotating shafts, it can appear due to presence of internal friction in material they are made of. Internal friction can destabilise such systems when subjected to permanent energy supply maintaining constant rotation speed. It is manifested by occurrence of additional rotary motion of the shaft when the angular velocity becomes sufficiently high. The additional precession can be of soft or hard character, depending on type of bifurcation observed in the system. Application of piezoelectric elements is expected to affect character of the self-excited vibration, as terms corresponding to their action are present in the non-linear part of equations of motion. The analysis proves that stabilisation method based on piezoelectric elements strongly effects near-critical vibration of rotating shafts as it makes the bifurcation, in vertically rotating shafts always supercritical, subcritical if only gain factor in the control system is great enough.

1. Introduction

The last several years have been characterised by an animated interest of scientific researchers and engineers in the so-called smart materials and structures that in contradistinction to classical ones can adapt their properties to varying operating conditions according to a given algorithm of controlling. Smart systems combine mechanical properties with non-mechanical ones, most often with electric, magnetic, thermal, or sometimes, optical fields of interaction. The most popular smart structures employ elements controllable by easy-to-transduce electric signal. Predominantly, piezoelectric elements made of lead zirconate titanate or polyvinylidene fluoride are applied.

The present paper examines dynamics of such a smart mechatronic system, in which the mechanical part is represented by rotating shaft and the electric one is posed by piezoceramics (PZT) elements attached to outer surface of the shaft. The purpose of making use of the piezoelectric elements is to actively stabilise the shaft; i.e. to increase critical angular velocity at which dynamic stability is lost and the shaft undergoes self-excitation. Appearance of self-excited vibration in rotating shafts results from presence of internal friction in material of the shafts. The internal friction leads to additional precession performed by the shaft in the critical point. Mathematically, the shaft exhibits Hopf's bifurcation – static equilibrium position evolves into a periodic solution (limit cycle). This phenomenon was thoroughly studied by Kurnik [1], who also investigated effect of other factors on the dynamic stability of rotating shafts and their non-linear, near-critical behaviour [2, 3]. This paper takes up the problem of stability of rotating shafts enhanced by making use of piezoelectric elements introduced to the system. Concept of piezoelectric stabilisation has been quite well recognised in beam-like systems, just works by Tylikowski [4] and Pietrzakowski [5] to mention. Kurnik and Przybyłowicz [6] examined similar approach based on control with proportional and velocity feedback towards a system with non-conservative load. In each case, the main intention was to actively damp or stabilise those systems. In the present paper, the goal remains the same, yet this time special emphasis is put on non-linear analysis. Near-critical behaviour of the shaft is of main interest. Derived equations of motion indicate that coefficients describing active stabilisation realised by the piezoelements appear next to the non-linear term. Moreover, these terms do not vanish for zeroed bifurcation parameter (angular speed) what satisfies one of the necessary conditions of Hopf's theorem. Application of piezoelectric stabilisation is the expected to affect the near-critical of the rotating shaft. In the considered model the shaft rotates in a vertical plane (or horizontal with gravity forces neglected). In such a case, the shaft is known to exhibit always supercritical bifurcation – a safe one as amplitude of the self-excitation grows slowly with increasing angular speed of the shaft [7]. Additional stabilisation introduced to the system can change the situation by creating possibility of appearance of subcritical bifurcation – much more dangerous one, being in fact a catastrophic loss of stability, where even a slight disturbance below the critical threshold makes the shaft jump onto the limit cycle of high amplitude (first limit cycle in this case is orbitally unstable). The paper is to discuss conditions in which such a situation takes place, if at all.

2. Assumptions and model of the system

Model of the considered system consists of a flexible simply supported shaft of length L, stiffness EJ, mass per unit length ρA having piezoelectric elements glued to its outer surface. The piezoelements have length l_{pe}, and are placed between coordinates x_1, x_2, see Fig. 1. Perfect attachment is assumed in the model (no gluing interlayer taken into account). The shaft rotates with a constant angular velocity ω, and gravity force is neglected.

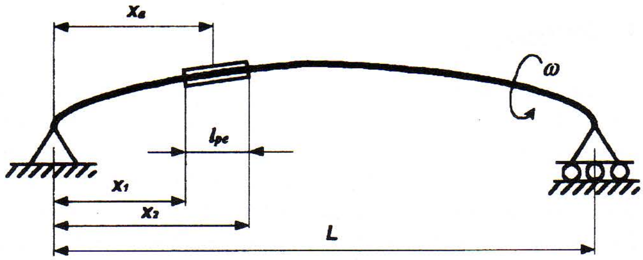

Fig. 1. Model of the shaft

This means that the initial equilibrium position of the shaft is trivial. For the thus formulated model equations of motion are [6]:

$$\rho A \frac{\partial^2 y}{\partial t^2} + c\frac{\partial y}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\partial M_z}{\partial x}\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2}\right] = 0$$
$$\rho A \frac{\partial^2 z}{\partial t^2} + c\frac{\partial z}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\partial M_y}{\partial x}\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2}\right] = 0 \quad (1)$$

where:

$$M_z = EJ\left(k_y + \beta \dot{k}_y + \beta\omega k_z\right) + C_y(x_1, x_2)$$
$$M_y = EJ\left(k_z + \beta \dot{k}_z - \beta\omega k_y\right) + C_z(x_1, x_2) \quad (2)$$

where k_y and k_z stand for the curvatures of the shaft in $x-y$ and $x-z$ planes, respectively, $C_{(.)}(x_1, x_2)$ denotes additional bending moment introduced by the piezoelectric elements, β is the time constant of the Kelvin-Voigt model reflecting presence of the internal friction in the shafts material, and c is coefficient of external damping. The piezoelectric elements work in the following way: the sensors measure internal bending moment appearing in transversely vibrating rotor and send the corresponding electric signal to actuators after appropriately programmed transformations in the electronic unit. The actuators, according to the converse piezoelectric effect, produce bending moment resulting from longitudinal elongation and contraction of the piezoelements situated in opposite sides of the rotor surface. The thus generated moment is to oppose the internal bending moment appearing in the rotor cross-section. In order to measure and generate the bending moment during rotation a segmentation of the sensing and actuating elements is required. This segmentation should be dense enough to localise precisely the plane in which vibration initiates.

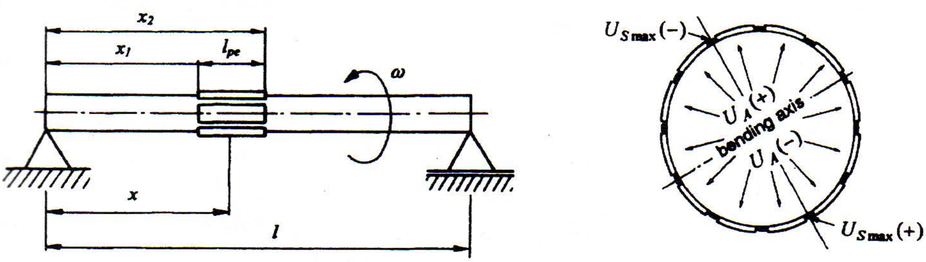

Fig. 2. Shaft with glued piezoelements and its cross-sectional view

By making use of constitutive equations describing the direct and converse piezoelectric effect, see Nye [8], one can find explicit expressions for the bending moments developed by the piezoelements $C_{(.)}(x_1, x_2)$. They are as follows [6]:

$$C_{(.)}(x_1, x_2) = k_d \gamma \left\{ \left.\frac{\partial^2 (.)}{\partial x \partial t}\right|_{x_2} - \left.\frac{\partial^2 (.)}{\partial x \partial t}\right|_{x_1} \right\} \left[H(x-x_1) - H(x-x_2)\right], \quad \gamma = \frac{8(E_{pe}d_{31})^2 r^3 h_{pe}}{\pi \in_0 \in_{pe} |x_2 - x_1|} \quad (3)$$

where (.) can be completed with either y or z transverse displacement of the shaft. In equation (3) the following denote: E_{pe} – Young's modulus of the piezoelectric material, d_{31} – its electromechanical coupling constant ($d_{31} = 170 \times 10^{-12}$ m/V), h_{pe} – thickness of the piezoelements, $\in_0 \in_{pe}$ – their absolute dielectric permittivity, k_d – gain factor applied in the control system, r – radius of the shaft. As the piezoelectric sensors and actuators do not cover the entire length of the shaft but placed in between the coordinates x_1 and x_2 the Heaviside function $H(.)$ is used in equation (3). Details of origin of formula (3) are given [6].

3. Domain of stability

In order to investigate stability of the considered system equations of motion (1) completed with (2) and (3) should be transformed first into a set of ordinary differential equations by means of orthonormalising discretisation. Having this discretisation done

$$\int_0^L L_i\{y[F(x), w_1(t)],\ z[F(x), w_2(t)]\} F(x)\, \mathrm{d}x = 0\,, \quad i = 1, 2 \tag{4}$$

being based on the first eigenmode $F(x) = \sin\frac{\pi x}{L}$ with $w_1(t)$ and $w_2(t)$ arbitrary time functions, and where $L_i\{.\}$ denotes the right-hand sides of equation of motion (1) one obtains the following expression:

$$\dot{\mathbf{u}} = \mathbf{f}(\mathbf{u}, \omega, k_d) \tag{5}$$

where $\mathbf{u} = [u_1, u_2, u_3, u_4]^{\mathrm{T}}$, and $u_1 = w_1$, $u_2 = \dot{w}_1$, $u_3 = w_2$, $u_4 = \dot{w}_2$. Linear approximation contains matrix $\mathbf{A}$, $\dot{\mathbf{u}} = \mathbf{A}(\omega, k_d)\mathbf{u}$, on the grounds of which one can examine stability of the solution of equations (5). The gain factor k_d in parentheses in $\mathbf{A} = \mathbf{A}(\omega, k_d)$ is to underline the presence of terms describing action of the piezoelectric elements in the linear approximation. Application of the Hurwitz criterion leads directly to characteristic equation of the fourth order $\Im_4 = 0$. Four complex and conjugate roots constituting solution of this equation decide about stability of the considered system, to be precise the root of the greatest real part decides about it. In Fig. 3 a blow-up of the diagram showing trajectory of the deciding eigenvaule is presented. When the control system is disabled the decisive eigenvalue is placed at the point O for $\omega = 0$. Increasing rotation speed makes the eigenvalue move towards point A, at which it intersects the imaginary axis. This means loss of stability. Flutter-type self-excitation occurs with the initial vibration

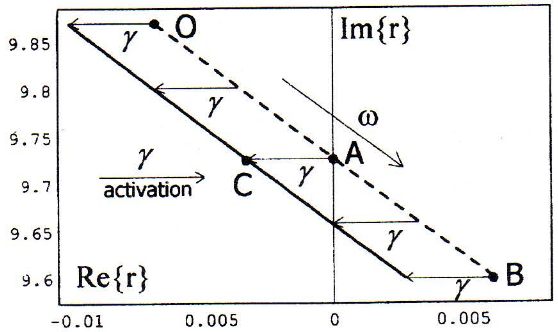

Fig. 3. Trajectory of the eigenvalue for disabled and enabled control system

frequency Ω_0 equal to ordinate of the intersection point. There are then two possibilities: further increase of the rotation speed develops the thus originated vibration (trajectory tends to point B) or the control system can be enabled what shifts the dashed line (see Fig. 3) towards left (point C in the continuous line). This shift (for a fixed ω at ω_{cr}) means stabilisation as the real part of the decisive eigenvalue becomes negative again. The goal is reached – desired stabilisation has been achieved.

4. Non-linear approximation

Analysis of the linearised system has revealed promising results. Stability effect has been obtained and the method seemed to be only limited by dielectric and mechanical strength of the piezoelements themselves. Admittedly, linear analysis does not explore the problem thoroughly. The question is if ever-supercritical bifurcation observed in shafts without stabilisation remains supercritical when stabilised or it can become subcritical in certain conditions. Moreover, in what conditions. To answer these questions it is necessary to formulate a non-linear approximation of equations of motion (5). By assuming that the system is only geometrically non-linear and the curvature is reflected by a power series truncated at terms of the third order, i.e.:

$$k_y = \frac{\partial^2 y}{\partial x^2}\left[1 - \frac{3}{2}\left(\frac{\partial y}{\partial x}\right)^2\right], \quad k_z = \frac{\partial^2 z}{\partial x^2}\left[1 - \frac{3}{2}\left(\frac{\partial z}{\partial x}\right)^2\right] \tag{6}$$

one obtains the third order non-linear approximation:

$$\begin{aligned}
&\rho A y_{,tt} + c y_{,t} + EJ[y_{,xxxx} - 1.5 y_{,xxxx} y_{,x}^{2} - 8 y_{,x} y_{,xx} y_{,xxx} - 3 y_{,xx}^{3} + \beta(y_{,xxxxt} - y_{,xxxxt} y_{,x}^{2} + \\
&- 3 y_{,x} y_{,xx} y_{,xxx} - 9 y_{,xt} y_{,xx} y_{,xxx} - 9 y_{,x} y_{,xxt} y_{,xxx} - 9 y_{,x} y_{,xx} y_{,xxxt} - 9 y_{,xx}^{2} y_{,xxt}) + \beta\omega(z_{,xxxx} + \\
&- 1.5 z_{,xxxx} z_{,xt}^{2} + 0.5 z_{,xxxx} y_{,xt}^{2} - 9 z_{,x} z_{,xx} z_{,xxx} - 3 z_{,xx}^{3} + y_{,x} y_{,xx} z_{,xxx})] + \gamma k_d [C_{y,xx}(x_1, x_2)(1 + \\
&+ 0.5 y_{,x}^{2} + C_{y,x}(x_1, x_2) y_{,x} y_{,xx}] = 0 \\
&\rho A z_{,tt} + c z_{,t} + EJ[z_{,xxxx} - 1.5 z_{,xxxx} z_{,x}^{2} - 8 z_{,x} z_{,xx} z_{,xxx} - 3 z_{,xx}^{3} + \beta(z_{,xxxxt} - z_{,xxxxt} z_{,x}^{2} + \\
&- 3 z_{,x} z_{,xx} z_{,xxx} - 9 z_{,xt} z_{,xx} z_{,xxx} - 9 z_{,x} z_{,xxt} z_{,xxx} - 9 z_{,x} z_{,xx} z_{,xxxt} - 9 z_{,xx}^{2} z_{,xxt}) - \beta\omega(y_{,xxxx} + \\
&- 1.5 y_{,xxxx} y_{,xt}^{2} + 0.5 y_{,xxxx} z_{,xt}^{2} - 9 y_{,x} y_{,xx} y_{,xxx} - 3 y_{,xx}^{3} + z_{,x} z_{,xx} y_{,xxx})] + \gamma k_d [C_{z,xx}(x_1, x_2)(1 + \\
&+ 0.5 z_{,x}^{2} + C_{z,x}(x_1, x_2) z_{,x} z_{,xx}] = 0
\end{aligned} \tag{7}$$

Non-linear equations of motion (7) can be then transformed into ordinary differential equations by application of Galerkin's discretisation (4) and introduction of the new variables u_1, ... u_4. Having this done equations (7) can be implicitly written down as:

$$\dot{\mathbf{u}} = \mathbf{f}(\mathbf{u}, \omega, k_d) = \mathbf{A}(\omega, k_d)\mathbf{u} + \mathbf{N}(\mathbf{u}, \omega, k_d) \tag{8}$$

where $\mathbf{f} = [f_1, f_2, f_3, f_4]$ represent right-hand sides of the discretised equations of motion where the linear and non-linear parts were separated. The angular velocity ω and the gain factor k_d were inserted into parentheses to emphasise that both linear and non-linear approximation depends on those factors. The explicit form of the matrix of the linear part is given as:

$$\mathbf{A}(\omega,k_d)=\begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & a_{22}(k_d) & a_{23}(\omega) & 0 \\ 0 & 0 & 0 & 1 \\ a_{41}(\omega) & 0 & a_{43} & a_{44}(k_d) \end{bmatrix} \tag{9}$$

where

$$\begin{aligned} a_{21} &= a_{43} = -\frac{\pi^4}{2} \\ a_{22} &= a_{44} = -\frac{1}{2}\left(\delta+\pi^4\xi\right)-\pi\gamma_d\left[\cos\pi x_2-\cos\pi x_1\right] \\ a_{23} &= -a_{41} = -\omega\beta\frac{\pi^4}{2} \end{aligned} \tag{10}$$

and

$$\delta = c\frac{k_t}{\rho A},\quad \xi=\frac{\beta}{k_t},\quad \gamma_d=\gamma k_d\frac{k_t}{\rho A L^3},\quad k_t=L^2\sqrt{\frac{\rho A}{EJ}} \tag{11}$$

while the non-linear part is:

$$\mathbf{N}(\mathbf{u},\omega,k_d)=\begin{bmatrix} 0 \\ \mu_1(\omega)u_3(u_3^{\,2}-u_1^{\,2})+\mu_2(k_d)u_1^{\,2}u_2 \\ 0 \\ \mu_1(\omega)u_1(u_3^{\,2}-u_1^{\,2})+\mu_2(k_d)u_3^{\,2}u_4 \end{bmatrix} \tag{12}$$

where

$$\begin{aligned} \mu_1 &= \frac{3}{16}\pi^6\omega\beta \\ \mu_2 &= \frac{3}{8}\xi\pi^6-\frac{3}{2}\pi^4\gamma_d\left(\cos^3\pi x_2-\cos^3\pi x_1\right)\left(\cos\pi x_2-\cos\pi x_1\right) \end{aligned} \tag{13}$$

5. Bifurcating solution

The derived non-linear approximation of the equations of motion enables one to construct a bifurcating solution describing properties of self-excited vibration appearing in neighbourhood of the critical point $\omega=\omega_{cr}$. The bifurcating solution is to be then formulated in a convenient manner described by Kurnik [6] and based on the approach by Iooss and Joseph [9]. The first approximation of the periodic, bifurcating solution has the following form:

$$\mathbf{u}(\varepsilon,t)=2\varepsilon\,\mathrm{Re}\{\mathbf{q}e^{i\Omega t}\} \tag{14}$$

where ε is norm of the solution (measure of the bifurcating vibration amplitude) and Ω frequency of the bifurcating vibration. These two quantities are defined by:

$$\Omega = \Omega_0 + \frac{1}{2}\Omega_2\varepsilon^2 \qquad \text{and} \qquad \varepsilon = \sqrt{2\frac{\omega - \omega_{cr}}{\omega_2}} \tag{15}$$

where Ω_0 is the initial frequency of the self-excited (bifurcating) vibration, discussed in the linear analysis and Ω_2, ω_2 – coefficients to be determined. According to [6] they are:

$$\omega_2 = -\frac{\mathrm{Re}\,\Xi_2}{3\left.\dfrac{\mathrm{d}\,\mathrm{Re}\{r\}}{\mathrm{d}\omega}\right|_{\omega_{cr}}}, \qquad \Omega_2 = \omega_2 \left.\frac{\mathrm{d}\,\mathrm{Im}\{r\}}{\mathrm{d}\omega}\right|_{\omega_{cr}} + \frac{1}{3}\mathrm{Im}\,\Xi_2 \tag{16}$$

As the non-linear part of equation (7) lacks terms of the second order, see also equation (12), the term Ξ_2 in (16) is then given by:

$$\Xi_2 = 3\sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4}\sum_{l=1}^{4} c_{ijkl}\bar{q}_i^{\,*} q_j q_k \bar{q}_l , \qquad c_{ijkl} = \frac{\partial f_i(\mathbf{0},\omega_{cr},\gamma_d)}{\partial u_j\,\partial u_k\,\partial u_l} \tag{17}$$

where f_i are the right-hand side functions of equations of motion.

Definitions (16), (17) as well as the bifurcating solution (14) include vectors denoted by $\mathbf{q}$ and $\mathbf{q}^*$, which are the eigenvectors corresponding to the following eigenproblems:

$$\{\mathbf{A}(\omega_{cr},k_d) - \mathrm{i}\Omega_0\mathbf{I}\}\mathbf{q} = \mathbf{0} \tag{19}$$

and the adjoint one

$$\{\mathbf{A}^{\mathrm{T}}(\omega_{cr},k_d) + \mathrm{i}\Omega_0\mathbf{I}\}\mathbf{q}^* = \mathbf{0} \tag{20}$$

After simple transformations one finds the vectors $\mathbf{q}$, $\mathbf{q}^*$ to be:

$$\mathbf{q} = \begin{bmatrix} 1 \\ 0 \\ \dfrac{-a_{21} - \Omega_0^2}{a_{23}} \\ \dfrac{a_{22}\Omega_0^2}{a_{23}} \end{bmatrix} + \mathrm{i}\Omega_0 \begin{bmatrix} 0 \\ 1 \\ -\dfrac{a_{22}}{a_{23}} \\ \dfrac{-a_{21} - \Omega_0^2}{a_{23}} \end{bmatrix} \tag{21}$$

$$\mathbf{q} = \left\{ \begin{bmatrix} \dfrac{a_{22}a_{43} + 2\Omega_0^2}{a_{23}} \\ \dfrac{-a_{43} - \Omega_0^2}{a_{23}} \\ -a_{41} \\ 1 \end{bmatrix} + \mathrm{i}\Omega_0 \begin{bmatrix} \dfrac{\Omega_0^2 + a_{44} - a_{22}^2}{a_{23}} \\ \dfrac{a_{44}}{a_{23}} \\ -1 \\ 0 \end{bmatrix} \right\} D \tag{22}$$

where the complex constant D is chosen in such a way, so that the orthonormalisation condition $\langle \mathbf{q},\mathbf{q}^* \rangle = \sum_i q_i \bar{q}_i = 1$ is fulfilled. This requires that the constant D must be:

$$D = \frac{1}{q_1^{\,*} + q_2^{\,*}\bar{q}_2 + q_3^{\,*}\bar{q}_3 + \bar{q}_4} \tag{23}$$

To complete constructing of the bifurcating solution one needs to find the derivatives c_{ijkl} in (17). They are as in the following:

$$\begin{aligned} &c_{2333} = -c_{4111} = 6\mu_1 \\ &c_{2211} = c_{2121} = c_{2112} = c_{4433} = c_{4343} = c_{4334} = 2\mu_2 \\ &c_{2311} = c_{2131} = c_{2113} = -c_{4133} = -c_{4313} = -c_{4331} = -2\mu_2 \end{aligned} \tag{24}$$

The last problem related with the bifurcation analysis is the problem of orbital stability of the obtained bifurcating solution. The solution is either stable or unstable, depending on sign of the Floquet exponent σ. The exponent is defined by:

$$\sigma(\varepsilon) = -\left.\frac{\mathrm{d}\,\mathrm{Re}\{r\}}{\mathrm{d}\omega}\right|_{\omega_{cr}} \omega_2 \varepsilon^2 + O(\varepsilon^3) \tag{25}$$

where $O(\varepsilon^3)$ denotes the terms corresponding to higher orders of ε (negligible with respect to those of ε^2). When $\sigma < 0$ then the bifurcating solution respecting in fact a limit cycle is orbitally and asymptotically stable (supercritical bifurcation), when $\sigma > 0$ it is unstable (subcritical bifurcation).

6. Results of numerical simulations

On the grounds of the developed bifurcating solution numerical simulations have been carried out. The main purpose was to evaluate coefficients appearing in the series describing bifurcating vibration and responsible for evolution of its frequency and amplitude with growing angular velocity of the shaft. Another important factor highly effecting stability of the bifurcating solution is the derivative of real part of the decisive eigenvalue in the critical point: $(\mathrm{Re}\{r(\omega_{cr})\})_{,\omega}$. It has turned out that $(\mathrm{Re}\{r(\omega_{cr})\})_{,\omega}$ remains positive for any value of the bifurcating parameter ω (also for values far from the critical threshold ω_{cr}) and the applied gain factor k_d. This means that stability of the bifurcating solution depends only on sign of the coefficient ω_2. A diagram presenting behaviour of ω_2 versus the gain factor k_d is shown in Fig. 4. One can clearly see that only very small value of k_d secures stability of the bifurcating solution ($k_d < 0.01$). This is a disadvantageous situation where on the one hand the rotating shaft becomes more stable but character of loss of the stability dramatically changes on the other. In terms of engineering practice this means that the shaft undergoing active stabilisation by piezoelectric elements should be protected from excessive growth of the rotation speed so that to avoid contact with direct neighbourhood of the newly obtained critical threshold. That would threaten the system with subcritical bifurcation, a ver dangerous case when the rotor equilibrium position jumps over the first unstable limit cycle to rest on the successive and

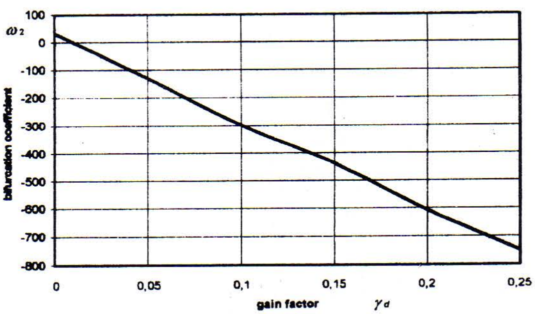

Fig. 4. Bifurcation parameter ω_2 vs. gain factor $\gamma_d = \gamma k_d$

stable one, but of much greater amplitude. The stability is lost in a catastrophic way, and it can appear even below the critical speed. Obviously, the critical speed itself is increased.

In Fig. 5 the bifurcation coefficient Ω_2 versus the applied gain factor k_d is shown. It rules behaviour of frequency of the self-excited vibration near the critical point. First approximation of the bifurcating solution enables obtaining only the linear function expressing development of this frequency for a narrow region of variability of ω. Exemplary frequencies of the bifurcating solution corresponding to some gain factors k_d are presented in Fig. 6.

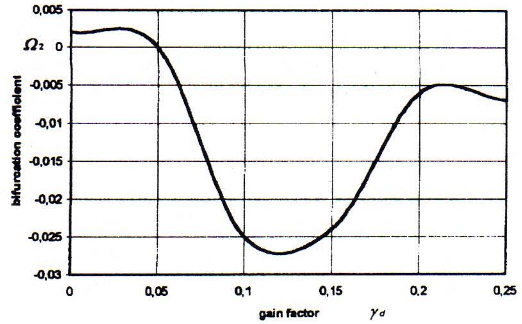

Fig. 5. Bifurcation coefficient Ω_2 vs. gain factor $\gamma_d = \gamma k_d$

Calculated values of ω_2 have enabled determination of amplitudes of the bifurcating solution. In the case of super-critical bifurcation, observed for slightly stabilised shafts, the trivial solution remains stable until ω_{cr} is reached and then self-excited vibration occurs with amplitude parabolically growing with ω (see the continuous line denoted by $\gamma_d = 0$, $\omega_{cr} = 640$ in Fig. 7). Greater gain factors make the bifurcation subcritical, which means that a small disturbation due to an external mechanical stimulus exceeding the ordinate of a given dashed line shown in Fig. 7 make the static equilibrium position bifurcate into the successive limit cycle of higher amplitude (not marked in the figure – this requires the second approximation of the bifurcation solution to be determined). Such a situation can happen even for ω below ω_{cr} what is especially dangerous.

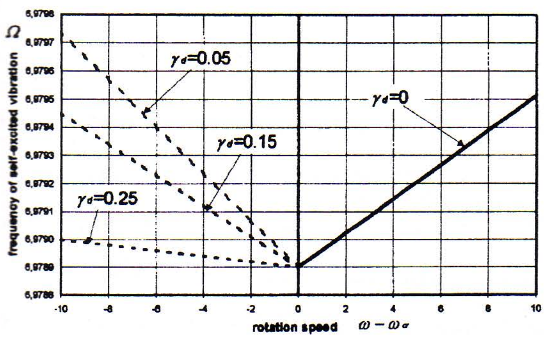

Fig. 6. Frequency of the bifurcating solution

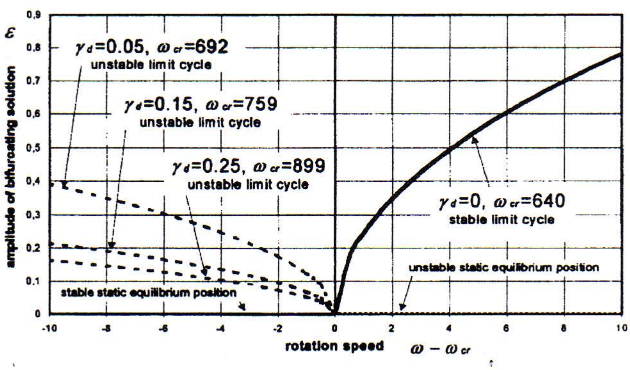

Fig. 7. Amplitude of the bifurcating solution

7. Concluding remarks

The applied method of active stabilisation of a rotating shaft by making use of piezoelectric elements made of lead zirconate titanate (PZT) has proved to be an efficient tool in shifting critical rotation speed towards greater values. As it is shown in Fig. 3 switching on the control system makes trajectory of the decisive eigenvalue move leftwards, i.e. where real parts are smaller (negative). Non-linear analysis discloses however, that application of stronger gain factors endangers the system with subcritical bifurcation. This entails hard self-excitation and sudden growth of the vibration amplitude in neighbourhood of the critical speed, even for $\omega < \omega_{cr}$. Thus the system is stabilised by increasingly growing gain factor on the one hand but on the other the system should be kept increasingly far from the newly obtained critical threshold to avoid hard self-excitation.

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