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ANALYSIS OF THE POSITIONING ACCURACY OF AN *NM7M-MAR* ROBOT MANIPULATOR

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Abstract

The results of a numerical analysis of the positioning accuracy of an *NM7M-MAR* manipulator gripping device in the main working space of the robot are presented. The concept of the positioning analysis involves the problem of the gripping device position accuracy and changes in the orientation determined in the local reference frame of the gripping device and caused by positioning inaccuracies of links in manipulator kinematic pairs. The positioning accuracy of a *RPPP* type robot manipulator with four generalised co-ordinates with an additional degree of freedom in the form of rotation of the gripping device around its axis of fixation has been analysed. The presented engineering analysis has been conducted in order to describe analytically positioning properties of an actual *NM7M-MAR* robot in its main working space. It is an introduction to the analysis of manipulator mechanical vibrations.

1. Introduction

An influence of the arrangement of angles and positions in individual robot modules on a positioning error, or in other words on an error of the position and orientation of the gripping device, has been presented as a jacobian matrix. Individual columns of this matrix include components of differential displacement and rotation vectors of a system of co-ordinates connected with the gripping device and caused by a differential change in the generalised co-ordinate in the proper kinematic pair. Elements of the robot jacobian matrix have been determined by means of the method developed by R.P. Paul [3]. This method was employed in [1,3] to analyse the positioning accuracy of a six-degree-of-freedom robot manipulator with the I-class kinematic pairs of the rotational type. Some examples of manipulator jacobian matrices were presented in [4].

An analysis of the positioning accuracy of the gripping device has been carried out for an *NM7M-MAR* robot which is in the possession of the Laboratory of Manipulators of the Division of Machine Dynamics. The *NM7M-MAR* robot is a module robot built on the basis of the *NM7M* robot developed at the R & D Centre of Fundamentals of Technology and Machine Design in Warsaw. In comparison with the *NM7M* robot, the *NM7M-MAR* robot is equipped additionally with an *MLL40* linear displacement module. This has allowed for an enlargement of the main working space of the robot, but simultaneously has affected the gripping device positioning accuracy. The analysis presented here is an introduction to the analysis of vibrations of robot links during its operation.

A structural and motion analysis, as well as identification of the working space of the *NM7M-MAR* robot has been carried out in Section 2 of the present paper. A general scheme of the structure of the jacobian matrix of the robot including both kinematic pairs of the rotational and translatory motion type has also been presented. The theory of the robot jacobian matrix has been applied to the *NM7M-MAR* robot. In Section 3 a numerical analysis based on the theoretical characteristics of an actual *NM7M-MAR* robot has been presented. Section 4 includes the conclusions resulting from the numerical analysis.

2. Theoretical Support

An analysis of mechanical vibrations of complex dynamic systems such as industrial robot manipulators is one of basic problems of dynamics. An accuracy of manipulator gripping device positioning in a defined point of the main working space belongs to basic characteristics of its quantitative evaluation. The accuracies of robot gripping device positioning met in industrial robot operation manuals usually fall within the range from ± 0.01mm to ± 5mm. Highest accuracies are required in precision works, for instance during assembly. A positioning error of the manipulator gripping device depends on many factors such as, for example, accuracies of the relative arrangement of links in kinematic pairs that follow from the way the robot is controlled and driven, manipulative dimensional error tolerance of its individual elements, elastic deformations and clearances in kinematic pairs caused by errors in manufacturing, assembly, as well as in wear and tear.

An accuracy analysis consists in determination of the position and orientation tolerance of the manipulator gripping device when the dimensions, position and arrangement tolerances of links in kinematic pairs are given. To analyse an accuracy, linear and angular displacements of individual manipulator links and limits and singular points of the working space have to be known.

Let us consider a robot manipulator with 'n' generalised co-ordinates. Then, a vector of generalised co-ordinates assumes the following form:

$$\bar{q} = [\, q_1 \,, q_2 \,, \dots \,, q_i \,, \dots \,, q_n \,]^T \tag{1}$$

A vector of disturbances of generalised co-ordinates of the manipulator takes the form:

$$d\bar{q} = [\, dq_1 \,, dq_2 \,, \dots \,, dq_i \,, \dots \,, dq_n \,]^T \tag{2}$$

The matrix $A_{i;i-1}$ that determines the position and orientation of a link 'i' with respect to the system of co-ordinates of a link 'i-1', by Denavit-Hartenberg transformation, assumes the well-known form:

$$A_{i;i-1} = \begin{bmatrix} a_{x_{i;i-1}} & b_{x_{i;i-1}} & c_{x_{i;i-1}} & d_{x_{i;i-1}} \\ a_{y_{i;i-1}} & b_{y_{i;i-1}} & c_{y_{i;i-1}} & d_{y_{i;i-1}} \\ a_{z_{i;i-1}} & b_{z_{i;i-1}} & c_{z_{i;i-1}} & d_{z_{i;i-1}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where:

$$A_{i;i-1} = \begin{bmatrix} \cos(q_i) & -\sin(q_i)\cdot\cos(\alpha_i) & \sin(q_i)\cdot\sin(\alpha_i) & l_i\cdot\cos(q_i) \\ \sin(q_i) & \cos(q_i)\cdot\cos(\alpha_i) & -\cos(q_i)\cdot\sin(\alpha_i) & l_i\cdot\sin(q_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & \lambda_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

whereas:

q_i - angle of the relative arrangement of the links that form a rotational pair;

λ_i - displacement along the rotational pair axis;

l_i, α_i - distance and angle of the relative torsional deflexion of the kinematic pair axes;

$\bar{d}\ (d_x, d_y, d_z)$ - vector of the position of the gripping device centre in the inertial system of co-ordinates of the manipulator;

$\bar{c}\ (c_x, c_y, c_z)$ - unit vector of the longitudinal symmetry axis of the gripping device in the inertial system of co-ordinates;

$\bar{b}\ (b_x, b_y, b_z)$ - unit vector of the orientation of the gripping device in the inertial system of co-ordinates;

$\bar{a} = \bar{b} \times \bar{c}$

For small changes in the position and orientation brought about by differential changes dq_i of the generalised co-ordinate q_i, a differential increment of the transformation matrix $dA_{i;i-1}$ will occur. This differential can be expressed as a derivative of the matrix $A_{i;i-1}$ with respect to the generalised co-ordinate q_i:

$$dA_{i;i-1} = \frac{\partial A_{i;i-1}}{\partial q_i}\cdot dq_i = \Delta_i^{i-1}\cdot A_{i;i-1} \tag{5}$$

Depending on the type of the kinematic pair and the way the systems of co-ordinates are selected, the differential matrix Δ_i^{i-1} assumes different forms. For instance:

- for the I-class kinematic pair of the rotational type and for the rotation around the axis 'z' of the local system of co-ordinates (Denavit-Hartenberg transformation):

$$\frac{\partial A_{i;i-1}}{\partial q_i} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \cos(q_i) & -\sin(q_i)\cos(\alpha_i) & \sin(q_i)\sin(\alpha_i) & l_i\cos(q_i) \\ \sin(q_i) & \cos(q_i)\cos(\alpha_i) & -\cos(q_i)\sin(\alpha_i) & l_i\sin(q_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & \lambda_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \bar{\Delta}_i^{i-1}\cdot A_{i;i-1} \tag{6}$$

- for the I-class kinematic pair of the translatory type and for the displacement along the axis 'z' of the local system of co-ordinates:

$$\Delta_i^{i-1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & dq_i \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

Robot modules usually have only one degree of freedom in the form of changes in the link natural co-ordinate that describes the reciprocal position and orientation of links. Therefore, elements of the differential matrix Δ_i^{i-1} are linearly dependent on changes dq_i. Thus, Eq. (5) assumes the form:

$$dA_{i;i-1} = \overline{\Delta}_i^{i-1} \cdot A_{i;i-1} \cdot dq_i \tag{8}$$

The matrix $\overline{\Delta}_i^{i-1}$ is a zero-one matrix.

Differential changes in the position and orientation of the manipulator gripping device are caused by differential changes in manipulator generalised co-ordinates, Eq. (1). An effect of differential changes dq_i on a differential increment of the transformation matrix of the position and orientation of the gripping device (changes in the gripping device position and orientation) in the inertial reference frame of the robot has been described by means of the differential matrix $dA_{n;0}$.

$$\begin{aligned} dA_{n;0} &= \sum_{i=1}^{n} A_{1;0} \cdot A_{2;1} \cdot \ldots \cdot A_{i-1;i-2} \cdot \Delta_i^{i-1} \cdot A_{i;i-1} \cdot \ldots \cdot A_{n;n-1} = \\ &= A_{n;0} \cdot \sum_{i=1}^{n} A_{n;i-1}^{-1} \cdot \Delta_i^{i-1} \cdot A_{n;i-1} = A_{n;0} \cdot \sum_{i=1}^{n} \Delta_i^{n} \end{aligned} \tag{9}$$

The matrix Δ_i^n shows differential changes of the position and orientation of the gripping device in its local system of co-ordinates, caused by differential changes of the 'i-th' generalised co-ordinate of the robot manipulator.

$$\Delta_i^n = A_{n;i-1}^{-1} \cdot \Delta_i^{i-1} \cdot A_{n;i-1} \tag{10}$$

$A_{n;i-1}$ - position and orientation of the local system of co-ordinates of the gripping device ('n' system of co-ordinates) with respect to the 'i-1' system.

A general form of disturbances in the position and orientation in the 'i-th' kinematic pair is expressed by the differential matrix Δ_i^{i-1}.

$$\Delta_i^{i-1} = \begin{bmatrix} 0 & -\delta_{z_{i,i-1}} & \delta_{y_{i,i-1}} & K_{x_{i,i-1}} \\ \delta_{z_{i,i-1}} & 0 & -\delta_{x_{i,i-1}} & K_{y_{i,i-1}} \\ -\delta_{y_{i,i-1}} & \delta_{x_{i,i-1}} & 0 & K_{z_{i,i-1}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad A_{n;i-1} = \begin{bmatrix} a_{x_i} & b_{x_i} & c_{x_i} & d_{x_i} \\ a_{y_i} & b_{y_i} & c_{y_i} & d_{y_i} \\ a_{z_i} & b_{z_i} & c_{z_i} & d_{z_i} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

Comparing the sides of Eq. (10), we obtain differential changes in positions and orientations of the gripping device caused by a differential disturbance of the '*i*-th' generalised co-ordinate:

$$
\begin{aligned}
K_{x_{i,n}} &= \bar{a}_i \cdot [(\bar{\delta}_{i,i-1} \times \bar{d}_i) + \bar{K}_{i,i-1}] \\
K_{y_{i,n}} &= \bar{b}_i \cdot [(\bar{\delta}_{i,i-1} \times \bar{d}_i) + \bar{K}_{i,i-1}] \\
K_{z_{i,n}} &= \bar{c}_i \cdot [(\bar{\delta}_{i,i-1} \times \bar{d}_i) + \bar{K}_{i,i-1}] \\
\delta_{x_{i,n}} = \bar{a}_i \cdot \bar{\delta}_{i,i-1} \quad &, \quad \delta_{y_{i,n}} = \bar{b}_i \cdot \bar{\delta}_{i,i-1} \quad , \quad \delta_{z_{i,n}} = \bar{c}_i \cdot \bar{\delta}_{i,i-1}
\end{aligned}
\tag{12}
$$

where:

$\bar{K}_{i,n}$, $\bar{\delta}_{i,n}$ - vectors of, respectively, differential displacement and rotation of the gripping device reference frame ('*n*' system of co-ordinates) with respect to its axis, caused by a differential change dq_i in the '*i*-th' kinematic pair.

Differential changes in the position and orientation of the gripping device described in the system related to the gripping device are a sum of differential changes of vector components of the position and orientation caused by changes dq_i in subsequent kinematic pairs of the manipulator.

$$
\begin{aligned}
K_x &= \sum_{i=1}^{n} K_{x_{i,n}} \cdot dq_i \qquad K_y = \sum_{i=1}^{n} K_{y_{i,n}} \cdot dq_i \qquad K_z = \sum_{i=1}^{n} K_{z_{i,n}} \cdot dq_i \\
\delta_x &= \sum_{i=1}^{n} \delta_{x_{i,n}} \cdot dq_i \qquad \delta_y = \sum_{i=1}^{n} \delta_{y_{i,n}} \cdot dq_i \qquad \delta_z = \sum_{i=1}^{n} \delta_{z_{i,n}} \cdot dq_i
\end{aligned}
\tag{13}
$$

In general, the resultants of the displacement and change in the orientation of the gripping device in its local system of co-ordinates, written in the form of a jacobian of the working link, for a manipulator with 'n'=6 generalised co-ordinates (all co-ordinates are disturbed), are equal to:

$$
J = \left[\frac{\partial A_{n;0}}{\partial q_i}\right] = \begin{bmatrix} K_x \\ K_y \\ K_z \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \begin{bmatrix}
\frac{\partial(K_{x_{1,n}})}{\partial q_1} & \frac{\partial(K_{x_{2,n}})}{\partial q_2} & \cdots & \frac{\partial(K_{x_{i,n}})}{\partial q_i} & \cdots & \frac{\partial(K_{x_{n,n}})}{\partial q_n} \\
\frac{\partial(K_{y_{1,n}})}{\partial q_1} & \frac{\partial(K_{y_{2,n}})}{\partial q_2} & \cdots & \frac{\partial(K_{y_{i,n}})}{\partial q_i} & \cdots & \frac{\partial(K_{y_{n,n}})}{\partial q_n} \\
\frac{\partial(K_{z_{1,n}})}{\partial q_1} & \frac{\partial(K_{z_{2,n}})}{\partial q_2} & \cdots & \frac{\partial(K_{z_{i,n}})}{\partial q_i} & \cdots & \frac{\partial(K_{z_{n,n}})}{\partial q_n} \\
\frac{\partial(\delta_{x_{1,n}})}{\partial q_1} & \frac{\partial(\delta_{x_{2,n}})}{\partial q_2} & \cdots & \frac{\partial(\delta_{x_{i,n}})}{\partial q_i} & \cdots & \frac{\partial(\delta_{x_{n,n}})}{\partial q_n} \\
\frac{\partial(\delta_{y_{1,n}})}{\partial q_1} & \frac{\partial(\delta_{y_{2,n}})}{\partial q_2} & \cdots & \frac{\partial(\delta_{y_{i,n}})}{\partial q_i} & \cdots & \frac{\partial(\delta_{y_{n,n}})}{\partial q_n} \\
\frac{\partial(\delta_{z_{1,n}})}{\partial q_1} & \frac{\partial(\delta_{z_{2,n}})}{\partial q_2} & \cdots & \frac{\partial(\delta_{z_{i,n}})}{\partial q_i} & \cdots & \frac{\partial(\delta_{z_{n,n}})}{\partial q_n}
\end{bmatrix} \cdot \begin{bmatrix} dq_1 \\ dq_2 \\ \cdots \\ dq_i \\ \cdots \\ dq_n \end{bmatrix}
\tag{14}
$$

From Eq. (9) it follows that:

$$\frac{\partial A_{n;0}}{\partial q_i} = A_{n;0} \cdot \bar{\Delta}_i^n \tag{15}$$

The jacobian of the working link is composed of vectors of differential rotation and displacement resulting from differential changes dq_i of the manipulator generalised coordinates. In the case 'n'>6, we have an excess of degrees of freedom and in order to select one of possible solutions, we have to add an additional criterion that limits a number of solutions, for instance a criterion of the manipulator energy minimisation.

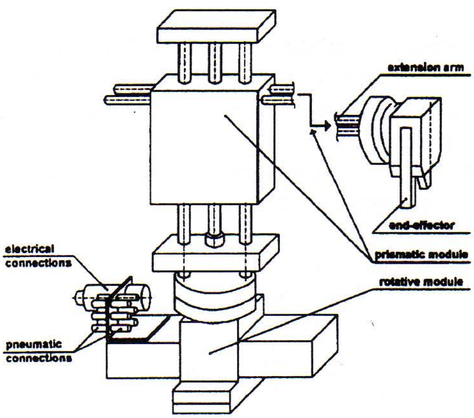


Fig.1. Scheme of the robot manipulator

An *NM7M-MAR* robot has been analysed from the viewpoint of its positioning accuracy. This manipulator consist of one rotational module and three linear ones. An extra degree of freedom is provided by a possibility of rotation of the gripping device around its fixation axis. An *NM7M-MAR* manipulator is designed to displace tiny objects in a continuous and repeatable way on assembly, processing, and control stands or to handle materials. An *NM7M-MAR* manipulator is a device that is characterised by a fixed duty cycle. Its working cycles depend on the way a controller is coded according to the needs of technological processes. The main working space and subsequent working strokes of the manipulator are shown in Fig. 3. The numbers correspond to the subsequent working strokes of the robot. An *NM7M-MAR* robot manipulator is composed of the following units: an angular displacement unit - a turn table, a vertical and horizontal linear displacement unit, an extension arm, and a gripping device. At the end of the manually adjusted extension arm there is a beam to which the gripping device unit is mounted with a possibility of its adjustment in any angular position with respect to the extension arm axis, Fig. 1.

A vector of generalised co-ordinates of the manipulator under analysis assumes the following form:

$$\bar{q} = [\, q_1 \,, q_2' \,, K \,, q_3' \,, K_1 \,]^T \tag{16}$$

A relation between the manipulator generalised co-ordinates and design constants is rendered by Eq. (17):

$$\begin{aligned} K &= S_{k_3} + K_m \\ q_2'(t) &= S_{k_4} + q_2(t) \\ q_3'(t) &= S_{k_5} + q_3(t) \end{aligned} \tag{17}$$

$S_{k_3}, S_{k_4}, S_{k_5}$ and S_{k_1}, S_{k_2}, S_{ch} - design constants of the actual robot.

A vector of disturbances of generalised co-ordinates of the manipulator takes the form:

$$d\bar{q} = [\, dq_1 \, , dq_2^{/} \, , dK \, , dq_3^{/} \, , dK_1 \,]^T \tag{18}$$

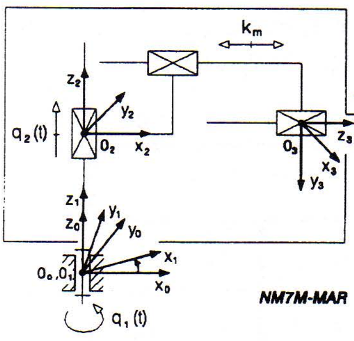


Fig. 2. Selection of local systems of co-ordinates of robot links

Figure 2 shows a relative arrangement of individual reference frames of the manipulator. Matrix $A_{5;4}$ describes transformation matrix of co-ordinates for the assumed type of the robot gripping device (a pneumomechanical jaw gripping device has been assumed). The angular position of the gripping device with respect to the extension arm is determined by an angle K_1. The gripping device assumes the vertical position for the angle $K_1 = 0$.

The position and orientation of the gripping device in the inertial system of co-ordinates of the robot is rendered by the following matrix:

$$A_{5;0} = A_{1;0} \cdot A_{2;1} \cdot A_{3;2} \cdot A_{4;3} \cdot A_{5;4} \tag{19}$$

The elements of the orientation and position matrix of the gripping device in the inertial system of co-ordinates of the base assume the form:

$$\begin{gathered} i = 1 \\ \bar{a}_1 = [\, -\sin(q_1) \cdot \sin(K_1) \, , \cos(q_1) \cdot \sin(K_1) \, , -\cos(K_1) \,]^T \\ \bar{b}_1 = [\, -\sin(q_1) \cdot \cos(K_1) \, , \cos(q_1) \cdot \cos(K_1) \, , \sin(K_1) \,]^T \\ \bar{c}_1 = [\, \cos(q_1) \, , \sin(q_1) \, , 0 \,]^T \\ \bar{d}_1 = [\, [K + q_3^{/}(t)] \cdot \cos(q_1) - S_{ch} \cdot \sin(q_1) \cdot \sin(K_1) \, , \\ [K + q_3^{/}(t)] \cdot \sin(q_1) + S_{ch} \cdot \cos(q_1) \cdot \sin(K_1) \, , q_2^{/}(t) + S_{k_1} - S_{k_2} - S_{ch} \cdot \cos(K_1) \,]^T \end{gathered} \tag{20}$$

The components of the position and orientation vectors of the system of co-ordinates connected with the gripping device with respect to the system connected with the link 'i' are:

$$\begin{gathered} i = 2 \\ \bar{a}_2 = [\, 0 \, , \sin(K_1) \, , -\cos(K_1) \,]^T \quad , \quad \bar{b}_2 = [\, 0 \, , \cos(K_1) \, , \sin(K_1) \,]^T \\ \bar{c}_2 = [\, 1 \, , 0 \, , 0 \,]^T \\ \bar{d}_2 = [\, K + q_3^{/}(t) \, , S_{ch} \cdot \sin(K_1) \, , q_2^{/}(t) + S_{k_1} - S_{k_2} - S_{ch} \cdot \cos(K_1) \,]^T \end{gathered} \tag{21}$$

$$\begin{gathered} i = 3 \\ \bar{a}_3 = [\, 0 \, , \sin(K_1) \, , -\cos(K_1) \,]^T \quad , \quad \bar{b}_3 = [\, 0 \, , \cos(K_1) \, , \sin(K_1) \,]^T \\ \bar{c}_3 = [\, 1 \, , 0 \, , 0 \,]^T \quad , \quad \bar{d}_3 = [\, K + q_3^{/}(t) \, , S_{ch} \cdot \sin(K_1) \, , S_{k_1} - S_{k_2} - S_{ch} \cdot \cos(K_1) \,]^T \end{gathered} \tag{22}$$

$$i = 4$$

$$\bar{a}_4 = [\, -\sin(K_1)\,,\ \cos(K_1)\,,\ 0\,]^T \quad , \quad \bar{b}_4 = [\, -\cos(K_1)\,,\ -\sin(K_1)\,,\ 0\,]^T \tag{23}$$

$$\bar{c}_4 = [\, 0\,,\ 0\,,\ 1\,]^T \quad , \quad \bar{d}_4 = [\, -S_{ch}\cdot \sin(K_1)\,,\ S_{ch}\cdot \cos(K_1)\,,\ q_3'(t)\,]^T$$

$$i = 5$$

$$\bar{a}_5 = [\, \cos(K_1)\,,\ \sin(K_1)\,,\ 0\,]^T \quad , \quad \bar{b}_5 = [\, -\sin(K_1)\,,\ \cos(K_1)\,,\ 0\,]^T \tag{24}$$

$$\bar{c}_5 = [\, 0\,,\ 0\,,\ 1\,]^T \quad , \quad \bar{d}_5 = [\, S_{ch}\cdot \cos(K_1)\,,\ S_{ch}\cdot \sin(K_1)\,,\ 0\,]^T$$

Table I

Type of the robot module	Constant quantities of the module	Module variable	
angular displacement module	$\bar{K}_{1,0} = 0$	$\bar{\delta}_{1,0} = \bar{k}_o \cdot \delta_{z_{1,0}}$	$\delta_{z_{1,0}} = dq_1$
linear displacement module	$\bar{\delta}_{2,1} = 0$	$\bar{K}_{2,1} = \bar{k}_1 \cdot K_{z_{2,1}}$	$K_{z_{2,1}} = dq_2'$
linear displacement module (manual)	$\bar{\delta}_{3,2} = 0$	$\bar{K}_{3,2} = \bar{i}_2 \cdot K_{x_{3,2}}$	$K_{x_{3,2}} = dK$
linear displacement module	$\bar{\delta}_{4,3} = 0$	$\bar{K}_{4,3} = \bar{k}_3 \cdot K_{z_{4,3}}$	$K_{z_{4,3}} = dq_3'$
angular displacement of the gripping device	$\bar{K}_{5,4} = 0$	$\bar{\delta}_{5,4} = \bar{k}_4 \cdot \delta_{z_{5,4}}$	$\delta_{z_{5,4}} = dK_1$
$\bar{k}_i$ - unit vector of the axis z of the 'i-th' system of co-ordinates; $\bar{i}_2$ - unit vector of the axis x of the system $X_2Y_2Z_2$.			

The terms of the position and orientation matrix of the robot gripping device with respect to the robot inertial reference frame have been employed to determine a jacobian matrix of the manipulator. An influence of the disturbance in the 'i-th' module on the components of the position and orientation vectors of the gripping device, expressed in the local system of co-ordinates of the gripping device, is as follows:

$$\bar{K}_{1,5} = [\,(a_{y_1}\cdot d_{x_1} - a_{x_1}\cdot d_{y_1})\cdot \delta_{z_{1,0}}\,,\ (b_{y_1}\cdot d_{x_1} - b_{x_1}\cdot d_{y_1})\cdot \delta_{z_{1,0}}\,,\ (c_{y_1}\cdot d_{x_1} - c_{x_1}\cdot d_{y_1})\cdot \delta_{z_{1,0}}\,]^T$$

$$\bar{\delta}_{1,5} = [\, a_{z_1}\cdot \delta_{z_{1,0}}\,,\ 0\,,\ 0\,]^T$$

$$\bar{K}_{2,5} = [\, a_{z_2}\cdot K_{z_{2,1}}\,,\ b_{z_2}\cdot K_{z_{2,1}}\,,\ 0\,]^T \qquad \bar{K}_{3,5} = [\, 0\,,\ 0\,,\ c_{x_3}\cdot K_{x_{3,2}}\,]^T \qquad \bar{K}_{4,5} = [\, 0\,,\ 0\,,\ c_{z_4}\cdot K_{z_{4,3}}\,]^T$$

$$\bar{\delta}_{i,5} = [\, 0\,,\ 0\,,\ 0\,]^T \qquad for \qquad i = 2\,,\ 3\,,\ 4$$

$$\bar{K}_{5,5} = [\,(a_{y_5}\cdot d_{x_5} - a_{x_5}\cdot d_{y_5})\cdot \delta_{z_{5,4}}\,,\ (b_{y_5}\cdot d_{x_5} - b_{x_5}\cdot d_{y_5})\cdot \delta_{z_{5,4}}\,,\ 0\,]^T \qquad \bar{\delta}_{5,5} = [\, 0\,,\ 0\,,\ c_{z_5}\cdot \delta_{z_{5,4}}\,]^T$$

(25)

The terms of the jacobian matrix include the components of the differential displacement and rotation vectors of the local frame of the gripping device, resulting from differential changes in generalised co-ordinates of the robot.

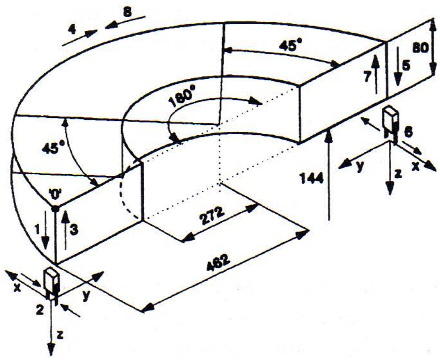


Fig. 3. Main working space of the *NM7M-MAR* robot. The angular position of the gripping device $K_1=0$

The differential changes in the position and orientation of the gripping device described in the system connected with the gripping device will be a sum of components of the differential displacement and rotation of the gripping device caused by differential changes of generalised co-ordinates of kinematic pairs.

The column 'i' of the jacobian matrix corresponds to the partial derivative of the matrix $A_{5;i-1}$ with respect to the generalised co-ordinate q_i. Taking into account Eqs. (13), (14) (18), Table I, and as a result of the mathematical analysis carried out for the manipulator under consideration, we obtain:

$$\begin{bmatrix} K_x \\ K_y \\ K_z \\ \delta_x \\ \delta_y \\ \delta_z \end{bmatrix} = \begin{bmatrix} \frac{\partial(K_{x_{1,5}})}{\partial q_1} & \frac{\partial(K_{x_{2,5}})}{\partial q_2'} & 0 & 0 & \frac{\partial(K_{x_{5,5}})}{\partial K_1} \\ \frac{\partial(K_{y_{1,5}})}{\partial q_1} & \frac{\partial(K_{y_{1,5}})}{\partial q_2'} & 0 & 0 & \frac{\partial(K_{y_{5,5}})}{\partial K_1} \\ \frac{\partial(K_{z_{1,5}})}{\partial q_1} & 0 & \frac{\partial(K_{z_{3,5}})}{\partial K} & \frac{\partial(K_{z_{4,5}})}{\partial q_3'} & 0 \\ \frac{\partial(\delta_{x_{1,5}})}{\partial q_1} & 0 & 0 & 0 & 0 \\ \frac{\partial(\delta_{y_{1,5}})}{\partial q_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial(\delta_{z_{5,5}})}{\partial K_1} \end{bmatrix} \cdot \begin{bmatrix} dq_1 \\ dq_2' \\ dK \\ dq_3' \\ dK_1 \end{bmatrix} \tag{26}$$

The components of the positioning vectors, that is of the vectors of the position and orientation of the robot gripping device in its system of co-ordinates, written as a function of the components of vector (18), are:

$$\begin{gathered} K_x = (a_{y_1} \cdot d_{x_1} - a_{x_1} \cdot d_{y_1}) \cdot dq_1 + a_{z_2} \cdot dq_2' + (a_{y_5} \cdot d_{x_5} - a_{x_5} \cdot d_{y_5}) \cdot dK_1 \\ K_y = (b_{y_1} \cdot d_{x_1} - b_{x_1} \cdot d_{y_1}) \cdot dq_1 + b_{z_2} \cdot dq_2' + (b_{y_5} \cdot d_{x_5} - b_{x_5} \cdot d_{y_5}) \cdot dK_1 \\ K_z = (c_{y_1} \cdot d_{x_1} - c_{x_1} \cdot d_{y_1}) \cdot dq_1 + c_{x_3} \cdot dK + c_{z_4} \cdot dq_3' \\ \delta_x = a_{z_1} \cdot dq_1 \quad , \quad \delta_y = b_{z_1} \cdot dq_1 \quad , \quad \delta_z = c_{z_5} \cdot dK_1 \end{gathered} \tag{27}$$

The accuracy of the gripping device position, expressed as a function of vector (18), is rendered by:

$$K = \sqrt{K_x^2 + K_y^2 + K_z^2} \tag{28}$$

While analysing Eqs. (27), (20)-(24), we determine relations of the components of the vectors of the position and orientation changes of the gripping device as a function of the robot generalised co-ordinates. Thus, considering Eq. (16), we obtain:

$$\begin{gathered} K_x = K_y = f([0,0,K,q_3',K_1]^T) \\ K_z = f([0,0,0,0,K_1]^T) \\ \delta_x = \delta_y = f([0,0,0,0,K_1]^T) \\ \delta_z = const. \end{gathered} \qquad (29)$$

Relation (29) presents an effect of the robot configuration on the components of the position and orientation vectors of the gripping device caused by disturbances in the generalised co-ordinates. This relation allows for identification of the main working space from the viewpoint of the robot positioning accuracy.

Co-ordinates of the points in the main working space, values of the manipulator generalised co-ordinates and corresponding accuracies of their position and orientation of the gripping device for the assumed accuracies of link positions in joints have been determined numerically.

3. Numerical analysis of the positioning accuracy of the *NM7M-MAR* robot

The aim of the analysis performed is to determine possibilities of application of the robot in technological processes, depending on the accuracy needed. The ranges of the rotation angle and linear displacements have been provided by the manufacturer. The selected design constants have been obtained by means of direct measurements of robot elements.

Table II

Motion module of the robot	Repeatability of the module positioning
MKR 90-180	±0°05'
MLL 80	±0.05 mm
extension of the arm	±0.05 mm
MLL 40	±0.05 mm
rotation of the gripping device	±0°05'

The errors in arrangements of robot individual generalised co-ordinates have been assumed according to the catalogue data on the motion tolerance of modules of the robot under consideration, provided by the manufacturer, Table II. In the case of a manual linear and angular module of the gripping device, values of precise positioning of links have been assumed. These values correspond to the values of the positioning accuracy of remaining displacement modules.

Graphical presentation of the numerical simulation results

The analysis of the positioning accuracy has been carried out in the whole main working space of the manipulator. The diagrams presented below show the accuracy of the position and orientation of the gripping device as a function of selected generalised co-ordinates of the manipulator. The tolerances of the manipulator gripping device position and orientation, presented here in a graphical form, have been determined in the local system of co-ordinates of the gripping device. Figure 4 presents changes in the accuracy of the manipulator gripping device position along the direction of 'x' and 'y' axes (local system of co-ordinates of the gripping device) as a function of the sum of the generalised co-ordinates

K and $q_3^{\prime}$ for the whole main working space. The diagram refers to the case when the angular position of the gripping device with respect to the extension arm equals $K_1=0$.

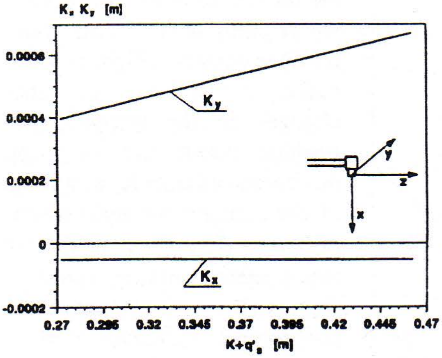


Fig. 4. Tolerance of the position of the gripping device along the direction of 'x' and 'y' axes. Orientation angle of the gripping device $K_1=0$

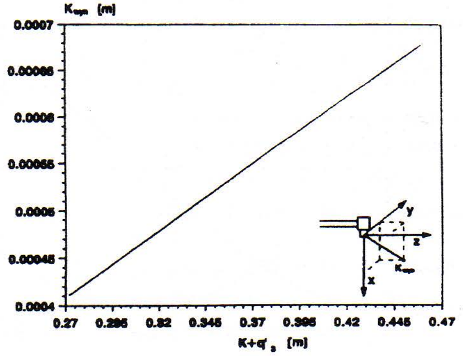
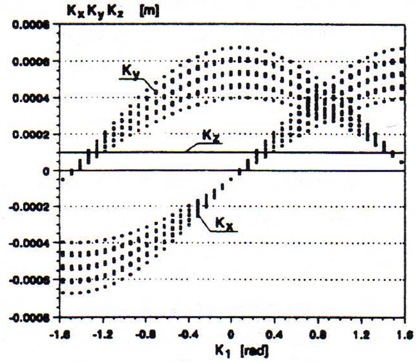


Fig. 5. Error in the position of the gripping device. Orientation angle of the gripping device $K_1=0$



Rys. 6. Changes in the components of the position vector of the gripping device as a function of the rotation angle of the fixation axis of the gripping device

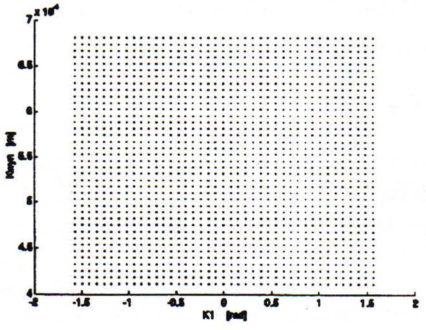


Fig. 7. Tolerance of the position of the gripping device as a function of the rotation angle of the fixation axis

Figure 5 shows an error of the position of the gripping device centre determined in its local system of co-ordinates as a function of the sum of the generalised co-ordinates K and $q_3^{\prime}$ for the gripping device rotation angle equal to $K_1=0$. An increase in the position error as a function of the generalised co-ordinates K and $q_3^{\prime}$ can be seen clearly. Changes of the position vector co-ordinates of the gripping device in its system of co-ordinates as a function of the angle K_1 are presented in Fig. 6. Subsequently, disturbances in the gripping device orientation in the main working space of the *NM7M-MAR* robot are shown in Fig. 8. Figures 6 and 8 allow for a selection of the angular position of the gripping device from the viewpoint of optimum distributions of the components of the position and orientation error of the gripping device for a given technological process. Figure 7 represents an effect of the

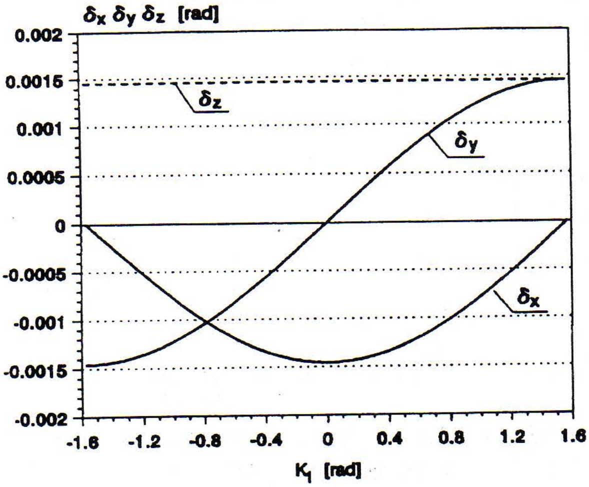


Fig. 8. Tolerances of the components of the orientation vector of the gripping device as a function of the rotation angle of the fixation axis of the gripping device.

gripping device orientation on the positioning accuracy determined in the local system of co-ordinates of the gripping device. This accuracy remains constant. Figures 9 and 10 make it possible to analyse changes in the gripping device position vector and the gripping device co-ordinate K_x as a function of the manipulator generalised co-ordinates K, K_1, q_3' in the whole robot main working space. The figures allow for evaluation of the positioning accuracy of the gripping device from the viewpoint of its technological tasks in any point of the working space.

The analysis curried out, as well as the conclusions that follow from it allow us to identify the robot main working space from the viewpoint of the gripping device positioning accuracy.

4. Conclusions

In order to estimate the accuracy tolerance of positioning and orientation of the *NM7M-MAR* robot gripping device, the method for determination of a jacobian matrix introduced by Paul into the robot dynamics analysis has been employed.

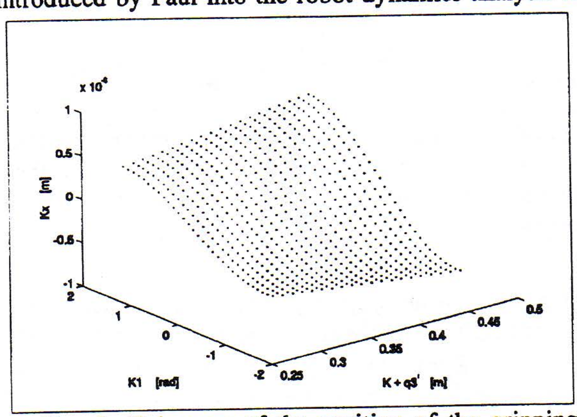


Fig. 9. Tolerance of the position of the gripping device along the direction of '*x*' axis as a function of the generalised co-ordinates K, K_1, q_3'

The resultant vectors of displacement and rotation provide a quantitative evaluation of these tolerances. The jacobian matrix method allows for identification of the robot main working space with respect to its gripping device position and orientation accuracy. Thus, it provides a possibility to use a robot in different stages of technological processes, such as assembly, materials handling, depending on a required accuracy.

An addition of an automatic linear horizontal displacement module has resulted in a change of the robot main working space, including its enlargement and a displacement 'outside' the original space. It has been also followed by deterioration of the robot positioning accuracy. The positioning accuracy for the

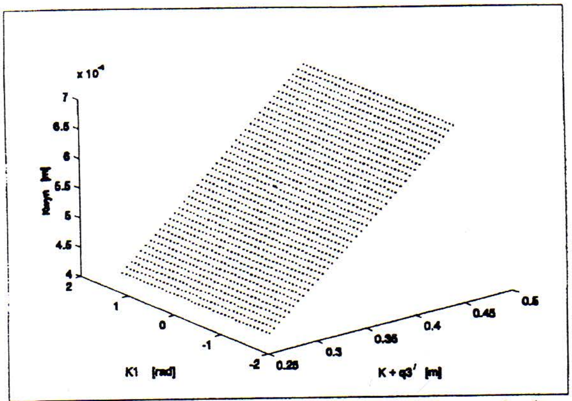


Fig. 10. Tolerance of the position of the gripping device as a function of the generalised co-ordinates K, K_1, q_3'

NM7M-MAR robot found analytically varies within ±0.7 mm and it is worse than for the *NM7M* robot for which it is ±0.2 mm, according to the data provided by its manufacturer.

In the main working space the position and orientation accuracy of the manipulator gripping device depends on its configuration, i.e. on the values of generalised co-ordinates. The presented diagrams enable us to evaluate the positioning accuracy in the robot working space region as a function of generalised co-ordinates. The basic influence on the gripping device position and orientation accuracy is exerted by the robot generalised co-ordinate K_1. It has been found that the positioning accuracy of the robot gripping device decreases along the axis 'z' of the local system of co-ordinates of the gripping device. Detailed conclusions are to be found above in the descriptions of the figures. The results obtained can be applied by manipulator users as well as by designers in order to increase the robot positioning accuracy through changes in its design.

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