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THE DAMPED VIBRATING STRINGS UNDER STOCHASTIC FORCES

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Abstract

The solution of the stochastic wave equation under damping coefficient and stochastic forces is considered in this paper using the finite sine transform method. The equation represents a damped vibrating string under a stochastic force. The ensemble average and variance of the string deflection are computed in general expressions and then relaxed to the case of the modulated white noise. A case study is considered to illustrate the reliability on the present analysis.

Introduction

Stochastic partial differential equations are a major part in the theory of the stochastic differential equations because of the wide applications they represent in engineering and science, the difficulties which should be solved and the great number of methods and techniques described in both analytical and numerical algorithms .Many researchers investigated the techniques and applications of the stochastic partial differential equations, for example see [1-14].

The random vibrations of strings was the subject of many investigators as an application on the stochastic wave equation. Vibrations determined by random forces having the structure of white noise or stationary and differentiable processes were analyzed by Cabana [15, 16] and Orsingher [17, 18]. Funaki, in a very lengthy paper [19], investigated the random motion of an elastic string by using the theory of infinite dimensional stochastic differential equation.A stochastic partial differential equation describing forced vibrations of a string subjected to random perturbations is studied by Elshamey [20]. The unstable simple modes of the nonlinear strings were analyzed in [21].

Some special cases were analyzed in [15,16] while Orsingher [18] considered the displacement of an infinitely long idealized string excited by white noise. Orsingher gave upper and lower bounds on the maximum displacement of a point on the string. Funaki [19] derived a basic equation which describes the random motion of a string using Ito equation and studied its several properties in Hilbert space. Elshamy [20] analyzed the large deviations of the random vibrations of a string using an external force which depends nonlinearly on the vibrations.

In this investigation, a linearly damped string under stochastic force is analyzed using the finite sine transform. General expressions for the ensemble average and variance of the random vibrations are obtained related to a general stochastic process as an external force. Then, random vibrations and some related moments are obtained under modulated white noise.

Problem formulation and solution methodology

The stochastic damped vibrating string equation with random force F(x,t;w) is given by

$$u_{tt}(x,t;w) = \alpha^2 u_{xx}(x,t) - \beta u_t(x,t) + F(x,t;w) \tag{1}$$

where u (x,t) is the deflection of the string. The constant $\alpha^2 = t/\rho$ represents the ratio of the tension of the string to its linear density. β is an arbitrary propotionality constant. (x,t) $\in$(0,L). (0, ∞) and w $\in$(Ω, B, P): a probability complete space in which Ω is the abstract space of the elementary events w. B is a σ- filed of subsets of Ω and P is a probability measure. The string is fixed at the ends, x = 0 and x = L, i.e.

$$u(0,t) = 0 \quad , \quad u(L,t) = 0 \tag{2}$$

The form of the motion of the string will depend on the initial deflection (deflection at t = 0) and on the initial velocity (velocity at t = 0). Let:

$$u(x,0) = f(x) \quad , \quad u_t(x,0) = g(x) \tag{3}$$

We shall proceed step by step as follows:

Step-1: Applying the method of the finite sine transform [22]. A non- homogeneous ordinary differential equation with initial conditions is finally obtained.

Step-2: Solving the previous ordinary differential equation.

Step-3: using Fourier series in order to get a solution of equation (1) .

Step-4: Computing statistical moments of the solution process .

The general solution

Step 1: Let us assume that the terms of equation (1) can be expanded as Fourier sine series as follows :

$$u_{tt} = \sum_{n=1}^{\infty} A_n(t)\sin(\frac{n\pi x}{L}) \tag{4}$$

where

$$A_n(t) = \frac{2}{L}\int_0^L u_{tt}(x,t)\sin(\frac{n\pi x}{L})dx \tag{5}$$

$$u_{xx} = \sum_{n=1}^{\infty} B_n(t)\sin(\frac{n\pi x}{L}) \tag{6}$$

where

$$B_n(t) = \frac{2}{L}\int_0^L u_{xx}(x,t)\sin(\frac{n\pi x}{L})dx \tag{7}$$

$$u_t = \sum_{n=1}^{\infty} C_n(t)\sin(\frac{n\pi x}{L}) \tag{8}$$

$$C_n(t) = \frac{2}{L}\int_0^L u_t(x,t)\sin(\frac{n\pi x}{L})dx \tag{9}$$

and

$$\mathrm{F(x,t;w)} = \sum_{n=1}^{\infty} F_n(t;w)\sin(\frac{n\pi x}{L}) \tag{10}$$

where

$$F_n(t;w) = \frac{2}{L}\int_0^L F(x,t;w)\sin(\frac{n\pi x}{L})dx \tag{11}$$

Substituting equations (4) , (6) , (8) and (10) into equation (1),the following is obtained :

$$\sum_{n=1}^{\infty}(A_n(t)-\alpha^2 B_n(t)+\beta C_n(t)-F_n(t;w))\sin(\frac{n\pi x}{L})=0 \tag{12}$$

The following relationships can be used :

$$A_n(t)=\frac{d^2U_n(t)}{dt^2} \tag{13}$$

$$B_n(t)=-\left[\frac{n\pi}{L}\right]^2 U_n(t)-\frac{2n\pi}{L}\left[u(0,t)+(-1)^{n+1}u(L,t)\right] \quad , \qquad n=1,2,3, \tag{14}$$

$$C_n(t)=\frac{dU_n(t)}{dt} \tag{15}$$

where $U_n(t)$ is the sine transform of u (x , t) ,i.e.:

$$u(x,t)=\sum_{n=1}^{\infty}U_n(t)\sin(\frac{n\pi x}{L}) \tag{16}$$

Substituting equation (2) into equation (14) ,we obtain

$$B_n(t)=-\left[\frac{n\pi}{L}\right]^2 U_n(t) \qquad n=1,2,3, \tag{17}$$

Substituting equations (14) , (15) and (17) into equation (13) ,we get :

$$\sum_{n=1}^{\infty}\left[U_n''(t)+\beta U_n'(t)+\gamma_n U_n(t)-F_n(t;w)\right].\sin(\frac{n\pi x}{L})=0 \tag{18}$$

$$\gamma_n=(\frac{\alpha n\pi}{L})^2 \tag{19}$$

Since this is an identity in x , the coefficient must be zero ; that is

$$U_n''(t)+\beta U_n'(t)+\gamma_n U_n(t)=F_n(t;w). \qquad n=1,2,... \tag{20}$$

Assuming the expandability of f (x) and g (x) as Fourier sine-series,i.e.

$f(x) = \sum_{n=1}^{\infty} U_n(0).\sin(\frac{n\pi x}{L})$,then

$$U_n(0) = \frac{2}{L}\int_0^L f(x)\sin(\frac{n\pi x}{L})dx \tag{21}$$

$$g(x) = \sum_{n=1}^{\infty} U'(0).\sin(\frac{n\pi x}{L})$$

from which

$$U_n'(0) = \frac{2}{L}\int_0^L g(x).\sin(\frac{n\pi x}{L})dx \tag{22}$$

Denoting $U_n(0)$ and $U_n'(0)$ as Φ_n and Ψ_n respectively, the following O.D.E. is obtained.

$$U_n''(t) + \beta U_n'(t) + \gamma_n U_n(t) = F_n(t;w) \tag{23}$$

I.C.:

$$U_n(0) = \Phi_n \tag{24}$$

$$U_n'(0) = \Psi_n \quad , \qquad n = 1,2,3... \tag{25}$$

<u>Step-2</u> : The roots of the auxiliary equation of (23) are $(-\beta \pm \sqrt{\beta^2 - 4\gamma_n})/2$.
Let us put $r = (\beta - \sqrt{\beta^2 - 4\gamma_n})/2$, $j = (\beta + \sqrt{\beta^2 - 4\gamma_n})/2$ (26)
and suppose that the first k values of n produce the overdamped solutions with real characteristic roots $(-r_1,-j_1),(-r_2,-j_2),......,(-r_k,-j_k)$ .For the rest of the values of n, the underdamped solutions, with the following complex characteristic roots [23] $-p_{k+1} \pm iq_{k+1}$, $-p_{k+2} \pm iq_{k+2}$, ...are the independent solutions of equation (23). Finally, the general solution of equation (23) takes the following form [24]:

$$U_n(t) = M_n e^{-r_n t} + N_n e^{-J_n t} + \frac{e^{-r_n t}}{-r_n + j_n}\int_0^t e^{r_n \tau} F_n(\tau;w)d\tau + \frac{e^{-j_n t}}{-j_n + r_n}\int_0^t e^{j_n \tau} F_n(\tau;w)d\tau ....n = 1,2,...,k$$

$$= e^{P_n t}(\bar{M}_n \cos q_n t + \bar{N}_n \sin q_n t) + \frac{e^{P_n t}\sin q_n t}{q_n}\int_0^t e^{P_n \tau} F_n(\tau;w)\cos q_n \tau d\tau - \tag{27}$$

$$\frac{e^{-P_n t}\cos q_n t}{q_n}\int_0^t e^{P_n \tau} F_n(\tau;w).\sin q_n \tau d\tau \quad , \qquad n = k+1, k+2,.....$$

where :

$$M_n = \frac{j_n\Phi_n - \Psi_n}{j_n - r_n},$$
$$N_n = \frac{r_n\Phi_n - \Psi_n}{r_n - j_n} \quad , \quad n = 1,2,\ldots,k \tag{28}$$

$$\bar{M} = \Phi_n$$
$$\bar{N} = \frac{\Psi_n + p_n\Phi_n}{q_n} \quad , \quad n = k+1, k+2,\ldots \tag{29}$$

Step-3 :Using the previous results of the last two steps the following general expression is obtained for the solution process under a general stochastic force:

$$u(x,t;w) = \sum_{n=1}^{\infty} U_n(t).\sin(\frac{n\pi x}{L}) = \sum_{n=1}^{k}\sin(\frac{n\pi x}{L}).[M_n e^{-r_n t} + N_n e^{-j_n t} + \frac{e^{-r_n t}}{-r_n + j_n}\int_0^t e^{r_n\tau}F_n(\tau;w)d\tau$$
$$+ \frac{e^{-j_n t}}{-j_n + r_n}\int_0^t e^{-j_n\tau}F_n(\tau;w)d\tau] + \sum_{n=k+1}^{\infty}\sin(\frac{n\pi x}{l})[e^{-p_n t}(\bar{M}_n\cos q_n t + \bar{N}_n \sin q_n t) +$$
$$\frac{e^{-p_n t}\sin q_n t}{q_n}\int_0^t e^{p_n\tau}F_n(\tau;w)\cos q_n\tau d\tau - \frac{e^{-p_n t}\cos q_n t}{q_n}\int_0^t e^{p_n\tau}F_n(\tau;w).\sin q_n\tau d\tau . \tag{30}$$

Step-4 : The ensemble average and variance of u (x , t ; w) are computed as the following

$$E[u(x,t;w)] = \sum_{n=1}^{k}\sin(\frac{nx\pi}{L}).[M_n e^{-r_n t} + N_N e^{j_n t} + \frac{e^{-r_n t}}{-r_n + j_n}\int_0^t e^{r_n\tau}E[F_n(\tau;w)]d\tau + \frac{e^{-j_n t}}{-j_n + r_n}\int_0^t e^{j_n\tau}E[F_n(\tau;w)d\tau]$$
$$+ \sum_{n=k+1}^{\infty}\sin(\frac{nx\pi}{L}).[e^{-p_n}(\bar{M}_n\cos q_n t + \bar{N}_n\sin q_n t) + \frac{e^{-p_n t}\sin q_n t}{q_n}\int_0^t e^{p_n\tau}E[F_n(\tau;w)]\cos q_n\tau d\tau -$$
$$\frac{e^{-p_n t}\cos q_n t}{q_n}\int_0^t e^{p_n\tau}E[F_n(\tau;w)]\sin q_n\tau d\tau], \tag{31}$$

where

$$E[F_n(\tau;w)] = \frac{2}{L}\int_0^t E[F(x,\tau;w).\sin(\frac{nx\pi}{L})dx, \tag{32}$$

and

$$Var[u(x,t;w)] = E[u(x,t;w)]^2 - [E(u(x,t;w))]^2, \tag{33}$$

where

$$E[u(x,t;w)]^2 = \sum_{n=1}^{k}\sum_{m=1}^{k}\sin(\frac{nx\pi}{L}).\sin(\frac{mx\pi}{L})[M_nM_me^{(-r_n-r_m)t} + N_nM_me^{(-j_n-r_m)t} + \frac{M_me^{(-r_n-r_m)t}}{-r_n+j_n}\int_0^t e^{r_n\tau}E[F_n(\tau;$$

$$w)]d\tau + \frac{M_me^{(-j_n-r_m)t}}{-j_n+r_n}\int_0^t e^{j_n\tau}E[F_n(\tau;w)]d\tau + M_nN_me^{(-r_n-j_m)t} + N_nN_me^{(-j_n-j_m)t} + \frac{N_me^{-r_nt}}{-r_n+j_n}\int_0^t e^{r_n\tau}E[F_n(\tau;w)].$$

$$d\tau + \frac{N_me^{-j_n}}{-j_n-r_n}\int_0^t e^{j_n\tau}E[F_n(\tau;w)]d\tau + \frac{M_ne^{(-r_n-r_m)t}}{-r_m+j_m}\int_0^t e^{r_m\tau}E[F_m(\tau;w)]d\tau + \frac{N_ne^{(-j_n-r_m)t}}{-r_m+j_m}\int_0^t e^{r_m\tau}E[F_m(\tau;w)]d\tau +$$

$$\frac{e^{(-r_n-r_m)t}}{(-r_n+j_n)(r_m+j_m)}\int_0^t\int_0^t e^{r_n\tau_1}e^{r_m\tau_2}E[F_n(\tau_1;w).F_n(\tau_2;w)]d\tau_1d\tau_2 + \frac{e^{(-j_n-r_m)t}}{(-j_n+r_n)(-r_m+j_m)}\int_0^t\int_0^t e^{j_n\tau_1}e^{r_m\tau_2},$$

$$E[F_n(\tau_1;w).F_m(\tau_2;w)]d\tau_1d\tau_2 + \frac{M_ne^{(-r_n-j_m)t}}{-j_m+r_m}\int_0^t e^{j_m\tau}E[F_n(\tau;w)]d\tau + \frac{N_ne^{(-j_n-j_m)t}}{-j_m+r_m}\int_0^t e^{j_m\tau}E[F_n(\tau;w)]d\tau +$$

$$\frac{e^{(-r_n-j_m)t}}{(-r_n+j_n)(-j_m+r_m)}\int_0^t\int_0^t e^{r_n\tau_1}e^{j_m\tau_2}E[F_n(\tau_1;w).F_m(\tau_2;w)]d\tau_1d\tau_2 + \frac{e^{(-j_n-j_m)t}}{(-j_n+r_m)^2}\int_0^t\int_0^t e^{j_n\tau_1}e^{j_m\tau_2}E[F_n(\tau_1;w).$$

$$F_m(\tau_2;w)]d\tau_1d\tau_2 + \sum_{n=k+1}^{\infty}\sum_{m=k+}^{\infty}\sin(\frac{nx\pi}{L}).\sin(\frac{mx\pi}{L})[e^{(-p_n-p_m)t}(\bar{M}_n\bar{M}_m\cos q_nt.\cos q_mt + \bar{N}_n\bar{M}_m\sin q_nt.$$

$$\cos q_mt + \bar{M}_n\bar{N}_m\cos q_nt.\sin q_mt + \bar{N}_n\bar{N}_m\sin q_nt.\sin q_mt) + \frac{e^{(-p_n-p_m)t}\sin q_nt}{q_n}(\bar{M}_m\cos q_m + \bar{N}_m\sin q_mt).$$

$$\int_0^t e^{p_n\tau}E[F_n(\tau;w)]\cos q_n\tau d\tau - \frac{e^{(-p_n-p_m)t}\cos q_nt}{q_n}(\bar{M}_m\cos q_mt + \bar{N}_m\sin q_mt)\int_0^t e^{p_n\tau}E[F_n(\tau;w)\sin q_n\tau d\tau +$$

$$\frac{e^{(-p_n-p_m)t}\sin q_mt}{q_m}(\bar{M}_n\cos q_nt + \bar{N}_n\sin q_nt)\int_0^t e^{p_m\tau}E[F_n(\tau;w)\cos q_m\tau d\tau + \frac{e^{(-p_n-p_m)t}\sin q_nt.\sin q_mt}{q_n\cdot q_m}.$$

$$\int_0^t\int_0^t e^{p_n\tau_1}e^{p_m\tau_2}E[F_n(\tau_1;w).F_m(\tau_2;w)]\cos q_n\tau_1\cos q_m\tau_2d\tau_1d\tau_2 - \frac{e^{(-p_n-p_m)t}\cos q_nt.\sin q_mt}{q_n\cdot q_m}.$$

$$\int_0^t\int_0^t e^{p_n\tau_1}e^{p_m\tau_2}E[F_n(\tau_1;w).F_m(\tau_2;w)]\sin q_n\tau_1.\cos q_m\tau_2d\tau_1d\tau_2 - \frac{e^{(-p_n-p_m)t}\cos q_mt}{q_m}(\bar{M}_n\cos q_nt +$$

$$\bar{N}_n\sin q_nt)\int_0^t e^{p_m\tau}E[F_m(\tau;w).\sin q_m\tau d\tau - \frac{e^{(-p_n-p_m)t}\sin q_nt.\cos q_mt}{q_n\cdot q_m}\int_0^t\int_0^t e^{p_n\tau_1}e^{p_m\tau_2}E[F_n(\tau_1;w).$$

$$F_m(\tau_2;w)]\cos q_n\tau_1.\sin q_m\tau_2d\tau_1d\tau_2 + \frac{e^{(-p_n-p_m)t}\cos q_nt.\cos q_mt}{q_n\cdot q_m}\int_0^t\int_0^t e^{p_n\tau_1}e^{p_m\tau_2}E[F_n(\tau_1;w).$$

$$F_m(\tau_2;w)].\sin q_n\tau_1.\sin q_m\tau_2d\tau_1d\tau_2, \tag{34}$$

in..which

$$E[F_n(\tau_1;w).F_m(\tau_2;w)] = \frac{4}{L^2}\int_0^L\int_0^L\sin(\frac{nx_1\pi}{L})\sin(\frac{mx_2)}{L}E[F(x_1,\tau_1;w).F(x_2,\tau_2;w)]dx_1dx_2, \tag{35}$$

$$p_n = p_m = \beta/2 \tag{36}$$

and

$$q_n = \sqrt{\frac{4n^2\pi^2\alpha^2}{L^2} - \beta^2} \tag{37}$$

Modulated white noise force

It is quite known that white noise n(t;w) defines an abstract stationary process whose power spectral density is constant over the whole spectrum . It is a zero mean Gaussian process and Dirac delta function correlation i .e .,

$$E[n(t;w)] = 0)) , \tag{38}$$

and

$$E[n(t_1;w).n(t_2;w)] = \delta(t_1 - t_2) . \tag{39}$$

In this section , the random force F(x ,t ; w) is assumed to take the following form :

$$F(x,t;w) = G(x,t).n(t;w) \tag{40}$$

Substituting (40) and (38) into (32),we get

$$E[F_n(\tau;w)] = \frac{2}{L}\int_0^t E[G(x,t)n(t;w)]\sin(\frac{nx\pi}{L})dx = 0 \tag{41}$$

Substituting (40) and (39) into (35) ,we get

$$E[F_n(\tau_1;w).F_m(\tau_2;w)] = \frac{4}{L^2}\int_0^L\int_0^L Z_n(x_1)Z_m(x_2).G(x_1,\tau_1).G(x_2,\tau_2).\delta(\tau_1-\tau_2)dx_1dx_2 \tag{42}$$

where

$$Z_n(x) = \sin(\frac{nx\pi}{L}) \tag{43}$$

Substituting (41) into (32) , we get

$$E[u(x,t;w)] = \sum_{n=1}^{k}\sin(\frac{nx\pi}{L})[M_n e^{-r_n t} + N_n e^{-J_n}] + \sum_{n=k+1}^{\infty}\sin(\frac{nx\pi}{L})[e^{-p_n t}(\bar{M}_n\cos q_n t + \bar{N}_n\sin q_n t)] .. \tag{44}$$

Substituting (41) and (42) into (34) ,we get

$$E[u(x,t;w)]^2=\sum_{n=1}^{k}\sum_{m=1}^{k}Z_n(x)Z_m(x)[M_nM_me^{(-r_n-r_m)t}+N_nM_me^{(-j_n-r_m)t}+M_nN_me^{(-r_n-j_m)t}+$$
$$N_nN_me^{(-j_n-j_m)t}+\frac{e^{(-r_n=r_m)t}}{(-r_n+j_n)(r_m+j_m)}\int_0^t e^{(r_n+r_m)\tau}\Delta_{n,m}.d\tau+\frac{e^{(-j_n-r_m)t}}{(-j_n+r_n)(-r_m+j_m)}\int_0^t e^{(j_n+r_m)\tau}\Delta_{n,m}d\tau$$
$$\frac{e^{(-r_n-j_m)t}}{(-r_n+j_n)(-j_m+r_m)}\int_0^t e^{(r_n+j_m)\tau}\Delta_{n,m}d\tau+\frac{e^{(-j_n-j_m)t}}{(-j_n+r_n)(-j_m+r_m)}\int_0^t e^{(j_n+j_m)\tau}\Delta_{n,m}d\tau]+\sum_{n=k+1}^{\infty}\sum_{m=k+1}^{\infty}Z_n(x)$$
$$.Z_m(x)[e^{(-p_n-p_m)t}(\bar{M}_n\bar{M}_m\cos q_nt.\cos q_mt+\bar{N}_n\bar{M}_m\sin q_n.\cos q_mt+\bar{M}_n\bar{N}_m\cos q_nt.\sin q_mt+$$
$$\bar{N}_n\bar{N}_m\sin q_nt.\sin q_mt)+\int_0^t e^{(p_n+p_m)\tau}\cos q_n\tau.\cos q_m\tau.\Delta_{n,m}d\tau-\frac{2e^{(-p_n-p_m)t}\sin q_nt.\cos q_mt}{q_n.q_m}\int_0^t e^{(p_n+p_m)\tau}.$$
$$\cos q_n\tau.\sin q_m\tau.\Delta_{n,m}d\tau+\frac{e^{(-p_n-p_m)t}\cos q_nt.\cos q_mt}{q_n.q_m}\int_0^t e^{(p_n+p_m)\tau}\sin q_n\tau.\sin q_m\tau.\Delta_{n,m}d\tau] \qquad (45)$$

where

$$\Delta_{n,m}=\frac{4}{L^2}\int_0^L\int_0^L Z_n(x_1).Z_m(x_2).G(x_1,\tau).G(x_2,\tau)dx_1dx_2 \qquad (46)$$

Case study

We take $u(x,0)=x, u_t(x,0)=0, L=1, \alpha=1, \beta=1$ and $G(x,t)=G(x)=x$. In this case, the ensemble average and variance take the following form respectively

$$E[u(x,t;w)]=\sum_{n=k+1}^{\infty}\sin(nx\pi).[e^{-\frac{t}{2}}(\bar{M}_n\cos q_nt+\bar{N}_n\sin q_nt)] \qquad (47)$$

where

$$\bar{M}_n=\frac{2}{n\pi}[\frac{1}{n\pi}.\sin n\pi-\cos n\pi] \qquad (48)$$

$$\bar{N}_n=\frac{2.[\sin n\pi-n\pi.\cos n\pi]}{(n\pi)^2\sqrt{(2n\pi)^2-1}};$$

$$q_n=\frac{\sqrt{(2n\pi)^2-1}}{2} \qquad (49)$$

$$
\begin{aligned}
Var[u(x,t;w)] = {} & \sum_{n=k+1}^{\infty} \sum_{m=k+1}^{\infty} \sin nx\pi . \sin mx\pi \Big\{ (e^{-t} - e^{t})[\bar{M}_n \bar{M}_m \cos q_n t . \cos q_m t + \bar{N}_n \bar{M}_m . \\
& \sin q_n t . \cos q_m t + \bar{M}_n \bar{N}_m \cos q_n t . \sin q_m t + \bar{N}_n \bar{N}_m \sin q_n t . \sin q_m t] + \frac{A_{n,m}}{2}[\frac{e^{t}}{1+(q_n - q_m)^2} . \\
& (\cos(q_n - q_m)t + (q_n - q_m). \sin(q_n - q_m)t) - \frac{1}{1+(q_n - q_m)^2} + \frac{e^{t}}{1+(q_n + q_m)^2} . (\cos(q_n + q_m)t + \\
& (q_n + q_m)\sin(q_n + q_m)t) - \frac{1}{1+(q_n + q_m)^2}] - 2A_{n,m}.B_{n,m}[\frac{e^{t}}{2}(\frac{\sin(q_n + q_m)t - (q_n + q_m)\cos(q_n + q_m)t}{1+(q_n + q_m)^2} \\
& - \frac{\sin(q_n - q_m)t - (q_n - q_m)\cos(q_n - q_m)t}{1+(q_n - q_m)^2}) - \frac{1}{2}(\frac{(q_n + q_m)}{1+(q_n + q_m)^2} + \frac{(q_n - q_m)}{1+(q_n - q_m)^2})] + \frac{A_{n,m}.C_{n,m}}{2} . \\
& [\frac{e^{t}}{1+(q_n - q_m)^2}(\cos(q_n - q_m).t + (q_n - q_m)\sin(q_n - q_m).t) - \frac{1}{1+(q_n - q_m)^2} - (\frac{e^{t}}{1+(q_n + q_m)^2} . \\
& \cos(q_n + q_m).t + (q_n + q_m)\sin(q_n + q_m).t) + \frac{1}{1+(q_n + q_m)^2}] \Big\}
\end{aligned}
\tag{50}
$$

where

$$A_{n,m} = \frac{4}{nm\pi^2}[\frac{\sin(n\pi)}{n\pi} - \cos(n\pi)].[\frac{\sin(m\pi)}{m\pi} - \cos(m\pi)] \tag{51}$$

$$B_{n,m} = \frac{e^{-t} \cos q_n t . \sin q_m t}{q_n . q_m} \tag{52}$$

and

$$C_{n,m} = \frac{e^{-t} \cos q_n t . \cos q_m t}{q_n . q_m} \tag{53}$$

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