MECHANICS AND MECHANICAL ENGINEERING
International Journal
Vol. 3 No. 1 1999

THE EFFECT OF SPATIAL MECHANISMS INTERACTING WITH THE SHAFT ON THE STATE OF THE BALANCING SHAFT

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Abstract

The overlock machine is an example of a machine of rotating elements interacting with spatial mechanisms. In the machine there are five interacting units, four of which are spatial mechanisms affecting the dynamic state of the whole machine. These mechanisms obtain their common drive from the main shaft so that they can interact synchronously. The links interacting directly with the drive shaft have an effect on the state of its unbalance. The balancing of the main vector of the inertial forces of the shaft can be obtained at a stage of design of the machine, on the basis of conditions of balancing.

Introduction

An overlock machine shown in a kinematic diagram, fig. 1, consists of units, five of which have an effect on the dynamic state of the machine. They are: the needle bar drive mechanism, the upper catcher drive mechanism, the lower catcher drive mechanism, the transport mechanism and the drive shaft unit. It results from an analysis of motion of these mechanisms that the unbalance of the needle bar mechanism and the drive shaft mechanism is the main source of vibrations of the machine – placed on elastic supports – in the vertical direction. On the other hand, the transport mechanism is responsible for vibrations in the horizontal direction. The needle bar crankshaft and the catcher mechanisms generate torsional vibrations along the longitudinal axis of the machine. The machine vibrations can be limited during two stages, namely during design, and after a prototype has been made – as a result of investigations of the operating machine. Generally speaking, the balancing of the machine with spatial mechanisms aims at balancing both the main vector of the inertial forces and the main vector of the moment of inertial forces. The balancing of only the main vector of inertial forces can, and should, be effected at a stage of design of the machine, as a result of selection of optimal shapes and sizes of movable elements of particular units, especially of those having large masses. Thus it is necessary to establish dynamic criteria which would make it possible to assess the state of balancing of the main vector of inertial forces of particular units – mechanisms of the machine. A constructor can use an appropriate algorithm to help him solve such a task by means of a computer. Such a method of design would

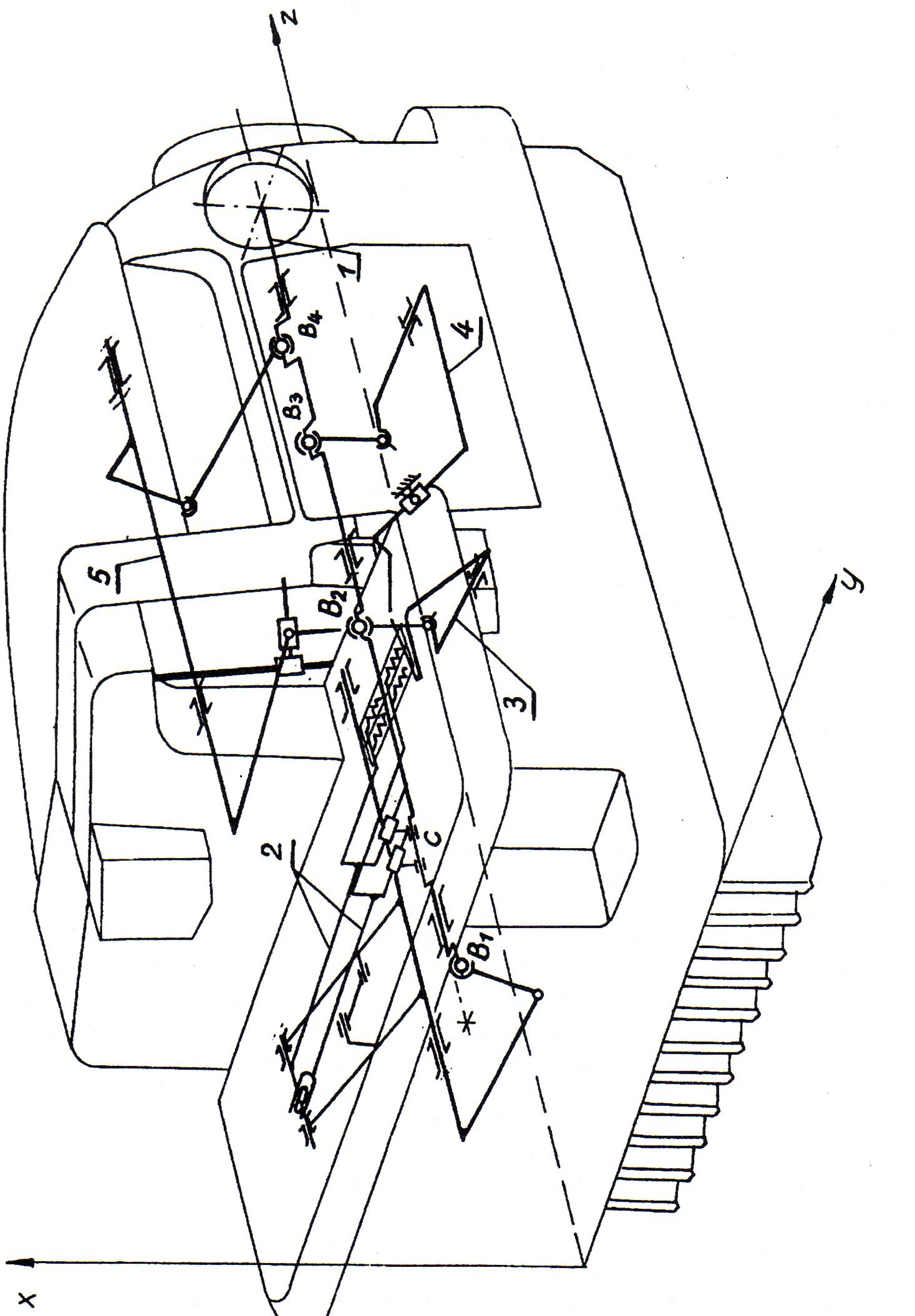

Fig. 1. A schematic diagram of the machine

allow the building of a prototype of the machine and its start-up, permitting, in turn, the balancing of the main vector of the moment of inertial forces to be investigated.

The balancing of the main vector of inertial forces of the machine will be possible if the main vectors of inertial forces of all the mechanisms affecting the dynamic state of the machine have been balanced. Conditions of balancing established for each mechanism should be taken as a criterion of the balancing of the main vector of inertial forces. These conditions also concern the main shaft of the machine along with the belt pulley.

The condition for balancing spatial mechanisms is that the total mass centre of all the mechanism links be constant during one motion cycle. This condition is correct assuming that the links are rigid bodies, and it serves to formulate a criterion of balancing of the main vector of the inertial forces.

The order of the mechanisms being considered has been established on the basis of the estimated assessment of their effect on the dynamic state of the machine. All the mechanisms have the same drive and interact in a cyclically synchronised manner. The fixed assembly positions of the mechanisms are shown in fig. 2.

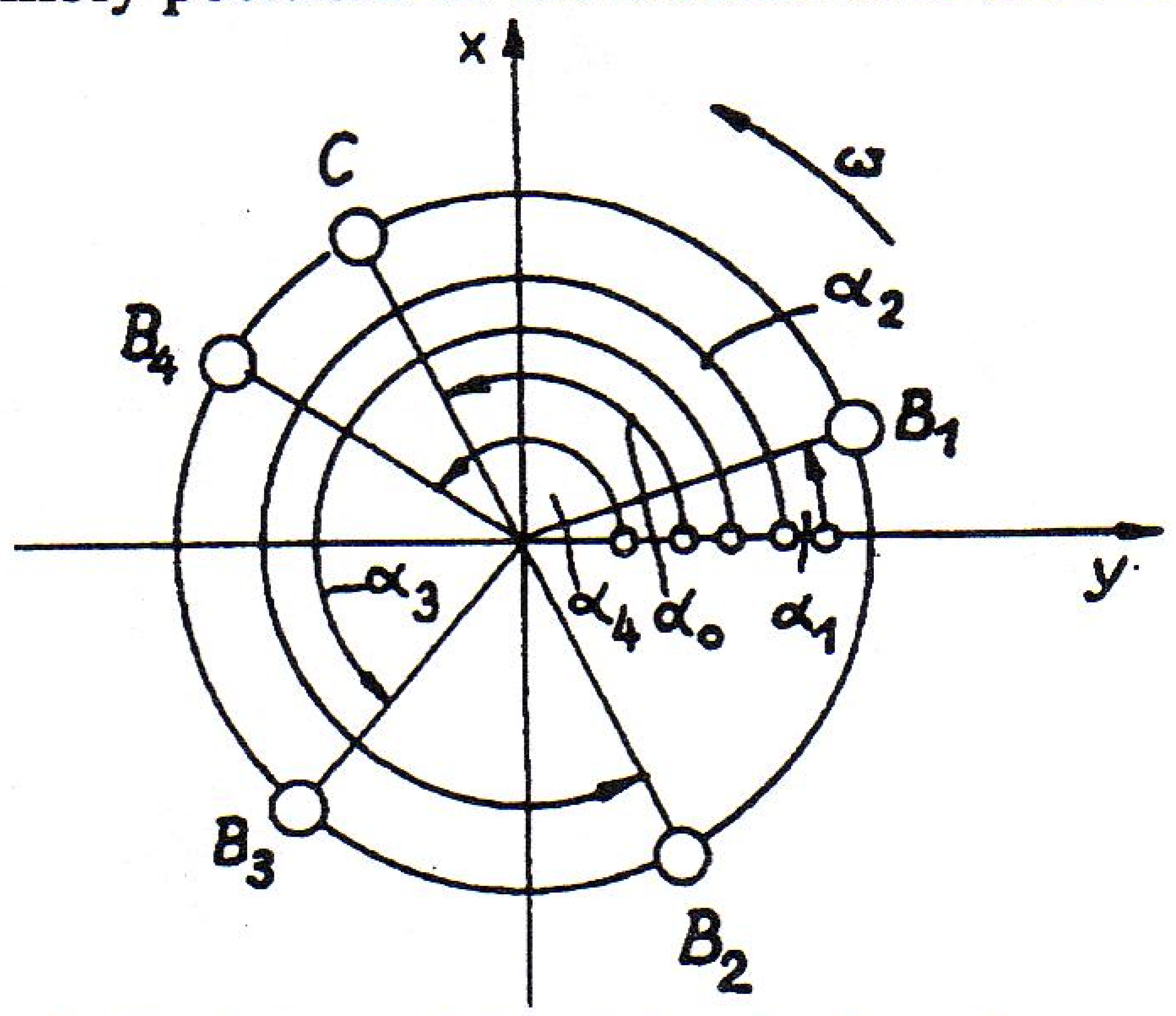

Fig. 2. Positions of the drive shaft necks

$\alpha_0 = 115^o$ – the angle of position of the circular cam of the lever of mechanism no. 2; $\alpha_1 = 20^o$ – the angle of position of the pin of mechanism no. 2; $\alpha_2 = 328^o$ – the angle of position of the circular cam pin of mechanism no. 3; $\alpha_3 = 238^o$ – the angle of position of the circular cam of mechanism no. 4; $\alpha_4 = 148^o$ – the angle of position of the crank pin of mechanism no. 5.

Points B_i and C correspond to the position of the kinematic pairs of the drive links. It is assumed that an instantaneous position of the main shaft is defined by the angle θ_1 ; then the angular positions of the drive links of particular mechanisms are determined by the following relationships:

$$\Theta_{1,1} = \Theta_1 + \alpha_0 \, , \Theta_{1,2} = \Theta_1 + \alpha_1, \Theta_{1,3} = \Theta_1 + \alpha_2, \qquad (1)$$

$$\Theta_{1,4} = \Theta_1 + \alpha_3 \, , \Theta_{1,5} = \Theta_1 + \alpha_4$$

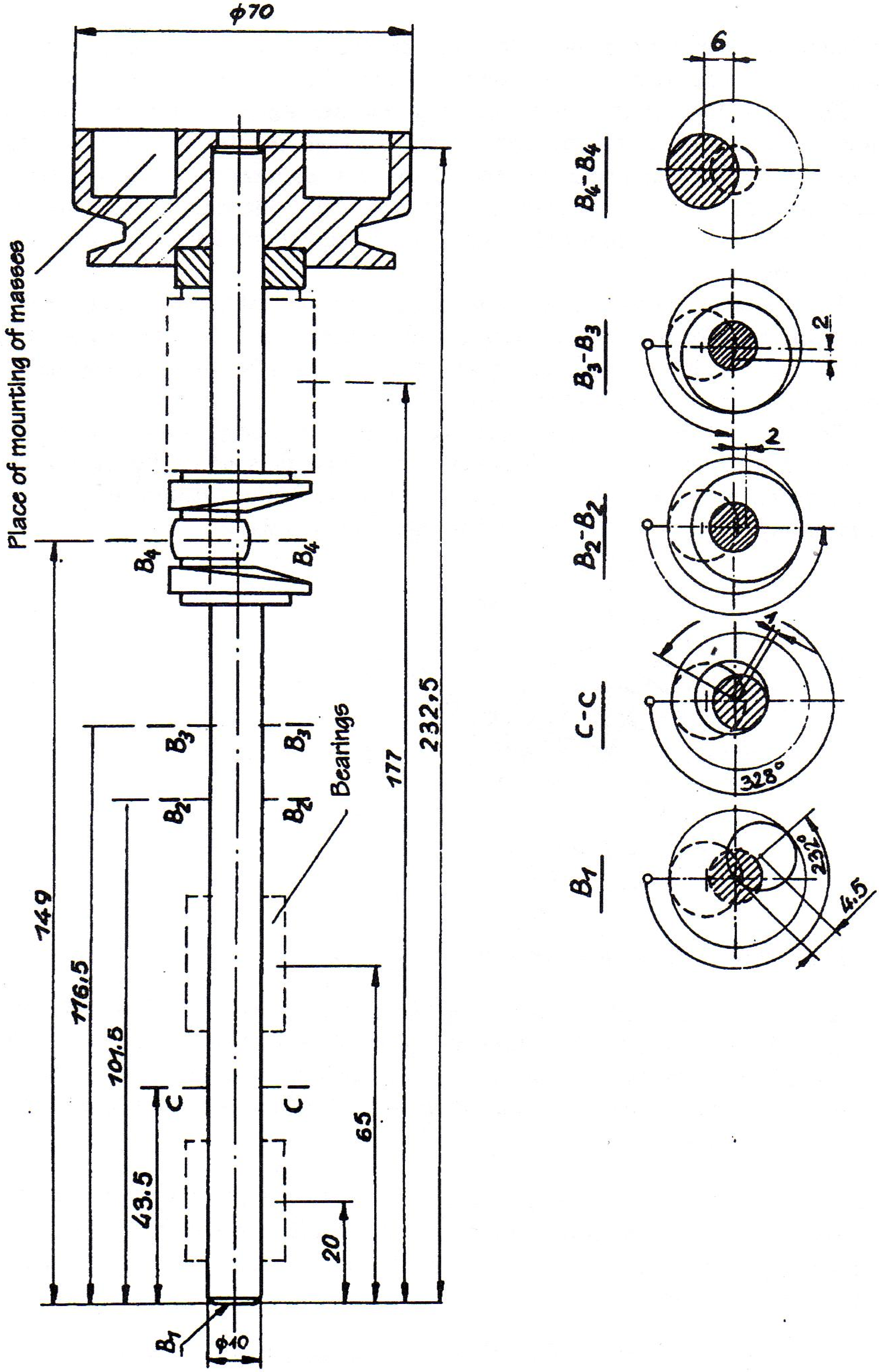

Fig. 3. Drive shaft unit

2. Drive shaft unit

The drive shaft unit shown in fig. 3 is made up of a multi-bearing rotor with one inside crank (section B_4) and four circular cams whose angular position relative to the shaft can be changed. The circular cams are mounted in places marked by cross-sections B_2, B_3, C, B_1.

The construction of the shaft makes it possible to distribute the drive to particular units of the machine, which is effected by the circular cams mentioned and the inside crank pin. The machine rotor composed of a belt pulley, a shaft of a diameter ϕ = 10 mm, and circular cams is included in flexible rotors as far as its dynamics are concerned. This rotor will operate over the second critical velocity at n_r = 4020 rpm and has an essential effect on the dynamic state of the machine.

3. Conditions for balancing of spatial mechanisms interacting with the shaft

The balancing of the main vector of inertial forces can, and should be effected at a stage of design of the machine. Therefore, it is necessary to know the criteria that allow an assessment of the dynamic state of the machine mechanisms to be made. Conditions of balancing established for each mechanism are the criteria of balancing of the main vector of inertial forces. The knowledge of the vector of position of the centres of masses of the mechanisms in the base system, [2], [3] is the basis for the formation of these conditions.

The vector $\mathbf{r}_{Sk,0}$ of position of the centre S of the mass of the chain in the base system is determined in its general form by the matrix equation [4]:

$$\begin{bmatrix} \mathbf{r}_{Sk,0} \\ 1 \end{bmatrix} = \frac{1}{M_k} \sum_{i=1}^{j} m_{i,k} \, \mathbf{T}_{0i,k} \begin{bmatrix} \mathbf{r}_{s,i} \\ 1 \end{bmatrix} \tag{2}$$

where: $\mathbf{r}_{s,i}$ is the vector of position of the centre of the mass link i in the system of co-ordinates of this link, $\mathbf{T}_{0i,k}$ is the matrix of transformation of the system i to the system 0 of the dimension 4 x 4 in the form:

$$\mathbf{T}_{0i,k} = \mathbf{A}_{1,k} \, \mathbf{A}_{2,k} \, \mathbf{A}_{j-1,k} \cdots \mathbf{A}_{j,k} \tag{3}$$

while $\mathbf{A}_{i,k}$ are matrices of elementary transformations, M_k is the mass of the whole chain and is equal to:

$$M_k = \sum_{i=1}^{j} m_{i,k} \tag{4}$$

if k = 1,2,3,4,5 is the number of the mechanism, while j is the number of the last movable link of a given mechanism.

From the matrix $\mathbf{T}_{0i,k}$ the matrix of orientation $\mathbf{B}_{0i,k}$ of the dimension 3 x 3 can be isolated.

Basing on relationships (2), (3) and (4), the author has determined conditions of balancing of spatial mechanisms of the machine, fig. 1, the satisfaction of which should ensure the balancing of the main vector of inertial forces of these mechanisms; these problems are discussed in publications [1], [2] and [3].

From an analysis of these conditions and calculation examples it results that the balancing of the drive shaft unit masses, the crankshaft masses in the system l, and part of the mass of the link interacting with the crankshaft defined in the system l would be possible if the conditions $\mathbf{W}_{I5}$ to $\mathbf{W}_{I5}$ were satisfied.

4. Conditions of balancing of the shaft

Out of the conditions of balancing of particular mechanisms [1], [2], [3], the conditions $\mathbf{W}_{Ik}$, formula (5), decide the balancing of the drive shaft unit.

$$\mathbf{W}_{I5} = \begin{bmatrix} -x_{1,5} \\ 0 \\ 0 \end{bmatrix} (m_{1,5} + m_{2,5} + m_{3,5}(1 - \tau_5)) + \mathbf{r}_{11,5} m_{1,5} + \mathbf{r}_{21,5} m_{2,5} = 0$$
$$\mathbf{W}_{I4} = \begin{bmatrix} -x_{1,4} \\ 0 \\ 0 \end{bmatrix} (m_{1,4} + m_{2,4} + m_{3,4}(1 - \tau_4)) + m_{1,4}\mathbf{r}_{11,4} + m_{2,4}\mathbf{r}_{21,4}) = 0 \qquad (5)$$
$$\mathbf{W}_{I3} = \begin{bmatrix} -x_{1,3} \\ 0 \\ 0 \end{bmatrix} (m_{1,3} + m_{2,3} + m_{3,3}(1 - \tau_3)\; m_{1,3}\mathbf{r}_{11,3} + m_{2,3}\mathbf{r}_{21,3} = 0$$
$$\mathbf{W}_{I2} = \begin{bmatrix} -x_{1,2} \\ 0 \\ 0 \end{bmatrix} (m_{1,2} + m_{2,2} + m_{3,2}(1 - \tau_2) + m_{4,2} + m_{5,2}) + m_{1,2}\mathbf{r}_{11,2} + m_{2,2}\mathbf{r}_{21,2} = 0$$

where: $x_{1,k}$ are the co-ordinates, $m_{i,k}$ are masses of the links of particular mechanisms, τ_k is the coefficient, $r_{11,k}$ is the radius of the centre of mass 1 in the system one, while $r_{21,k}$ is the radius of the centre of mass of crankshaft 2 in system l, with $\mathbf{r}_{2,1,k} = \mathbf{B}_{2,k}$ $\mathbf{r}_{22,k}$ for k = 2,3,4,5 are radii, of which only $\mathbf{r}_{21,5}$ changes during a motion cycle, whereas the remaining quantities are not time-dependent.

The condition $\mathbf{W}_{I5}$ will only be satisfied partially, since the vector $\mathbf{r}_{21,5}$ of position of the centre of the mass $m_{2,5}$ of link 2 in the system 1 of the mechanism is time-dependent, which results from the matrix $\mathbf{B}_{2,5}$. As the angles $\phi_{2,5}$ increase, there is an increase in $\Delta r_{21,5}$ and this increment can reach about 32%, depending on the tolerance of z_{T5}.

In the case of the needle bar mechanism, when the nominal parallelism and perpendicularity of the axes are maintained, the value of the angle $\phi_{2,5}$ for a given tolerance of the assembly z_{T5} is constant and a change in this angle depends only an a change in tolerance. The vector of position $\mathbf{r}_{21,k}$ of the centre of the mass $m_{2,5}$ of crankshaft 2 written in the system of the link 1 in one cycle of motion is determined by formula (6).

$$\mathbf{r}_{21} = \mathbf{B}_{2,5}\mathbf{r}_{22} \qquad (6)$$

where $$\mathbf{B}_{2,5} = \mathbf{B}_{2,5}(O_{2,5}, \phi_{2,5})$$

The magnitude of changes in the modulus of this radius is shown in fig. 4.

If any deviations from the parallelism and perpendicularity of the axes occurred, then the angle $\phi_{2,5}$ would vary in one cycle of motion at a given tolerance.

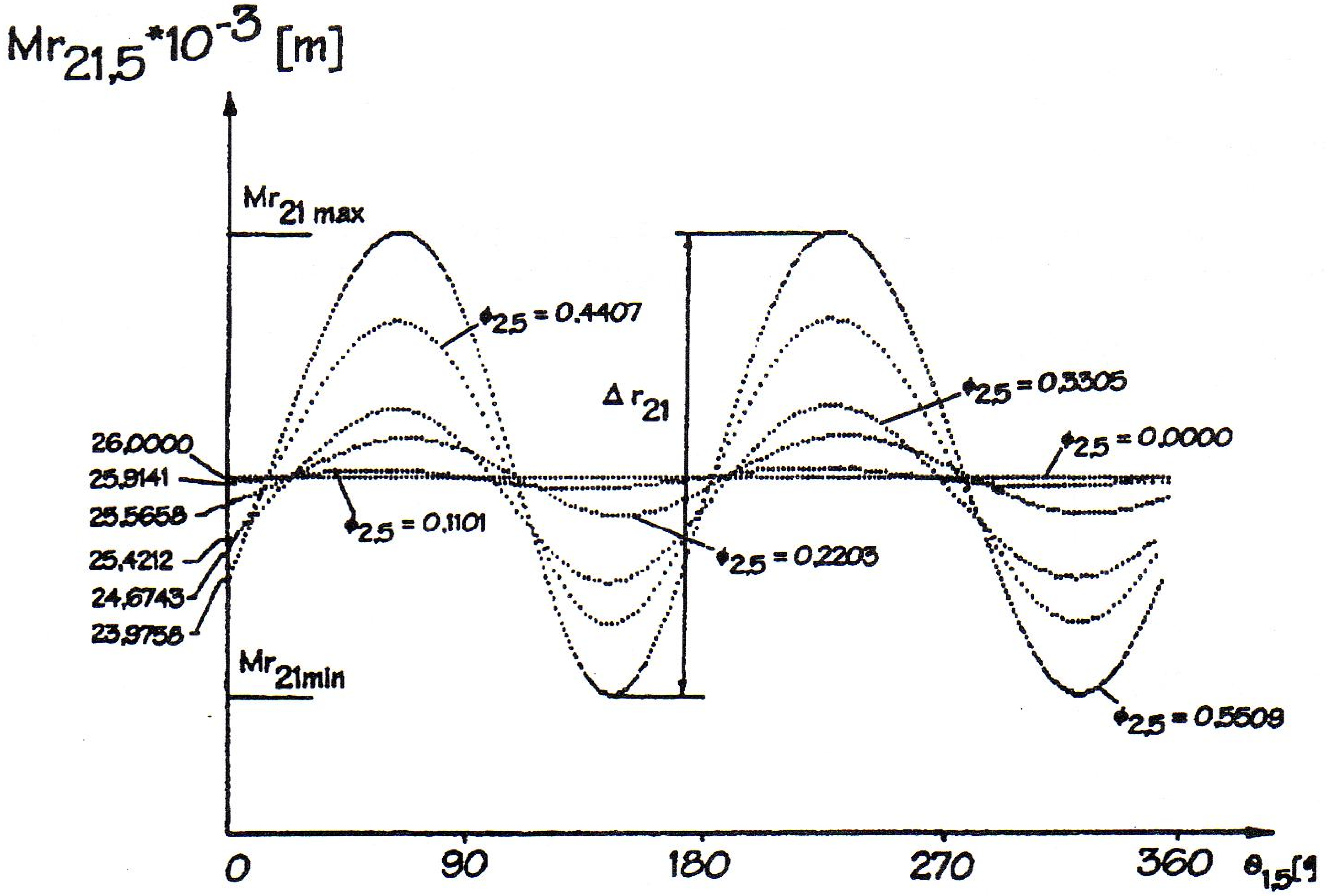

Fig. 4. A diagram of changes in the modulus of the vector $\mathbf{r}_{21,5}$ of the needle bar drive mechanism

The conditions $\mathbf{W}_{14}$ and $\mathbf{W}_{13}$ will only be satisfied because in the case of these mechanisms $\mathbf{r}_{21,4}$ and $\mathbf{r}_{21,3}$ are nearly constant. Changes in the angles $\phi_{2,4}$ and $\phi_{2,3}$ depend on the tolerance of the parallelism and perpendicularity of the axes with reference to the nominal values.

In the case of the upper catcher, a diagram of changes in the modulus of the radius $\mathbf{r}_{21,4}$ is shown in fig. 5.

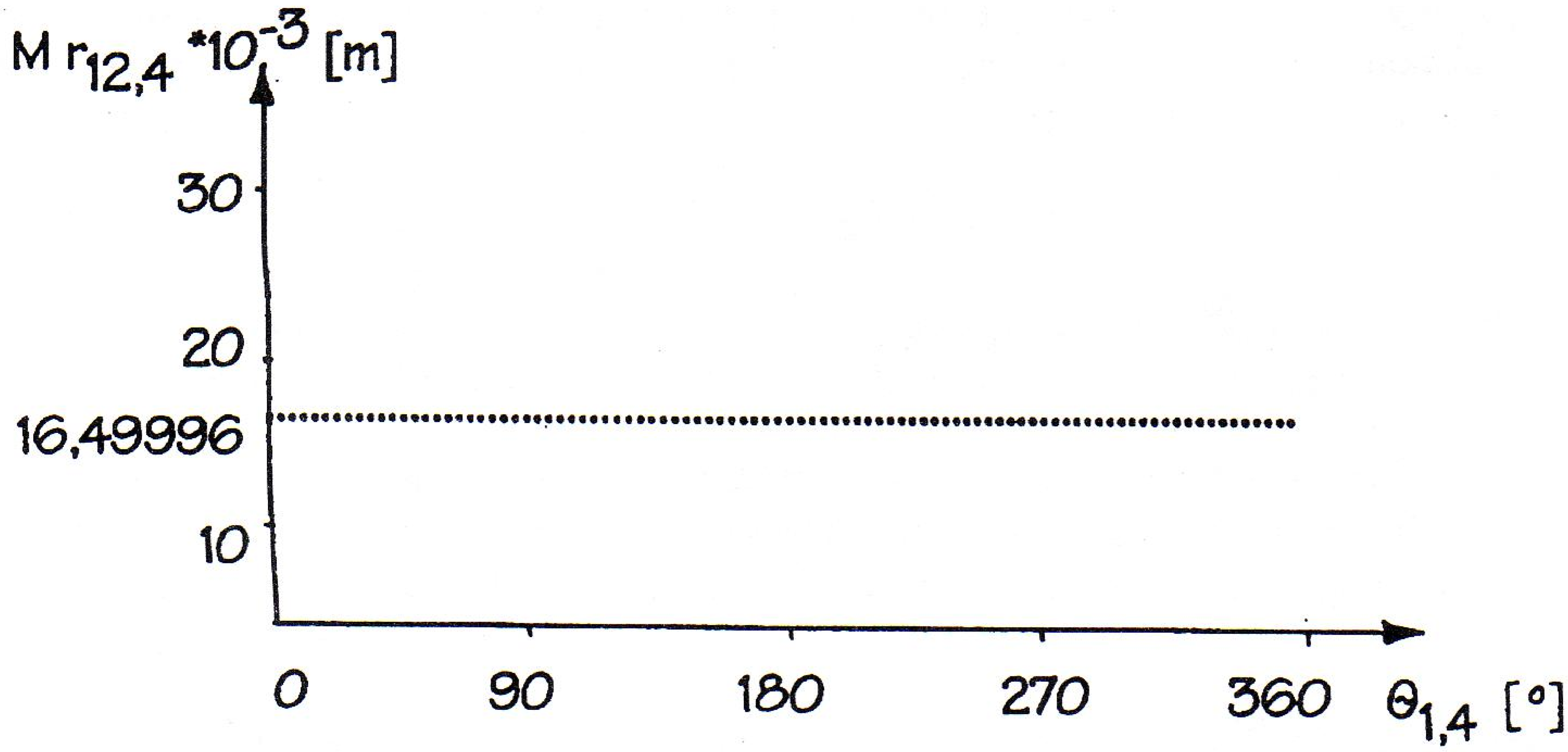

Fig. 5 A diagram of the modulus of the radius $\mathbf{r}_{21,4}$ of the upper catcher mechanism.

The modulus of the radius $\mathbf{r}_{21,3}$ proceeds as shown in fig. 6 and causes the shaft interacting with the lower catcher mechanism to be balanced completely in this case.

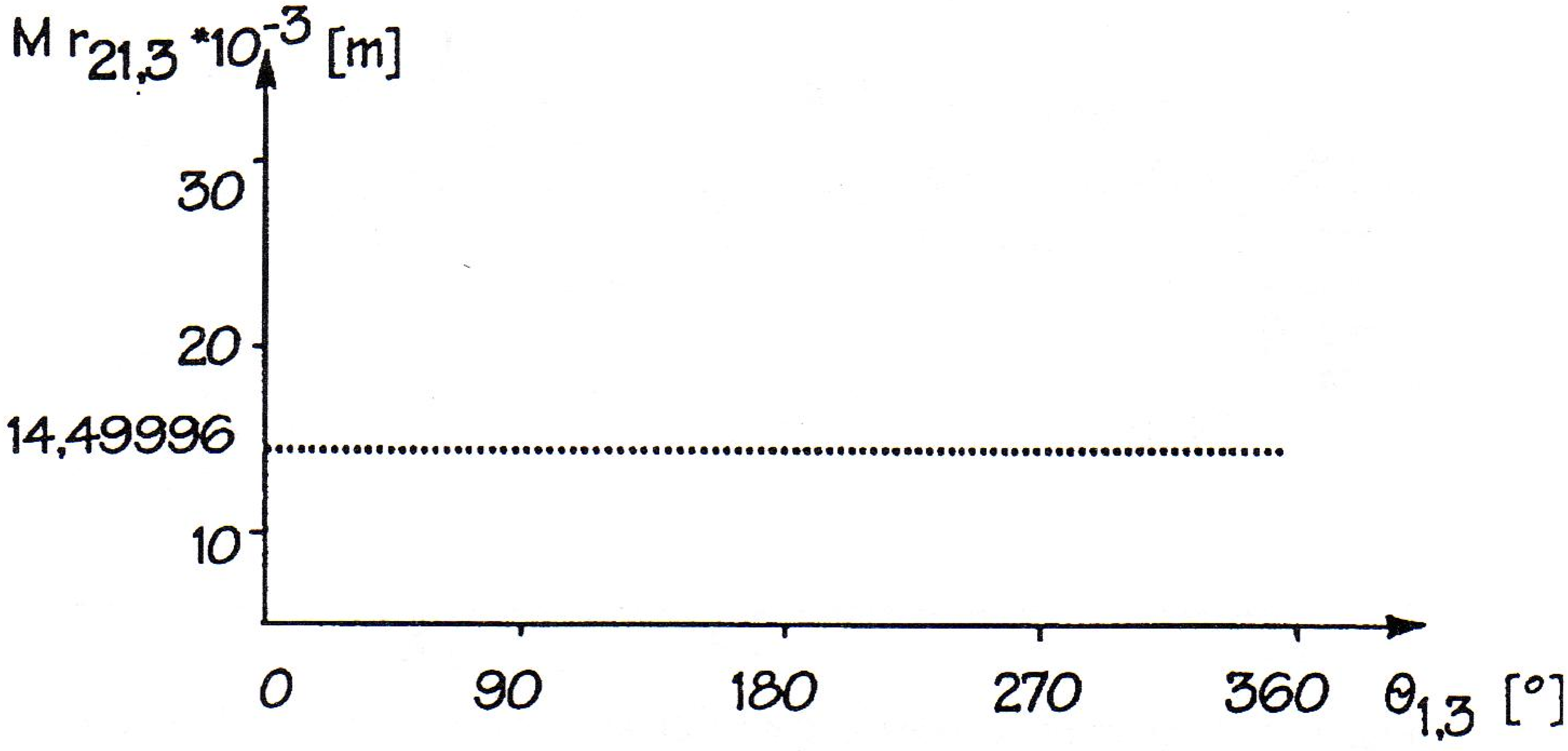

Fig. 6 A diagram of the modulus of the radius $\mathbf{r}_{21,3}$ of the lower catcher.

In the transport mechanism, the relationship concerning changes in the radius $\mathbf{r}_{21,2}$ is similar to those of mechanisms nos. 2 and 3.

The balancing of the rotor must be carried out in two stages: first, the masses of link 1 and, partially, the masses of links 2 of the mechanisms constituting units of the machine from nos. 2 to 5 should be balanced. As a result of balancing, the conditions $\mathbf{W}_{12}$, $\mathbf{W}_{13}$, $\mathbf{W}_{14}$, $\mathbf{W}_{15}$ defined by formulae (5).

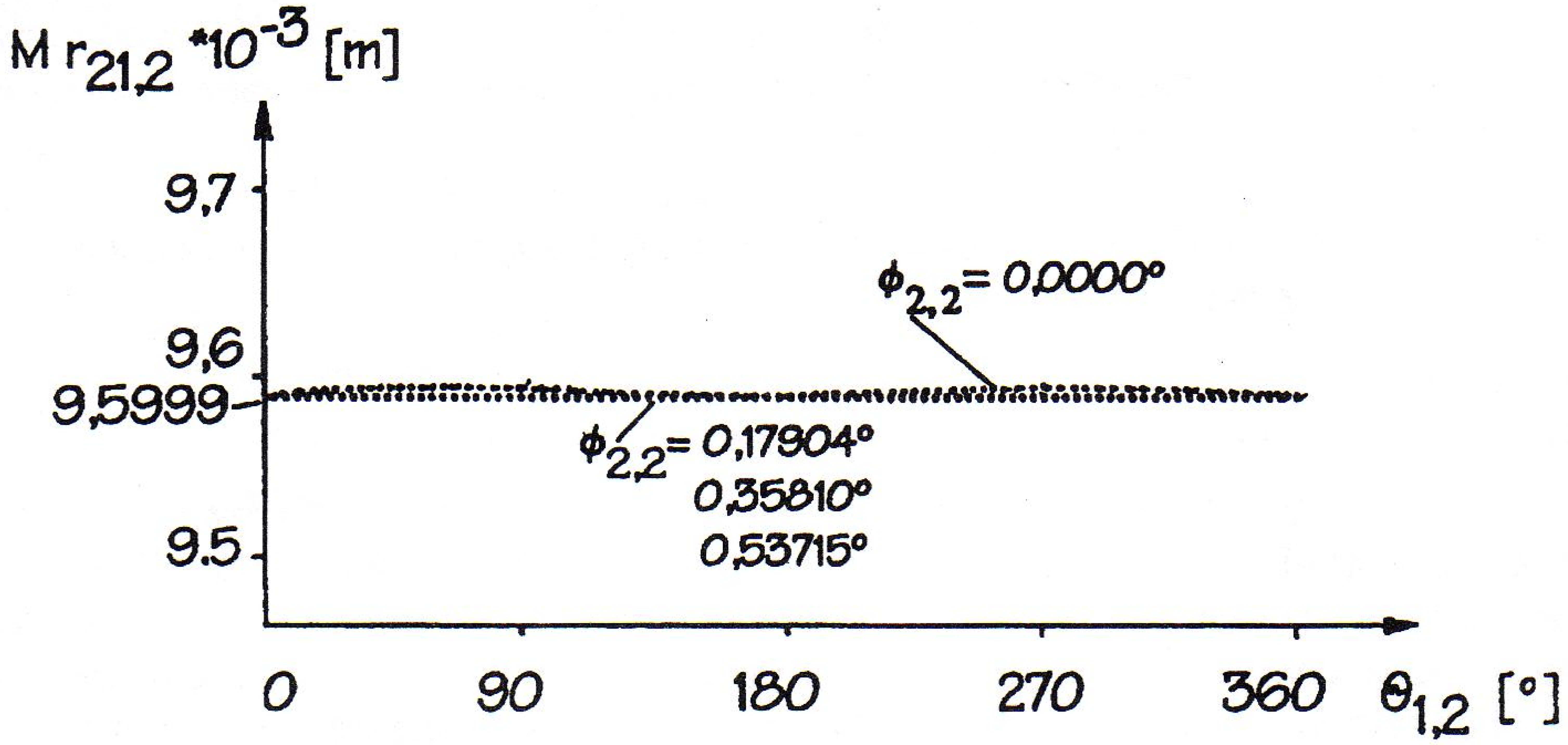

Fig. 7 A diagram of the modulus of the radius $r_{21,2}$ in the transport mechanism.

As has already been mentioned, the condition W_{15} will only be satisfied partially. Thus, there will be certain remainder unbalance because of lack of the complete balancing of the main vector of inertial forces. However, the dynamic load of the shaft bearings should be decreased. At the second stage, the vector of the main moment of inertial forces is balanced and this operation, called dynamic balancing, is carried out in motion by means of well-known methods and equipment.

The construction of the rotor provides for the possibility of so called additional balancing of the ready prototype of the machine. In this case, the correction masses indispensable for balancing should be mounted on the belt pulley in places of appropriate construction.

As is well-known, the inertial force of the mechanism is equal to zero at every instant of its motion only when the acceleration of the centre of the mechanism mass is zero. In the case of the machine mechanisms this means that the centre of mass of particular mechanisms must remain immovable relative to the base, i.e. the co-ordinates of the centre of mass are $x_{s,k}$ = constant, $y_{s,k}$ = constant, $z_{s,k}$ = constant. This happens only when the conditions of balancing are satisfied. The distribution of mass of the mechanisms thus balanced should cause the mechanism to remain at rest in its every position, under the action of gravity forces of the links at the vertical arrangement of the plane of motion.

The operation of such balancing can be carried out without having to investigate the mechanism motion, therefore such balancing should be performed during the design of the machine. If the conditions of balancing are not satisfied, additional balancing masses must be used according to the principle:

$$m_{ic} r_{ic} = m_i r_i + m_{ik} r_{ik}; \quad m_{ic} = \Sigma m_i + m_{ik} \quad \text{for } i = 1....n \text{ and } k = 2, 3, 4, 5. \tag{7}$$

for i=1...n and k = 2, 3, 4, 5

The static moment of the total mass m_{ic} and the vector radius r_{ic} is equal to the sum of the static moments of the movable links being balanced and the static moment of the correction mass and the correction vector radius. The total mass of the link is equal to the sum of the masses being balanced and the correction mass. This can lead to an increase in the overall dimensions of the mechanisms. Thus, particular links must obtain appropriate shapes during design.

4. Conclusion

From the formulae determining the conditions of balancing it follows that some vectors of position of the centres of masses of the links are expressed by the kinematic parameters of other links during a cycle of motion of the mechanism. This justifies the need for balancing only some links of the mechanism. It can be stated that links of rotational motion, whose vectors of position of the centres of masses are determined in the systems related to these links, and links of translational motion constituting kinematic pairs with links of rotational motion and in the systems of rotatable links should be balanced. From the conditions of balancing it also results that, e.g. part of the mass of the crankshafts m_{2k} and the inside cranks m_{lk} should be balanced in the system of link 1. Thus in practice, the balancing of the masses of the shaft inside cranks is brought about using the counterweight of the shaft inside crank, whereas the balancing of part of the crankshaft mass requires an appropriate mass constituting a counterweight. In the case of more complex shapes of links, a special algorithm for computer aided design should be used. These procedures should lead to the balancing of the main vector of inertial forces.

There remain the components of the main moment of inertial forces revealing in motion to be balanced. As is well-known, such balancing is referred to as dynamic balancing and can be carried out on a prototype of the machine when the vibrations of the operating machine can be measured. To reduce the vibrations of the prototype to the permissible level, so called additional balancing, by means of a specially designed belt pulley, which also serves as a flywheel, may suffice. In addition, the characteristics of the elastic support of the machine can be varied. If these prove to be insufficient, the dynamic balancing of the whole machine must be carried out.

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