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ALMOST PERIODIC VIBRATION EXCITED BY DRY FRICTION

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Abstract

This paper attends to the problem of a self-excited system with dry friction. The system is composed of mass that interact viscoelasticly with the drive, and by means of dry friction with a foundation. The objective of the experimental research on the system was to formulate a dry friction model that would describe the case of stick-slip vibration as well as almost-periodic vibration of steel-polyester pair. The model allows for the impact of the following parameters: velocity of motion, acceleration sign, time of adhesion and force rate. The paper includes computer simulation of the vibration of the considered system and comparison between the results of experimental and theoretical analysis.

Introduction

Engineering is a constantly advancing area which together with its friction-induced vibration theory is an essential element of nonsmooth dynamical system theory. Apart from engineering, friction-excited vibration can be frequently noticed in everyday situations although they are sometimes undesirable and avoided as a result of noise and wear. The wheel/rail contact issue accounts for the wavy outline of rail wear corrugations considering nonlinear nonsmooth friction law and shows that, according to the friction model, the rolling moment oscillates in time qualitatively different [1] and [2]. Since the method of discretisation had an impact on the outcome of the observations, some theoretical and experimental research enabling to distinguish numerical and physical effects should be carried out. This study, unlike some previous ones which examined simpler models of friction, deals with a more complex model and discusses the friction force as depending on: velocity of relative motion, acceleration, time of stick (adhesion), and force rate.

Investigations into chaotic solutions in the case of stick-slip vibration were approached in [3] and [4]. The solutions of experimental and theoretical research, in the case when adhesion does not occur on the interacting bodies surface as well, were given in [5].

The behaviour of the system depends on the analysed model of friction. The problem of forced vibration was discussed in [6]. Functions describing static and kinetic friction force have a vital influence on the occurrence of fixed-points, stability, ranges of self-excitation, bifurcation and chaos.

Obviously, accepting an improper friction model makes it impossible to obtain theoretical results that would confirm the experimental ones, an example here being manifestation of a stable form of out of phase vibration of a system of two degrees of freedom, which was not found in the experimental investigations [8]. Considering the above facts, an attempt was made to create a dry friction model that would meet the standards of steel-polyester pair. This paper illustrates the results of experimental research, as well as the procedure for modelling self-excited system with dry friction in the case of almost-harmonic vibration and stable slip. The research is the continuation of the studies of slip-stick vibration whose results were given in [2].

1. Model of self-excited system

A self-excited system that occurs in real mechanical systems is modelled a drive that moves at a constant velocity and interacts with a stiff mass. The mass and the drive are joined by viscoelastic element that is described by the Voigt model (Fig. 1). Moreover, the mass interacts frictionally with a foundation.

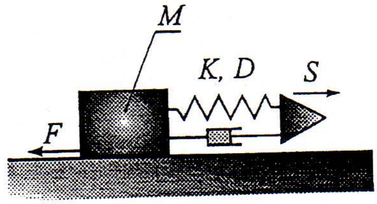

Fig. 1. Mechanical model of self-excited system.

Assuming that the state of equilibrium point is the state where the force in the spring equals zero, the equation of the motion of the investigated system will take the form:

$$MX'' + DX' + KX = F(), \tag{1}$$

where X - mass deflection from the state of equilibrium, K - spring stiffness, D - viscous damping, F - friction force between the mass M and the foundation, τ - time, $S>0$ - drive velocity, $X' = \frac{dX}{d\tau}$.

Let us introduce the following dimensionless quantities:

$$t = \frac{\tau}{\tau_0}, \quad x = \frac{KX}{F_0}, \quad d = \frac{D}{\sqrt{KM}}, \quad s = \frac{S\sqrt{KM}}{F_0}, \quad f = \frac{F()}{F_0}, \tag{2}$$

where $\dot{x} = \frac{dx}{dt}$, $\tau_0 = \sqrt{\frac{M}{K}}$, F_0 - friction unit (e.g. body weight or normal force), τ_0 - time unit.

The equation of motion (1) will then take the following form:

$$\ddot{x} + d\dot{x} + x = f(), \tag{3}$$

where $f()$ - function describing the friction model, d - dimensionless damping.

2. Function describing friction model

On the basis of the results of investigations into the system in the case of stick-slip vibration [2], there was assumed a preliminary function that would describe the friction model and would depend on the following physical parameters of the system: relative velocity of the mass and the drive w, static friction force f_s, and the sign of relative acceleration $\dot{w}$. Allowing for the assumptions mentioned above, the equation of motion (3) takes the form:

$$\ddot{x} + d\dot{x} + x = f(w, f_s, sign\ \dot{w}), \tag{4}$$

where $w = s - \dot{x}$, s - dimensionless drive velocity.

Let us assume static friction force to be the function of the following parameters:

$$f_s = f_s(t_s, \delta), \tag{5}$$

where $t_s = t_2 - t_1$, $\delta = \left.\frac{df_s}{dt}\right|_{t=t_2} = s$, t_s - time of adhesion of mass to foundation, t_1 - initial moment of adhesion, t_2 - final moment of adhesion, δ - force rate,

then the equation of motion of the investigated self-excited system will take the form:

$$\ddot{x} + d\dot{x} + x = f(w, t_s, \delta, sign\ \dot{w}). \tag{6}$$

Determining the function that would describe the friction model requires experimental research aiming at defining the parameters of a dry friction model. In the considered matter, i.e. steel-polyester friction pair, we will limit the range of velocity to the degree in which in the system there is almost-periodic vibration and stable slip.

3. Experimental investigations

Experimental research aiming at determining the parameters of the friction model were carried out using the research stand that consisted of two systems: mechanical, which was a set of mobile masses that were joined by means of springs and interacted frictionally with the conveyor belt, and optically-electronical measuring system, which was a system of visualization, aquisition and digital handling of data [2].

The measuring system is composed of: monochromatic camera with fast shutter, digitally controlled video recorder, the analog-to-digital converter card that transforms the video signal into digital form and dedicated software 'IPS' (Image Processing System). The stand enables the investigation of various frictional pairs for different values of the following parameters: number of masses, belt velocity, initial conditions, stiffness and mass.

As a result of the experiment, we achieve coordinates of the masses being in motion, on the basis of which, using the 'IPS' software, it is possible to determine dynamic characteristics of the mass motion, i.e. relations of displacement, velocity and acceleration in time, phase plane, acceleration spectra (actual and total). When applying the indirect measurement method, such relations allows to determine static and kinetic friction force as a function of parameters of the investigated system.

4. Measurements of vibration of investigated system

In order to define experimentally the static friction force as a function of parameters of the system (5), there was determined the value of friction force during loss of adhesion of the interacting bodies, with fixed time of adhesion t_s and various force rate δ. Then with variation of time t_s, after the velocity δ has been determined. The results of the measurements (after approximation) that are constitutive relations for the case of steel-polyester friction pair are shown in Fig. 2.

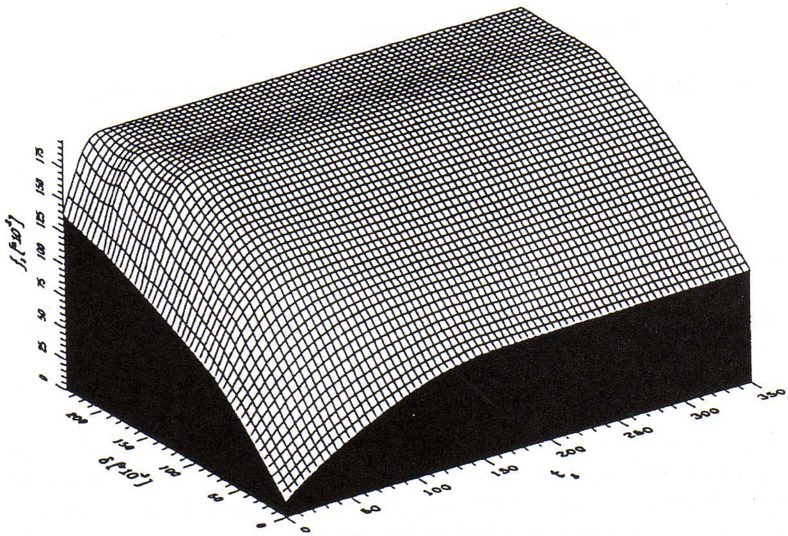

Fig. 2. Static friction force f_s as a function of adhesion time t_s and force rate δ.

As results from the plot in Fig. 2, when $t_s \to 0$ and stick-slip vibration vanishes, i.e. in the whole range of velocity of the mass motion the condition: $s > \dot{x}$ is fulfilled, the kinetic friction force has a predominant influence on the motion of the investigated system. Further investigations will concern the problem of establishing the dependence of kinetic friction force on relative velocity w and acceleration $\dot{w}$. To achieve that, dynamic characteristics of the investigated system were assigned, i.e. the dependence of displacement on time (Fig. 3) and phase trajectory at different velocities s. On the basis of the investigations within the scope of lower velocities, it was found that there occurs almost-harmonic vibration in the form of stable limit cycles which were determined with the use of the first representation of Poincaré. Together with the growth of the velocity s the amplitude of periodic almost-harmonic vibration decreases. At a certain value of velocity s, limit cycle attractor vanishes and there appears a qualitatively new attractor in the form of a stable fixed-point, so called stable slip. This transition is characteristic of the supercritical Hopf bifurcation.

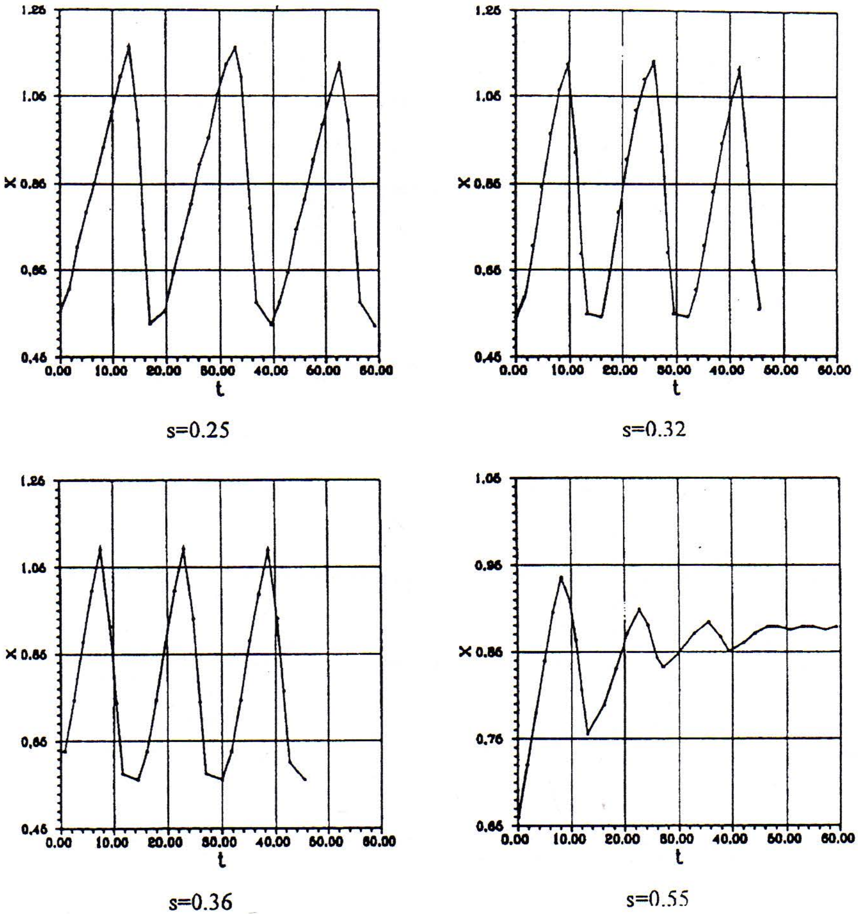

Fig. 3. Experimentally obtained displacement x of mass in the function of time t at given velocity s.

In order to establish the dependence of kinetic friction force f on relative velocity w, an indirect measurement method was used. The dependence was described by the equation (6) on the basis of experimentally achieved characteristics of the motion of the system in the case of almost-harmonic vibration. Assuming a piece-wise linear friction model and applying the least square method, approximation of kinetic friction force f was made as a function of velocity of relative motion w. Result of investigation is shown in Fig. 4. The kinetic friction force f as a piece-wise linear function of relative velocity w for various velocities s after approximation of the experimentally obtained results is shown in Fig. 5.

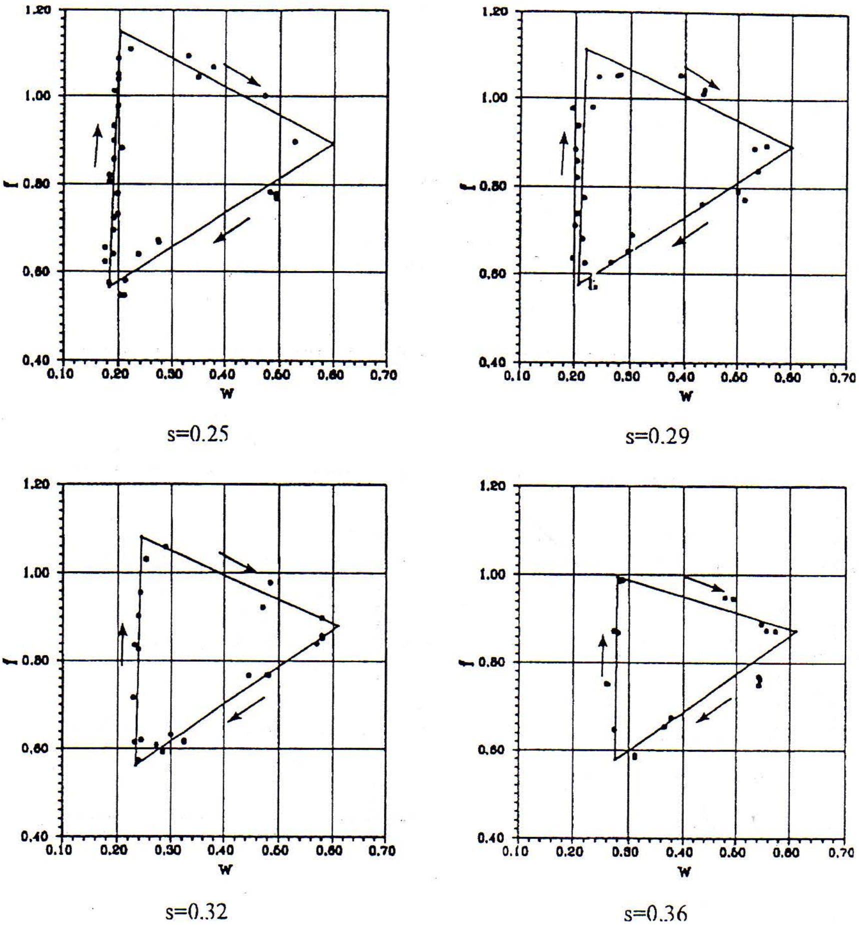

Fig. 4. Approximation of kinetic friction force f as a function of relative velocity w for various values of velocity s.

As a result of the above dependences, kinetic friction force at $\dot{w} > 0$ receives higher values than in the case of $\dot{w} < 0$, which is compatible with the previously obtained results of the analysis of the system in the case of stick-slip vibration [2]. Together with the increase of velocity s, corresponding values of kinetic friction force decrease and the values of relative velocities w at $\dot{w} = 0$ rise. In the whole range of relative velocity in which the vibration of the system occurs, the condition $w > 0$ is met. This condition differentiates between almost-harmonic and stick-slip vibration with $w \geq 0$. This means that the motion of the system depends mainly on kinetic friction force with the exception of the value of relative velocity with which kinetic friction force is the highest and satisfies the equation (Fig. 2):

$$f_{max}(w, sign\,\dot{w})\big|_{s=const} = f_s(t_s \to 0, \tilde{\omega}) . \tag{7}$$

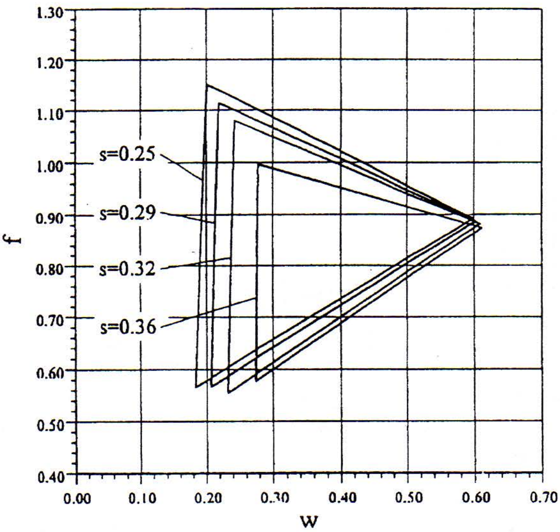

Fig. 5. Dependences $f(w, \text{sgn}\ \dot{w})|_{s=const}$ determined for various values of velocity s after approximation of experimental results.

Due to the non-occurrence of the slip phase during almost-harmonic vibration it is necessary to develop the dry friction model proposed previously and identified in the case of stick-slip vibration [2]. Generalization of the friction model should also give consideration to the existence of the mass stable slip which was noticed for higher values of velocity s. The goal of the following investigations is generalization of the present friction model and its parametric identification by use of computer simulation of the system.

5. Generalization of friction model

The results of experimental investigations: kinetic friction force as the function of relative velocity and acceleration sign (Fig. 5), as well as static friction force dependent on time of adhesion and force rate (Fig. 2), enable formulation of dry friction model in the case of steel-polyester pair. Generalised friction model should take into consideration not only stick-slip vibration [2] but also almost-harmonic oscillations and stable slip. On the basis of experimentally obtained results, properties and parameters analyses were carried out. This allowed initial acceptance of the following properties of the model:

- friction model is piece-wise linear,
- in the case of stick-slip vibration the angles of inclination of the corresponding linear parts of characteristic are dependent on the value of static friction force at the moment of the loss of adhesion,
- for almost-harmonic vibration and stable slip, the angles of inclination of the corresponding linear parts are described by the static friction force that satisfies condition (7).

The above assumptions make it possible to approve of the general shape of the dry friction model which is shown in Fig. 6. In the figure, elements of the friction characteristic that appear for almost-harmonic vibration are marked with a broken line.

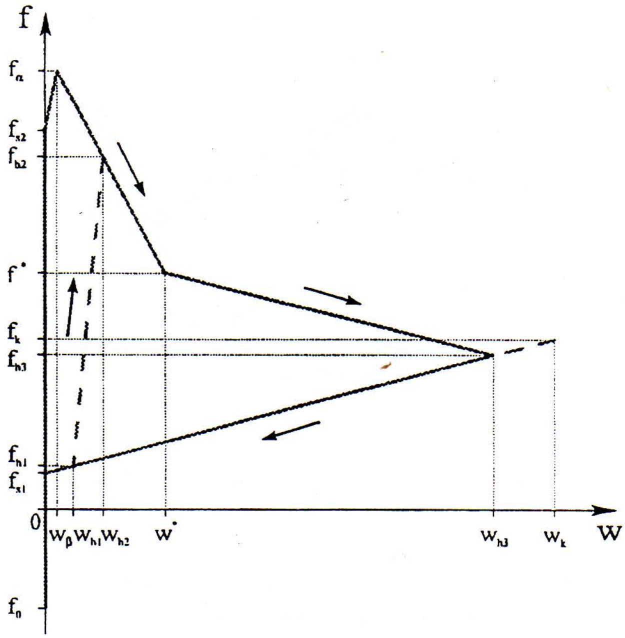

Fig. 6. General shape of dry friction model.

Allowing for different shapes of characteristics for assigned drive velocities s (Fig. 5), it demands that there should be introduced geometrical parameters of the model that are shown in Figs. 7-9 and fulfil the following simplifications:

- for various velocities of the drive motion, linear parts of characteristics correspond to the intersection points (A, B, C, D, respectively),
- points of upper intersections for different drive velocities are on a straight line $(l: f^* = a_i w^* + b_i)$,
- for given velocities of the drive motion, as presented in Fig. 6, coordinates $w_{max} = w_k$ (for stick-slip vibration) and $w_{max} = w_{h3}$, $w_{min} = w_{h1}$ (for almost-harmonic vibration and stable slip), have been obtained from the condition $\dot{w} = 0$,
- parameters α, β (Fig. 7) describe change of the friction force at the moment of adhesion loss,
- parameters occurring when $A \to \infty$, $C \to \infty$ or $D \to \infty$ have been denoted: φ, ψ, η, respectively (Fig. 7 and Fig. 9).

Geometrical parameters shown in Fig. 7 allow to describe the occurrence of stick-slip vibration in the analysed self-excited system. Abscissa of the C point satisfies the condition $w_C \neq 0$, when there appears almost-harmonic vibration at higher velocities in the system. Obtaining out of phase vibration needs additional parameters to be introduced, as presented in Figs. 8 and 9.

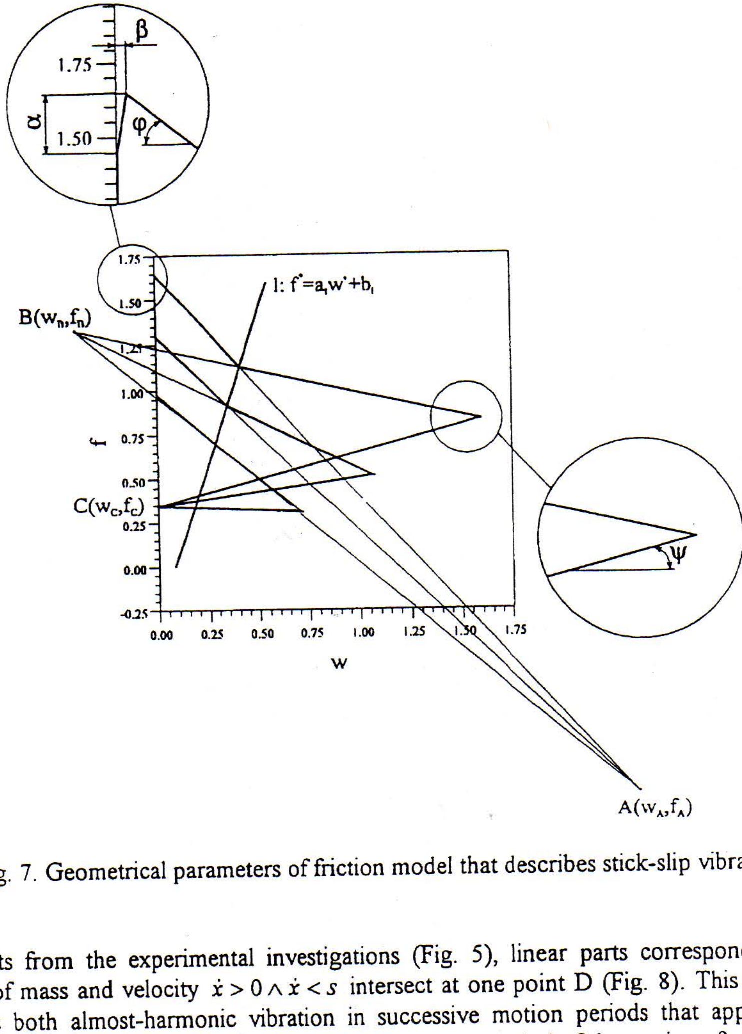

Fig. 7. Geometrical parameters of friction model that describes stick-slip vibration.

As results from the experimental investigations (Fig. 5), linear parts corresponding to the motion of mass and velocity $\dot{x} > 0 \wedge \dot{x} < s$ intersect at one point D (Fig. 8). This assumption concerns both almost-harmonic vibration in successive motion periods that approach limit cycle (Fig. 8a), and almost-harmonic vibration in the first period of the motion after the loss of adhesion on the contact surface (Fig. 8b). Together with the growth of drive velocity s, the value of the coordinate $w_{min} = w_{hl}$ increases to a certain critical value w^*_{hl}. In order to obtain a solution compatible with the results of experimental investigations at $w = w^*_{hl}$, point $D(w, f)$ was translated in infinity and, simultaneously, parameter η was introduced (Fig. 9). After the relative velocity value $w = w^*_{hl}$ has been exceeded, a stable slip of the mass occurs in the system.

Determining the numerical values of geometrical parameters and corresponding physical parameters of the friction model is the target to be met in the following analysis. The proper selection of parameters of the model should make it possible to receive compatibility between the fixed values of self-excited system and the values of the actual system, which were obtained experimentally. Computer simulation of the vibration of the investigated system was made use of to conduct a parametric identification in the case of initially suggested dry friction model.

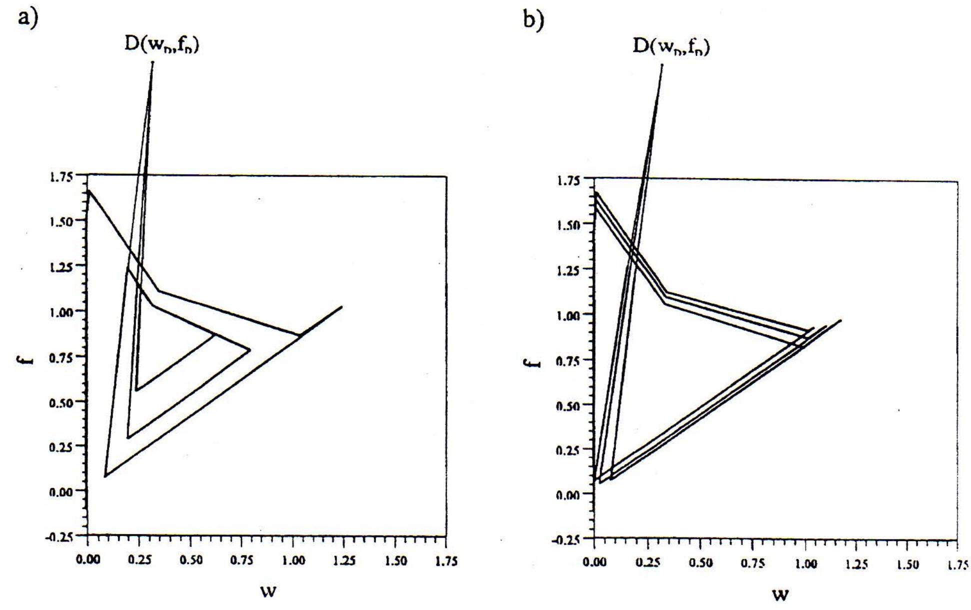

Fig. 8. Geometrical parameters of the friction model that describes almost-harmonic vibration: a) in successive motion periods, b) with different conditions of adhesion loss.

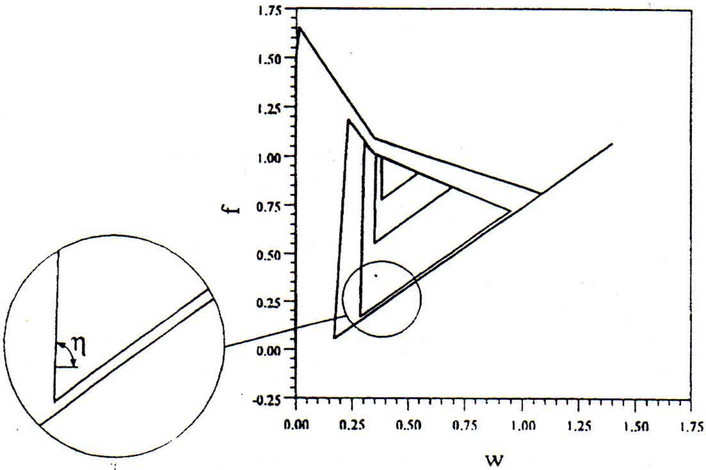

Fig. 9. Geometrical parameters of the friction model that describes stable slip of mass.

6. Identification of the friction model

Parametric identification tends to determine the numerical values assumed at the beginning on the basis of experimental investigations into the parameters of the friction model. The criterion of identification is compatibility between phase trajectories of the motion of the investigated system at different drive velocities *s* obtained theoretically, and the ones obtained experimentally. The compatibility concerns quantitative properties (corresponding values of displacement and velocity), as well as qualitative properties (occurrence of specified stationary solutions).

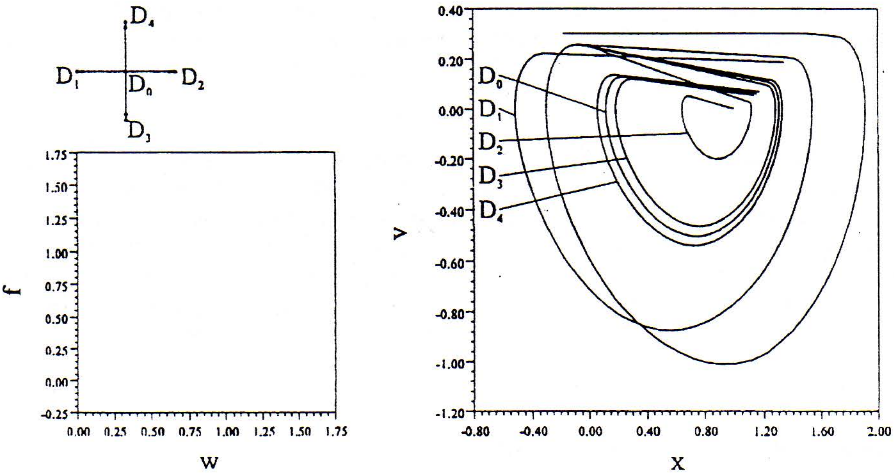

Fig. 10. Impact of translation of point D on trajectory of motion.

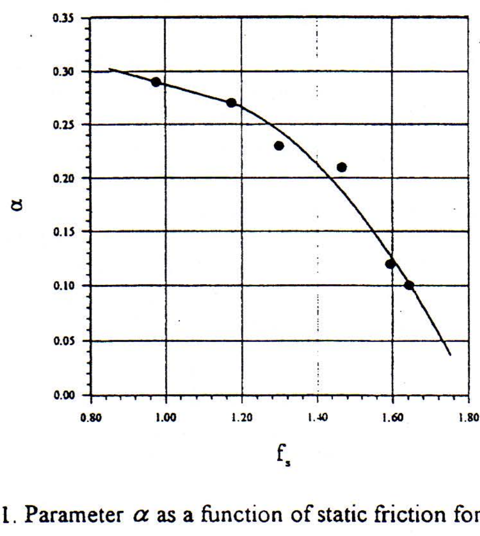

Fig. 11. Parameter α as a function of static friction force f_s.

What was studied during the identification, was the sensitivity of the trajectory of mass motion to changes of the friction model parameters at a fixed value of drive velocity and fixed initial conditions of the motion. The results of the investigations for stick-slip vibration are presented in [2], while translation of the point D and consequent changes of phase trajectories of the almost-harmonic motion are shown in Fig. 10. The parameter marked with D_0 corresponds to an average value initially accepted on the basis of experimental investigations. Similarly to stick-slip vibration, phase trajectories of almost-harmonic vibration take various shapes by reason of change of damping intensity during different phases of the motion of the system.

The identification of the influence of the friction model parameters on the shape of the system trajectory (stick-slip vibration, almost-harmonic vibration, stable slip), enabled to make an identification of the assumed dry friction model. As a result of the identification, the following parameters of dry friction model have been accepted: $f_s = f_s(t_s, \delta)$ (Fig. 2), $\alpha = \alpha(f_s)$ (Fig. 11), $\beta = 0.01$, $\varphi = -58^\circ$, $\psi = 39^\circ$, $\eta = 89^\circ$, B(-0.41;1.33), D(0.33;2.40), $1: f^* = 3.41w^* - 0.28$.

The last stage of the identification of the friction model was its verification, during which the compatibility between phase trajectories (obtained theoretically and experimentally) was investigated [9]. During the verification, some discrepancies were found which were the result of a limited precision of the measurement system and of simplifications of the theoretical analysis. After the identification, the friction model was used for computer simulation of the motion of the analysed self-excited system. The simulation aimed at investigating the properties of the system in a wide range of drive velocity s when there occur various types of oscillation.

7. Simulation of motion of self-excited system

Computer simulation of motion of the investigated self-excited system enables to examine its properties. Investigations at the velocities fixed during the experimental research make it possible to verify the correctness of the parametric identification and identification of the system features. Theoretical analysis in a wide range of velocities leads to a more complete knowledge of the properties of the vibrating system for different types of oscillations.

The previously made investigations of the stick-slip vibration whose results are given in [2], demonstrated the existence of a motion in the form of a stable limit cycle. For any initial conditions (except the conditions which meet the standards of limit cycle), after some time trajectories tend to stationary periodic solution for which stick-slip vibration take place. Together with the increase of drive velocity s we receive values for which the stick-slip vibration from the initial phase of the motion passes through non-stationary almost-periodic to the stationary solution. The solution can take the form of almost-harmonic vibration due to the growth of drive velocity s. Later on it can take the form of stable slip. The outcome of the computer simulation of the investigated self-excited system, which illustrates the occurrence of the above described types of oscillations and transitions between them, is presented in Fig. 12.

In relation to the adhesion at the first stage of the motion, static friction force f_s determines the motion of the system irrespective of the forms of vibration met at the later stages. The influence of the system history on its properties is conditioned by the analysed dry friction model whose parameters are characteristic of the so-called memory-dependent friction.

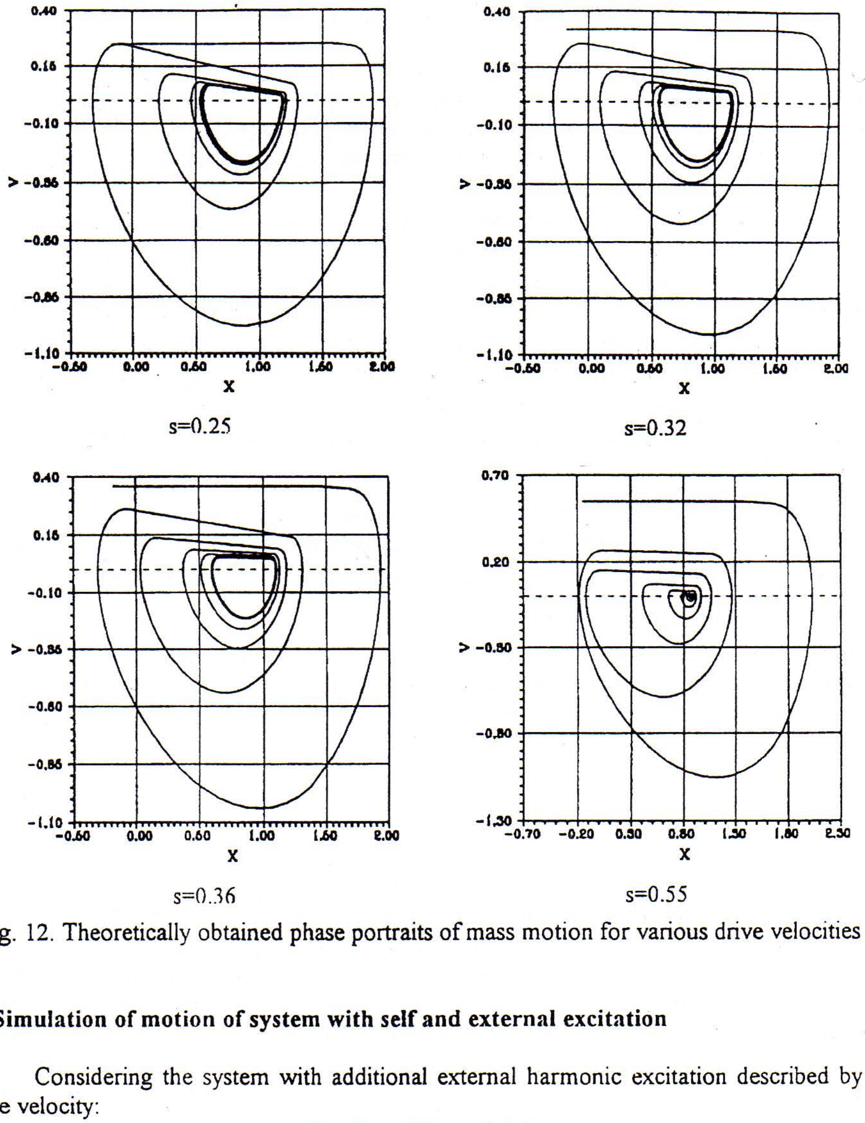

Fig. 12. Theoretically obtained phase portraits of mass motion for various drive velocities *s*.

8. Simulation of motion of system with self and external excitation

Considering the system with additional external harmonic excitation described by the drive velocity:

$$S = S_1 + \Omega U_0 \cos(\Omega\tau) \tag{8}$$

where S_1 - constant value, U_0 - excitation amplitude, Ω - excitation frequency.
the equation of the system motion reads

$$MX'' + DX' + KX = F(W) + KU_0 \cos\Omega\tau, \tag{9}$$

Using dimensionless quantities (2) the motion of the investigated system is described by the equation

$$\ddot{x} + dx + x = f(w) + u_0 \cos\eta t, \tag{10}$$

where

$$u_0 = \frac{KU_0}{F_0}, \quad \eta = \Omega\tau_0.$$

Motion of a simpler model alike the mentioned above was considered in [5].

Computer simulation of system motion described by equation (10) was conducted for various values of drive velocity s for which the system revealed stick-slip vibration, almost-harmonic motion or stable slip properly. The results of motion simulation within the range of velocity s, within stick-slip vibration occurs, for different excitation frequencies η, were presented in [10].

Fig. 13 depicts the behaviour of the system for drive velocity when stick mode does not occur in the whole range of motion. When limiting the range of velocity s to such a degree in which there appear almost-harmonic vibration (in Fig. 13 $s = 0.30$), at fixed value excitation amplitude u_0 and changeable value of frequency η the system exhibits existence of qualitatively different behaviour. For some values of excitation frequency the stationary system motion takes the form of one-periodic vibration, similar to the one appearing for the system without external excitation (cf. Fig. 12). Nevertheless, for another values of frequency η in the system two- ($\eta = 0.35$ in Fig. 13) or higher-periodic motion occurs, that lead to non-periodic vibration ($\eta = 1.25$).

For drive velocity $s = 0.40$ which determines the stable slip of the mass (see Fig. 13), change of excitation frequency η results also in appearing qualitatively different solutions. It can take the form of one-, two- ($\eta = 0.40$ in Fig. 13) higher- or non-periodic oscillations ($\eta = 1.60$) in the neighbourhood of equilibrium state.

As results from computer simulation of the system motion with simultaneous self and external excitation, one-, higher- and non-periodic (chaotic) solutions occurs both for the almost-harmonic vibration and stable slip. It allows to suppose that period doubling aiming at chaotic solution exists in the considered self-excited system.

9. Concluding remarks

Experimental research, modelling and computer simulation of vibration of a self-excited system for the accepted friction model, proved that drive velocity s has an essential influence on the behaviour of the system. Different stationary solutions for the given values of drive velocity is shown in Fig. 14. In the case of small velocities for which there are relaxation stick-slip oscillations in the system, the vibration is considerably influenced by static friction characteristic (Fig. 2). Together with the increase of drive velocity ($s > 0.25$), a transition between qualitatively new types of oscillations of various frequencies, characteristic of supercritical Hopf bifurcation, can be noticed in the system. A limit cycle solution of stick-slip vibration turns intermittently into stationary almost-harmonic vibration. Consequently, kinetic characteristic of the friction begins to have a decisive influence on the system motion. For further increase in velocity ($s > 0.40$), self-excited vibration vanishes and the system tends to stable slip which is a stationary state in which relative velocity takes a constant value equal to drive velocity. Despite certain ranges of velocity s for which static or kinetic friction force is more significant, instantaneous system motion depends on both characteristics of friction due to the sensitivity of the model to the system history, which is characteristic of the so-called memory-dependent friction models.

Introduction of additional harmonic excitation into frictionally excited system results in existing not only one-periodic but also higher-periodic and non-periodic (chaotic) motion. The afore-mentioned behaviour of the system appears both in the case of stick-slip and almost-harmonic vibration as well as for stable slip of the mass.

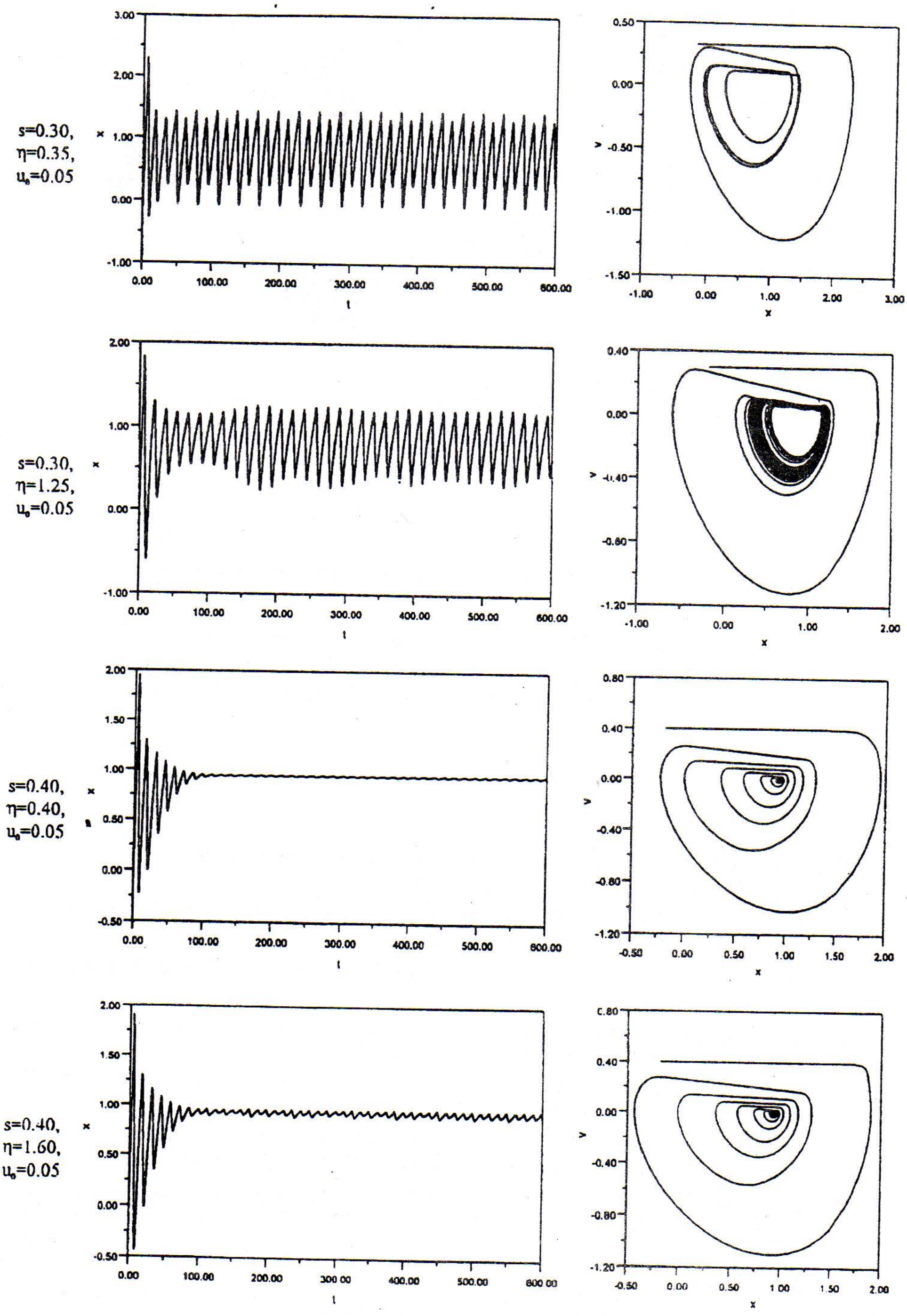

Fig. 13. Different types of mass motion for assigned drive velocity s, excitation frequency η and amplitude u_0.

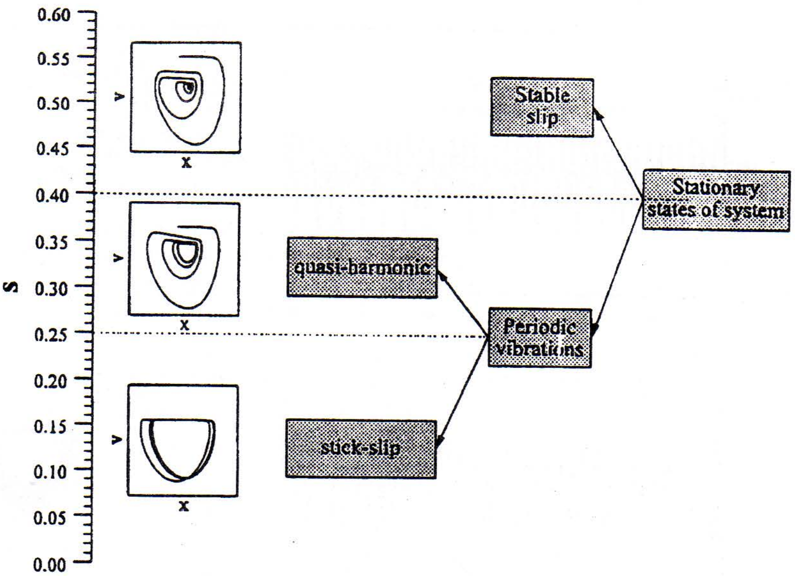

Fig. 14. Solutions illustrating behaviour of self-excited system with dry friction.

Further experimental and theoretical investigations into self-excited dry friction models, will tend to analyse vibration of systems of greater number of degrees of freedom including external damping. Such forms of the system behaviour as flip and fold bifurcation, chaos, will be incorporated in the study.

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