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SIZING OF THE RECIPROCATING (PISTON – CYLINDER) COMPRESSOR UNIT

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ABSTRACT

In this paper the authors presents their view point on the sizing of a single cylinder crank arrangement. Having this determined property, the overall multiple cylinder arrangement, and the proper speed can be determined for the required overall performance.

FORMULATION OF THE PROBLEM

Two fundamental parameters determine the main dimension status of one cylinder arrangement, namely:

1. The swept volume S is given by:

$$S = \frac{\pi}{4} D^2 .2e \qquad (I)$$

Where:

D piston diameter

e Crank length

2. Maximum piston force is given by:

$$F_{\max} = \frac{\pi}{4} D^2 P_{\max} = \frac{\pi}{4} D^2 \left(\frac{2e}{C}\right)^n \qquad (II)$$

Where:

C Clearance volume (determined by valve function)

n Index of compression

P_{max} Maximum pressure

P_o Inlet pressure

Equations (I), (2) can be solved for D , e to give,

$$2e = \frac{4S}{\pi}\frac{1}{D^2}$$

and

$$\frac{4F_{\max}}{\pi P_o}C^n = D^2\left(\frac{4S}{\pi}\frac{1}{D^2}\right)^n$$

i.e

$$\frac{4F_{\max}}{\pi P_o}C^n\left(\frac{\pi}{4S}\right)^n = D^{2(1-n)}$$

$$\therefore D = \left[\frac{4F_{\max}}{\pi P_o}C^n\left(\frac{\pi}{4S}\right)^n\right]^{\frac{1}{2(1-n)}} \qquad (1)$$

$$e = \frac{2S}{\pi}\left[\frac{4F_{\max}}{\pi P_o}C^n\left(\frac{\pi}{4S}\right)^n\right]^{-\frac{1}{(1-n)}} \qquad (2)$$

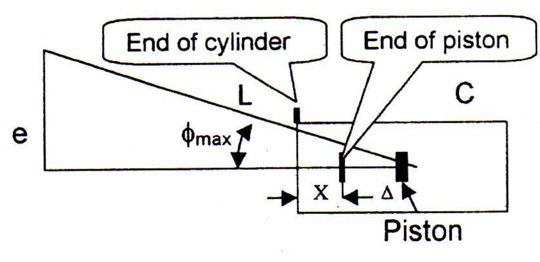
Example:

$S = 1$ lit
$F_{max} = 2$ ton
$P_o = 1$ kg/cm^2
$C = 1$ cm
$n = 1.25$

Equations (I), (2) gives,

D = 9 cm
, e = 8 cm

The length of connecting rod is determined by



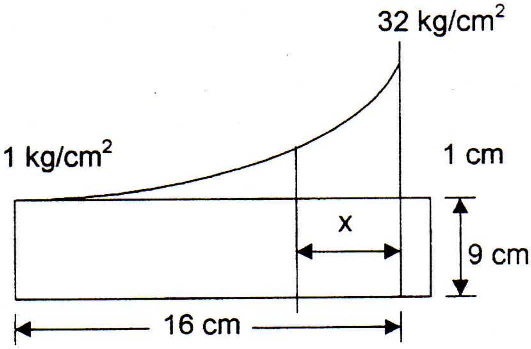
$$\frac{e}{l} = \sin \phi_{max} \qquad (3)$$

$$\tan \phi_{max} = \frac{\frac{D}{2}}{X + \Delta} \qquad (4)$$

$$X = \sqrt{l^2 - e^2} - (l - e) \quad (5)$$

Δ is determined by construction (for this example Δ is 6 cm)

Equations (3) , (4) , and (5) gives l = 24 cm

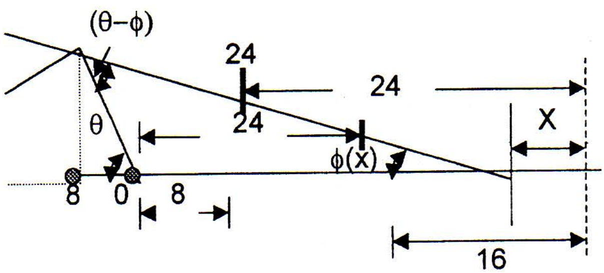


Torque

$$P_{max} = 1\left(\frac{16}{1}\right)^{1.25} = 32 \quad kg/cm^2 \,, T_{max} = 300\left(\frac{16}{1}\right)^{\frac{1.25-1}{1.25}} - 273 = 249\ ^{o}C$$

$$1 \ (16)^{1.25} = P\,(X+1)^{1.25}$$

$$\therefore P = \left(\frac{16}{X+1}\right)^{1.25} \qquad (6)$$



X + 24 cos ϕ = 8 cos θ -(8 + 24 + 2 x 8) (7)

8 sin θ = 24 sin ϕ (8)

Equations 6, 7, and 8 gives

$$P = \left(\frac{16}{X(\theta)+1} \right)^{1.25} \qquad (9)$$

Torque T on O is calculated as:

$$\frac{\pi}{4}(q)^2 \left[\frac{16}{X(\theta)+1} \right]^{1.25} \cos[\phi[\theta]]\sin(\theta-\phi).\frac{8}{100} kg.m \qquad (10)$$

Maximum torque can be determined by differentiation and for θ = π / 2 , the torque is estimated as 17 kg.m

DESIGN CALCULATION

D = 9 cm
e = 8 cm
l = 24 cm
P_{MAX} = 32 kg/cm^2

Minimum cylinder thickness t_{min}

$$\sigma_w = \sqrt{\left(\frac{P_{max} D}{2t_{min}}\right)^2 \left(\frac{P_{max} D}{4t_{min}}\right)^2 + P_{max}^2}$$

For cast iron cost σ_w = 10 kg/cm^2
D = 9 cm

P_{max} = 32 kg/cm^2 ⇒ gives t_{min} I) ⇒ 1 cm

Minium thickness of cylinder head thickness S_{min} accordingly

$$\left(\frac{S_{min}}{D}\right)^2 = \frac{3}{4}\frac{P_{max}}{\sigma_w} \qquad D=9cm$$

$$P_{max} = 32 kg/cm^2$$

$$\sigma_w = 10 kg/cm^2$$

$\Rightarrow$ *gives* $S_{min} = 1cm$ *II)*

Minimum pin diameter ϕ according to

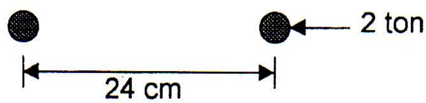
$$2\frac{\pi}{4}\phi^2\,\tau_{\max} = 2\,x10^3\,kg$$

$$\phi^2 = \frac{2}{\pi}\frac{2\,x10^3}{40} \qquad \tau_{\max} = 40\,kg/mm^2 \;\; for\ steel$$

$$\phi \;= 10\sqrt{\frac{10}{\pi}}\;cm$$

$$= \frac{10}{1.8} = 5.5\ cm$$

Minimum section of connecting rod



Safe for buckling

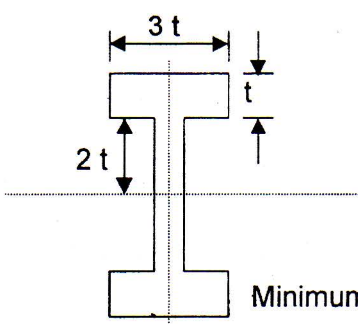
$$2\,x10^3 \,\langle\, \frac{\pi^2}{l^2}\;x\;20000\;E\,I \qquad (I\ \min iumum\ \ moment\ \ of\ \ inertia)$$

$$E = 20000\;kg/cm^2$$

$$2\,x10^3 \,\langle\, \frac{\pi^2}{(24)^2}\;x\;20000\,I$$

$$I \,\rangle\, \frac{(24)^2}{\pi^2}\;x\;\frac{2000}{20000} \,\rangle\, \frac{(24)^2}{\pi^2\;x\,10}$$

$$\rangle\frac{576}{100}\,\rangle 5.76\,cm^4$$



Minimum section

$$I_{60} = 4\left[\frac{3t\left(t/2\right)^3}{12} + t\int_{t/2}^{3t/2} y^2\,dy\right]$$

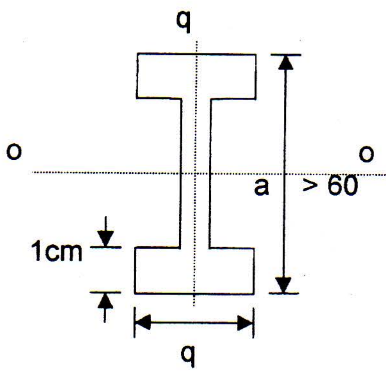
$$= 4\left[\frac{3}{12x8}t^4 + \frac{t}{3}\left[\left(\frac{3}{2}t\right)^3 - \left(\frac{t}{2}\right)^3\right]\right]$$

$$= 4t^4\left[\frac{3}{12x8} + \frac{1}{3}\left(\frac{27}{8} - \frac{1}{8}\right)\right]$$

$$= t^4\left[\frac{1}{4} + \frac{26}{6}\right] = 4.5t^4_{min}$$

$$\therefore \quad 4.5\,t^4_{mim} \rangle\ 5.76$$

$$t_{min} \rangle \sqrt[4]{\frac{5.76}{4.5}} \rangle\ 1\ cm$$



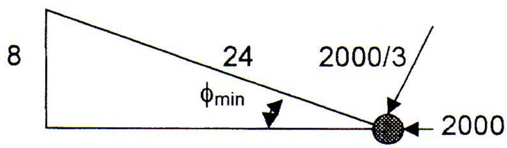
Check for compression

$$\sigma_w\ [Forged\ steel] = 30\,kg/mm^2$$

$$= 20x10^2 \rangle \frac{2x1000}{6+4}$$

$$2000 \rangle 200 \qquad \textit{Safe for compression}$$

Check for bending:



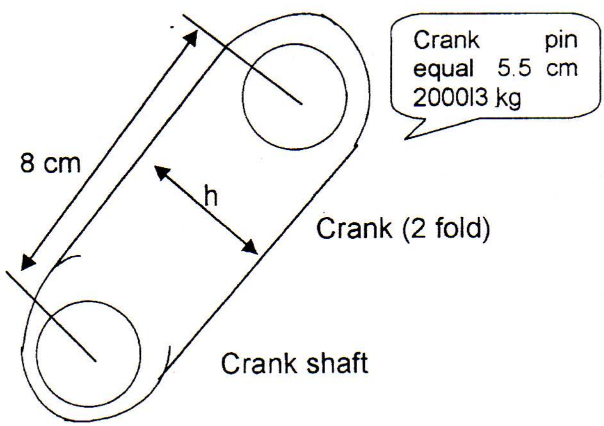
Sin ϕ_{min} = 1/3

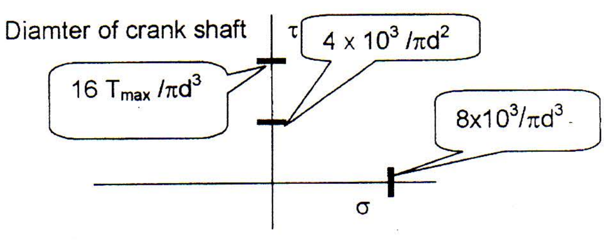
Maximum bending moment = (2000/3) x 24 = 16000 kg.cm

For lateral rigidity must choose I_{oo} although bending implies I_{qq}

$$Bending\ stress = \frac{M_{max} Y}{I} = \frac{16000 x\ 1.5}{5.76} 5.76 = 40\ kg/mm^2$$

(Must increase slightly the size of the main section), (Although over estimation of M_{max})



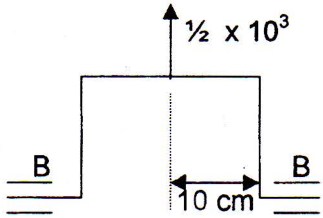


Crank Shaft:

$$T_{\max} \langle \frac{2000\sqrt{8}}{3}\frac{\sqrt{8}}{3}8$$

$$T_{\max} \langle \frac{64\,x\,2000}{3}\ kgcm$$

$$\langle 31.3\ x\,2000 = 62.5\ x\ 10^3\ kg\ cm$$



$$Shear\ stress = \tau$$

$$\frac{16\,x\dfrac{2000}{3}x\,8}{\pi\,d^3} + \frac{2000}{\dfrac{\pi}{4}d^2}$$

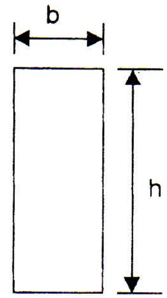
$$Bending\ stress\ \sigma = \frac{\dfrac{1}{2}x\,10^3\,x\,d/2}{\dfrac{\pi}{32}d^4}$$

$$\tau_w = 20\ kg/mm^2\ x\,10^2 = \sqrt{\tau^2 + \sigma^2}$$

$$gives\ d \cong 4\,cm$$

Crank arms:

Maximum bending moment / fold



$$Maximum\ bending\ must\ /\ fold\ (Forged\ steel)$$

$$= \frac{1000}{3} x\, 8 = 3000\ kg\,/\,cm$$

$$\sigma_w = \frac{3000\ \frac{h}{2}}{\frac{b\,h^3}{12}} = \frac{18000}{b\,h^2} = 40\, kg\,/\,cm^2$$

$$\therefore b\,h^2 = \frac{18000}{40} = 4.5\, x\, 100\, cm^3$$

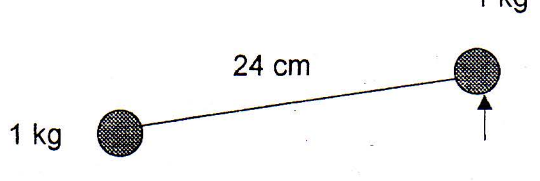
$$set\ b = 4.5\ cm\ ,\ h = 10\ cm$$

Intertia force stress at 1000 rpm

$$= \frac{1000\,\mathrm{x}\,2\,\pi}{60} = 100\ radian\ /\sec$$

$$Weight\ of\ connecting\ rod:$$

$$\gamma\ (24\,x\,10) = 7.8\,x\,240\, gram\ weight = 2\, kg$$



$$\text{Weight of pin} = \frac{\frac{\pi}{4}(5.5)^2 x 9 x 7.8}{1000} = 2.16\,kg$$

Weight of piston = 1 kg

Total weight of reciprocating parts = 1 + 1 + 2 = 4 kg

Inertia force of piston attachments = M ω^2 e [cos θ + e/l cos 2θ]

$$= \frac{4x(100)^2}{10} x 8\left(4 + \frac{8}{24}\right) = 320\,kg \quad at\,\theta = 0$$

Dynamicreactionat crankbearing

$$= \left[\frac{2x7.8x10^{-3}x4.5x10x5(100)^2 x\frac{8}{2}}{10-} \quad + \frac{1}{10}x(100)^2 x8\right] already\ crank\ outwar$$

$$= 8\left[2x7.8x4.5x10x8 + 10^3\right]$$

$$= 8[3808]kg = 30.4\ ton$$

Reduce speed to 100 rpm, inertia force = 0.3 ton

(Static design is safe)

Bearing:

$$w = \frac{0.5x\,1000}{2} \stackrel{?}{=} \frac{12\pi\,\alpha x 10^{-5} x 10 x 2^3}{(0.1)^2} \frac{\in B}{\left(2+\in^2\right)\sqrt{1-\in^2}}$$

$$\in = \frac{e}{C} = 0.5$$

$$\alpha = \frac{0.5\,x1000x(0.1)^2 x10^5}{2x12\pi x10x2^3 x0.25x10} = \frac{10}{48\,\pi\,x2^3\,x5} = 60$$

$$Gives\ \mu = 60x10^{-5}\ kg.\sec/cm^2 \qquad (thick\ grease)$$

COMMENTS

In this brief calculation the main features of the compressor sizing is presented. It gives the crude estimate of the main dimensions and limitations on design and is in no way a final design.

Different materials can be considered and results will be subject to corresponding changes.