INTERNAL DAMPING EFFECT ON THE DYNAMIC STABILITY OF AXIALLY MOVING WEB MODEL

Krzysztof Marynowski
Technical University of Łódź
Department of Machines Dynamics
Stefanowskiego 1/15, 90 - 924 Łódź, Poland
Tel. (42) 6312230, E-mail: kmarynow@ck-sg.p.lodz.pl

Abstract

Dynamic investigations of string-like and beam-like models of axially moving web are carried out in this paper. The string model material as the Bürgers element, and the beam model material as the Kelvin - Voigt element are considered. The results of numerical investigations show the solutions of the linearized and non-linear problems. Numerical results show axially traveling speed and internal damping effects on dynamic stability of axially moving web model.

1. Introduction

Axially moving continua in the form of thin, flat rectangular shape materials with small flexural stiffness, called a web, one can find in industry as band saws blades, power transmission belts, magnetic tapes and paper webs. Excessive vibrations of moving webs increase defects and can lead to failure of the web. The analysis of vibration and dynamic stability of such systems is very important for design of manufacturing devices.

A lot of the earlier works in this field focused on dynamic investigations of string-like and beam-like axially moving isotropic systems (e.g. [3], [4]). In all these works, the web material was taken to be linearly elastic. However, paper webs, new plastics and composite materials webs, which are used in industry need more realistic rheologic models. Many investigators studied linear viscoelastic models. Kovalenko [1] considered the problem of a column of constant stiffness with internal damping linearly proportional to the strain rate. Stevens [2] considered the stability of an initially straight, simply supported column subjected to axial load under the assumption that simple spring-dashpot models might adequately represent column material. Fung et al. [5] studied the transverse vibrations of an axially moving string subjected to initial stress. The string material was considered as the Kelvin - Voigt element in series with a spring.

In this paper the transverse vibrations and dynamic stability of an axially moving viscoelastic string model and beam model of the web are studied. The string model material as the Bürgers element (four-parameter model), and the beam model material as the Kelvin - Voigt element (two-parameter model) are considered. Galerkin's method is used to approximate the mathematical model in the form of nonlinear partial differential equations. Numerical results show axially traveling speed and material parameters effects on dynamic stability of the axially moving web model.

2. Equation of motion

An viscoelastic moving web of the length l is considered. The web moves at axial velocity c. The co-ordinates system and geometry are shown in Fig. 1.
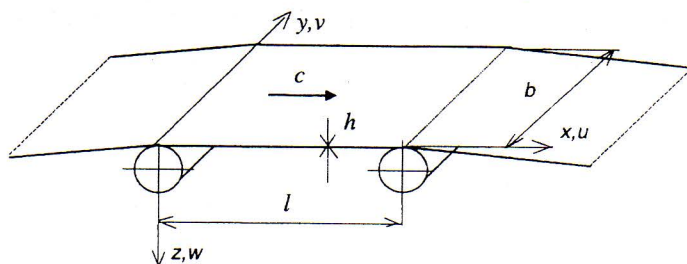

Fig. 1. Axially moving web

The problem of transverse vibrations of the axially moving continua in a state of uniform initial stress was investigated [6]. In the case of thin web, the results of earlier investigations show that the string and beam models can approximate the dynamic behaviour of the web. The equation of the beam model motion in the z direction is

$$\left(\frac{P}{A}+\sigma\right)w_{,xx}+\sigma_{,x}w_{,x}=\rho\left(w_{,tt}+2cw_{,xt}+c^2w_{,xx}-\frac{M_{,xx}}{A}\right) \tag{1}$$

where: A - the cross section area of the beam,

M - bending moment.

σ - perturbated axial stress,

ρ - mass density of the beam,

The equation of the string model motion in the z direction is

$$\left(\frac{P}{A}+\sigma\right)w_{,xx}+\sigma_{,x}w_{,x}=\rho(w_{,tt}+2cw_{,xt}+c^2w_{,xx}) \tag{2}$$

The uniform initial tension force P provides the required initial stress for the models materials. The nonlinear strain component in x direction is related to the displacement w by

$$\varepsilon(x,t)=\frac{1}{2}w_{,x}^2(x,t) \tag{3}$$

3. Differential constitutive equation

The one-dimensional constitutive equation of a differential type material obeys the relation

$$\Gamma\,\sigma=\Xi\,\varepsilon \tag{4}$$

where: Γ and Ξ are differential operators defined as

$$\Gamma=\sum_{j=0}^{R}a_j\frac{d^j}{dt^j}\;;\quad\Xi=\sum_{j=0}^{P}b_j\frac{d^j}{dt^j} \tag{5}$$

In the case of the beam the two-parameter viscoelastic model of material (Kelvin - Voigt element) was taken into account (Fig.2a). The differential constitutive equation of the beam model material can be written as

$$\sigma_x=E\varepsilon_x+\gamma\dot{\varepsilon}_x \tag{6}$$

Bending moment M is given $M=-EJ_z\,w_{,xx}-J_z\gamma\,w_{,xxt}$ (6a)
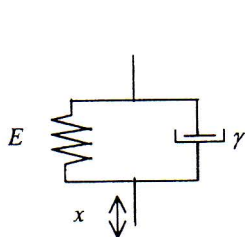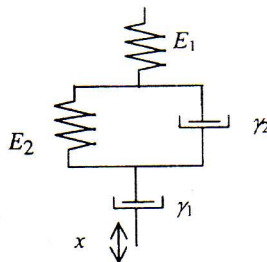
Fig.2a. Kelvin - Voigt element   Fig.2b. Bürgers element

In the case of the string the four-parameter viscoelastic model of material (Bürgers element) was taken into account (Fig.2b). The differential constitutive equation of the string model material can be written as

$$a_2 \ddot{\sigma}_x + a_1 \dot{\sigma}_x + a_0 = b_2 \ddot{\varepsilon}_x + b_1 \dot{\varepsilon}_x \qquad (7)$$

where:

$$a_2 = \gamma_1 \gamma_2 ; \quad a_1 = (E_1 + E_2)\gamma_1 + E_1 \gamma_2 ; \quad a_0 = E_1 E_2 ;$$
$$b_2 = E_1 \gamma_1 \gamma_2 ; \quad b_1 = E_1 E_2 \gamma_1 \qquad (8)$$

4. Mathematical models of the systems

To obtain mathematical description of the viscoelastic beam model one should multiply Eq (1) with operator Γ and using Eq (3) and (4) receives

$$w_{,tt} + 2cw_{,xt} + c^2 w_{,xx} + \frac{EJ_z}{\rho A_z} w_{,xxxx} + \frac{J_z \gamma}{\rho A_z} w_{,xxxxt} - \frac{P_0}{\rho A_z} w_{,xx} + \qquad (9)$$

$$-\frac{3}{2}\frac{E}{\rho} w_{,x}^2 w_{,xx} - 2\frac{\gamma}{\rho}(w_{,x} w_{,xt} w_{,xx} + + c w_{,x} w_{,xx}^2) - \frac{\gamma}{\rho}(w_{,x}^2 w_{,xxt} + c w_{,x}^2 w_{,xxx}) = 0$$

The boundary conditions:

$$w(0,t) = w(l,t) = 0; \qquad w_{,xx}(0,t) = w_{,xx}(l,t) = 0 \qquad (10)$$

Let the dimensionless parameters be

$$z = \frac{w}{h}; \quad \mu = \frac{x}{l}; \quad c_f = \sqrt{\frac{P_0}{A_z \rho}}; \quad s = \frac{c}{c_f} = c\sqrt{\frac{A_z \rho}{P_0}}; \quad \tau = t\frac{c_f}{l} = \frac{t}{l}\sqrt{\frac{P_0}{A_z \rho}} \qquad (11)$$

Substitution of Eq (11) into Eq (9) gives the dimensionless nonlinear equation of the viscoelastic beam model motion

$$z_{,\tau\tau} + 2s\, z_{,\mu\tau} + (s^2 - 1) z_{,\mu\mu} + s\, z_{,\mu} + \varepsilon_z\, z_{,\mu\mu\mu\mu} + b_w z_{,\mu\mu\mu\mu\tau} + \qquad (12)$$

$$-\frac{3}{2}a_g z_{,\mu}^2 z_{,\mu\mu} - a_w s(2 z_{,\mu\mu}^2 z_{,\mu} + z_{,\mu}^2 z_{,\mu\mu\mu}) - a_w(2 z_{,\mu} z_{,\mu\tau} z_{,\mu\mu} + z_{,\mu}^2 z_{,\mu\mu\tau}) = 0$$

where: $\quad b_w = \dfrac{J_z \gamma}{l^3 \sqrt{P_0 \rho A_z}} ; \quad \varepsilon_z = \dfrac{EJ_z}{P_0 l^2} \quad a_g = \dfrac{E h_z^2 A_z}{P_0 l^2} ; \quad a_w = \dfrac{\gamma h_z^2 A_z}{l^3 \sqrt{P_0 \rho A_z}} \qquad (13)$

To obtain mathematical description of the viscoelastic string model one should multiply Eq (2) with operator Γ and using Eq (3) and (4) receives

$$\rho a_0 w_{,tt} + 2\rho a_0 c w_{,xt} + c^2 \rho a_0 w_{,xx} + \rho a_1 w_{,ttt} + 3\rho a_1 c w_{,xtt} + 3\rho a_1 c^2 w_{,xxt} +$$
$$+ \rho a_1 c^3 w_{,xxx} + \rho a_2 w_{,tttt} + 3\rho a_2 c w_{,xttt} + 3\rho a_2 c^2 w_{,xxtt} + \rho a_2 c^3 w_{,xxxt} +$$
$$- \frac{P}{A_z} a_0 w_{,xx} - \frac{P}{A_z} a_1 w_{,xxt} + - \frac{P}{A_z} a_1 c w_{,xxx} - \frac{P}{A_z} a_2 w_{,xxtt} - \frac{P}{A_z} a_2 c w_{,xxxt} = \qquad (14)$$
$$= 2 b_1 w_{,x} w_{,xx} w_{,xt} + 2 b_1 c w_{,xx} w_{,x}^2 + 2 b_2 w_{,x} w_{,xx} w_{,xtt} + 2 b_2 c w_{,x} w_{,xx} w_{,xxt} + b_1 w_{,x}^2 w_{,xxt} +$$
$$+ b_1 c w_{,x}^2 w_{,xxx} + b_2 w_{,x}^2 w_{,xxtt} + b_2 c w_{,x}^2 w_{,xxxt}$$

Substitution of Eq (11) into Eq (14) gives the dimensionless nonlinear equation of the viscoelastic string model motion

$$z_{,\tau\tau\tau\tau} + 3 s z_{,\mu\tau\tau\tau} + (3 s^2 - 1) z_{,\mu\mu\tau\tau} + s(s^2 - 1) z_{,\mu\mu\mu\tau} + g_{1_2} z_{,\tau\tau\tau} + 3 g_1 s z_{,\mu\tau\tau} +$$
$$+ g_1 (3 s^2 - 1) z_{,\mu\mu\tau} + g_2 z_{,\tau\tau} + 2 g_2 s z_{,\mu\tau} + g_2 (s^2 - 1) z_{,\mu\mu} - g_1 (s - 1) z_{,\mu\mu\mu} = \qquad (15)$$
$$= g_3 z_{,\mu} z_{,\mu\mu} z_{,\mu\tau} + 2 g_3 z_{,\mu} z_{,\mu\mu}^2 + g_4 z_{,\mu} z_{,\mu\mu} z_{,\mu\tau\tau} + 2 g_4 s z_{,\mu} z_{,\mu\mu} z_{,\mu\mu\tau} + g_3 z_{,\mu}^2 z_{,\mu\mu\tau} +$$
$$+ g_3 s z_{,\mu}^2 z_{,\mu\mu\mu} + g_4 z_{,\mu}^2 z_{,\mu\mu\tau\tau} + g_4 s z_{,\mu}^2 z_{,\mu\mu\mu\tau}$$

where:

$$g_1 = \left(\frac{E_1 + E_2}{\gamma_2} + \frac{E_1}{\gamma_1} \right) \frac{l\sqrt{A\rho}}{\sqrt{P}}; \quad g_2 = \frac{E_1 E_2 l^2 A\rho}{P\gamma_1\gamma_2}; \quad g_3 = \frac{E_1 E_2 h^2 A\sqrt{A\rho}}{P\sqrt{P}\gamma_2 l^3}; \quad g_4 = \frac{E_1 A h^2}{Pl^2} \quad (16)$$

The problems represented by Eq (12) and Eq (14) together with boundary conditions (10) have been solve using the Galerkin method. The following finite series representation of the dimensionless transverse displacement has been assumed

$$z(\mu, \tau) = \sum_{i=1}^{n} \sin(i\pi\mu) q_i(\tau) \qquad (17)$$

where $q_i(\tau)$ is generalized displacement.

Substituting Eq (17) into Eq (12) or Eq (14) and using orthogonality condition one determines the set of ordinary differential equations. The Runge - Kutta method was used to integrate these equations and analyse the dynamic behaviour of the system.

5. Numerical results

Numerical investigations have been carried out for the beam model of the steal web. Parameters data: length $l = 1$m, width $b = 0.2$ m, thickness h = 0.0015 m, mass density ρ =7800 kg/m^3, Young's modulus along x: $E_x = 0.2 \ 10^{12}$ N/m^2, initial stress, $N_0 = 2500$ N/m.

At first the linearized damped system was investigated. To show dynamic behaviour of the web natural damped vibrations of the first generalized coordinate and phase plots for different values of axial speed of the beam model are shown in Figures 3 ÷ 6. In undercritical region of transport speeds ($c < c_{cr}$) one can observe free flexural damped vibrations around trivial equilibrium position (Fig.3). The critical axial speed value decreases when the internal damping increases. In supercritical transport speeds ($c > c_{cr}$) for small internal damping the web experiences divergent instability (Fig.4) and next flutter instability (Fig.6). Between these two instability regions there is a second stability area. The width of the second stable region is dependent on the internal damping of the web material. When the internal damping increases the width of the second stable region decreases. The time dependence of the first generalized coordinate value at the boundary of the second stable region is shown in Fig.5.

Next the non-linear damped model of the steel web was investigated. Numerical results in the form of the phase and transverse vibrations diagrams in various supercritical transport speed of the web are shown in Fig.7 and Fig.8. Though the analysis of the linearized system

predicts exponentially growing oscillations in divergence instability region of transport speeds, non-linear damped vibrations which tend to new equilibrium position occur (Fig.7).
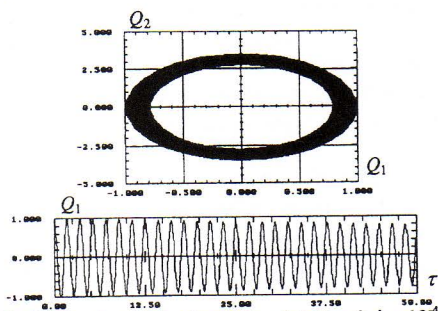

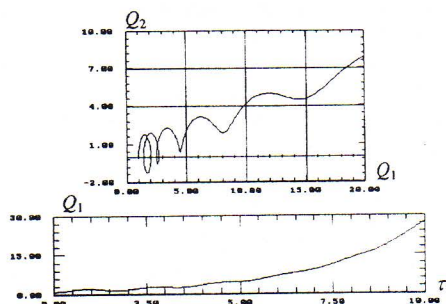Fig.3. The phase and vibrations plots ($s = 0$, $b_w = 10^{-4}$).

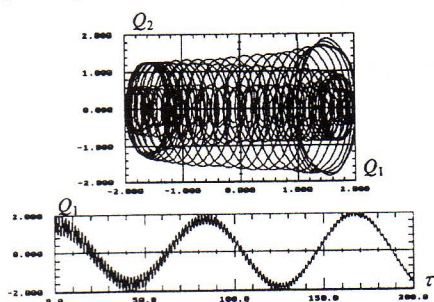Fig.4. The phase and vibrations plots ($s = 1.4$, $b_w = 10^{-4}$)

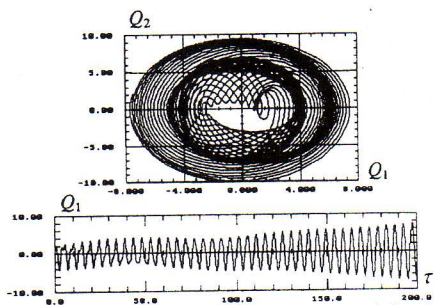Fig.5. The phase and vibr. plots ($s = 1.406$, $b_w = 10^{-5}$).

Fig.6. The phase and vibr. plots ($s = 1.449$, $b_w = 10^{-5}$)

Above the second critical speed of the linearized system the non-linear system experiences global motion between new equilibrium positions (Fig.8). At the same transport speed for different values of internal damping the system may reach various equilibrium positions.
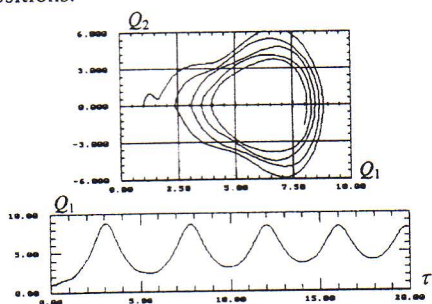

Fig.7. The phase and vibrations plots ($s = 1.3$, $b_w = 10^{-4}$)
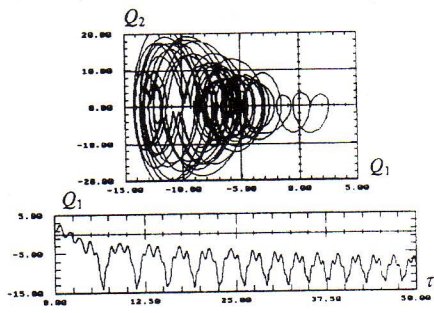

Fig.8. The phase and vibr. plots ($s = 1.54$, $b_w = 10^{-5}$)

6. Conclusions

Dynamic investigations of string-like and beam-like models of axially moving web are carried out in this paper. The string model material as the Bürgers element (four-parameter model), and the beam model material as the Kelvin - Voigt element (two-parameter model) are considered. The general forms of differential equations of transverse vibrations of the systems are derived with the differential constitutive law for their rheologic models.

Numerical investigations have been carried out for the beam model of the steal web. At first the linearized damped model was investigated. Investigations results show that the critical axial speed value decreases when the internal damping increases. In supercritical transport speeds for small internal damping the web experiences divergent instability and flutter instability. Between these two instability regions there is a second stability area. The width of the second stable region is dependent on the internal damping of the web material. When the internal damping increases the width of the second stable region more and more decreases and disappears.

Dynamic analysis of the non-linear damped system with constant axial stress shows in supercritical transport speed region non-trivial equilibrium positions bifurcate from the straight configuration of the web and global motion between coexisting equilibrium positions occurs. At the same transport speed for different values of internal damping the system may reach various equilibrium positions.

Acknowledgement

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