MECHANICS AND MECHANICAL ENGINEERING
International Journal
Vol. 5 No. 2 2001

NONLINEAR STABILITY PROBLEM OF FERROMAGNETIC FLUIDS WITH MASS AND HEAT TRANSFER EFFECT

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Abstract

The nonlinear theory of Kelvin-Helmholtz instability is employed to analyze the instability phenomena of ferromagnetic fluids. The effect of both the magnetic field and the mass and heat transfer at the interface on the instability is investigated. The method of multiple scale expansion is employed for the investigation. It is shown that, for the Rayleigh-Taylor problem, the mass and heat transfer has no effect. In absence of the magnetic field, the system cannot be stabilized by the finite amplitude effects for two semi-infinite fluid layers up to the third-order.

1. Introduction

Ferromagnetic fluids are colloidal dispersions of submicro-sized ferrite particles in a carrier or parent fluid such as kerosene. These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomena. Ferromagnetic fluids are not found in nature but are artificially synthesized. The two main features that distinguish ferrofluids from ordinary fluids are the polarization force and the body couple. In absence of a magnetic field the orientation of ferromagnetic particles is disordered by the thermal agitation, and coating prevents the particles from sticking to each other. Numerous applications for these fluids appear possible, including novel energy-conversion schemes, levitation devices and rotation seals. An authoritative introduction to this fascinating subject is provided in Rosensweig's book [1]. This monograph along with that of Bashtovoy et al. [2] reviews several applications of heat transfer through ferromagnetic fluids.

The magnetization of ferromagnetic fluids is a function of the magnetic field, the density, and the temperature of the fluid. Due to the variation of any of these quantities the force induced gives rise to mass and heat transfer by convection in ferrofluids. Convection can also be induced by surface tension, provided that the latter is a variable [3]. In view of the fact that mass and heat transfer is greatly enhanced due to convection, the magnetic convection problems offer new possibilities for applications in colling with motors, loudspeakers, transmission lines, and other equipments where a magnetic field is already present.

Various investigations of the instability of a ferromagnetic fluid (or simply a magnetic fluid) by linear and weakly nonlinear analysis, in the absence of mass and heat transfer, have been carried out by Singh et al. [4], Elhefnawy [5,6] and El -Dib [7]. They used the method of multiple scales to derive a pair of partial differential equations that describe the evolution of finite-amplitude wave-packets on the interface separating two magnetic fluids. These equations were combined to yield two alternate nonlinear Schrödinger equations and a nonlinear Klein-Gordon equation. The effect of mass and heat transfer on the motion of fluids has been treated extensively by many authors [8-15]. Hsieh [8] presented a formulation to deal with interfacial stability problem taking into account mass and heat transfer and applied it to discuss the Rayleigh-Taylor and Kelvin-Helmholtz instability problems. Hsieh [9] used the method of multiple scale expansion to study the nonlinear Rayleigh-Taylor stability of a liquid layer over a finite vapour layer. It is observed that the nonlinear effects can indeed increase the range of stability of the system when there is strong heat and mass transfer, while this is not the case for linear Rayleigh-Taylor instability [8].

The aim of this work is to study the nonlinear Kelvin-Helmholtz instability in magnetic fluids in the presence of both a tangential magnetic field and mass and heat transfer across the interface. The objective of this presentation is to investigate the nonlinear dynamic stability when the applied magnetic field is greater than the critical value of the field. The nonlinear stability analysis along with the linear instability results thus become necessary in order to find a band of physical parameters where the possible subcritical instabilities arise.

The basic equations governing the formulation of the two-dimensional problem are given in section 2, along the procedure for obtaining linear and successive nonlinear partial differential equations of various orders by means of multiple scales. The linear theory is reviewed in section 3, where the critical magnetic field is obtained. In section 4 we obtain the equation governing the evolution of the amplitude from the second-and third order theories, which leads to a Landau equation with complex coefficients. From the latter equation, the various stability criteria are obtained in section 5.

2. The governing equations

Consider two incompressible, inviscid magnetic fluid layers separated by an interface $z=\eta(x,t)$. We shall use subscripts 1 and 2 to denote variables in these two fluids, which occupy the regions $-h_1<z<\eta$ and $\eta<z<h_2$ respectively (h_1 and h_2 are the thicknesses of the two fluid layers). Let the temperatures at $z=-h_1$, $z=0$, and $z=h_2$ be T_1, T_0 and T_2 respectiveley. The two fluids are streaming with velocities u_1 and u_2 along the positive x direction. The magnetic field H_0 acts along the direction of the flow. Acceleration due to gravity g acts in the negative z direction. The fluid densities are ρ_1 and ρ_2 while μ_1 and μ_2 are the magnetic permeabilities of the two fluids. The motion is considered irrotational having velocity potential ϕ. We also assume that the quasi- static approximation is valid and the magnetic field is a curl-free vector having magnetic potential ψ.

The basic equations governing ϕ and ψ, on each side of the interface, are [16,17] :

$$\nabla^2\phi_1=\nabla^2\psi_1=0 \quad for \quad -h_1<z<\eta(x,t), \tag{2.1}$$

$$\nabla^2\phi_2=\nabla^2\psi_2=0 \quad for \quad \eta(x,t)<z<h_2, \tag{2.2}$$

$$where \ \nabla\equiv\left(\frac{\partial}{\partial x},0,\frac{\partial}{\partial z}\right)$$

The solutions for ϕ_j *and* ψ_j (j=1, 2) must satisfy the following conditions on the rigid boundaries:

$$\frac{\partial\phi_j}{\partial z}=\frac{\partial\psi_j}{\partial x}=0 \quad on \quad z=(-1)^j\, h_j \quad ,j=1,\ 2 \tag{2.3}$$

Also, at the free interface $z=\eta\ (x,t)$ the interfacial conditions are:

$$\frac{\partial\psi_1}{\partial x}+\frac{\partial\eta}{\partial x}\frac{\partial\psi_1}{\partial z}=\{(2)\} \tag{2.4}$$

$$\mu_1\left(\frac{\partial\psi_1}{\partial z}+H_0\frac{\partial\eta}{\partial x}-\frac{\partial\eta}{\partial x}\frac{\partial\psi_1}{\partial z}\right)=\{(2)\} \tag{2.5}$$

$$\rho_1\left(\frac{\partial\varphi_1}{\partial z}-\frac{\partial\eta}{\partial t}-u_1\frac{\partial\eta}{\partial x}-\frac{\partial\eta}{\partial x}\frac{\partial\varphi_1}{\partial x}\right)=\{(2)\} \tag{2.6}$$

$$\rho_1\left[g\eta+\frac{\partial\varphi_1}{\partial t}+u_1\frac{\partial\varphi_1}{\partial x}+\frac{1}{2}(\nabla\varphi_1)^2-\frac{1}{1+\left(\frac{\partial\eta}{\partial x}\right)^2}\left(\frac{\partial\varphi_1}{\partial z}-u_1\frac{\partial\eta}{\partial x}-\frac{\partial\eta}{\partial x}\frac{\partial\varphi_1}{\partial x}\right)\left(\frac{\partial\varphi_1}{\partial z}\right.\right.$$

$$\left.\left.-\frac{\partial\eta}{\partial t}-u_1\frac{\partial\eta}{\partial x}-\frac{\partial\eta}{\partial x}\frac{\partial\varphi_1}{\partial x}\right)\right]+\mu_1\left[H_0\frac{\partial\psi_1}{\partial x}+\frac{1}{2}\left(\frac{\partial\psi_1}{\partial z}\right)^2-\frac{1}{2}\left(\frac{\partial\psi_1}{\partial x}\right)^2\right.$$

$$\left.+2H_0\frac{\partial\eta}{\partial x}\frac{\partial\psi_1}{\partial z}-2\frac{\partial\eta}{\partial x}\frac{\partial\psi_1}{\partial x}\frac{\partial\psi_1}{\partial z}+H_0^2\left(\frac{\partial\eta}{\partial x}\right)^2-2H_0\left(\frac{\partial\eta}{\partial x}\right)^2\frac{\partial\psi_1}{\partial x}\right]=$$

$$\{(2)\}+\sigma\frac{\partial^2\eta}{\partial x^2}\left[1+\left(\frac{\partial\eta}{\partial x}\right)^2\right]^{-3/2} \qquad (2.7)$$

$$\rho_1\left(\frac{\partial\varphi_1}{\partial z}-\frac{\partial\eta}{\partial t}-u_1\frac{\partial\eta}{\partial x}-\frac{\partial\eta}{\partial x}\frac{\partial\varphi_1}{\partial x}\right)=\alpha(\eta+\alpha_2\eta^2+\alpha_3\eta^3), \qquad (2.8)$$

where the notation $\{(2)\}$ on the right-hand sides of these equations is used to denote the same expression as that on the left, except for changing the subscript 1 to 2. The coefficient of surface tension is denoted by σ. It is to be noted that according to the quasi-equilibrium approximation, the coefficients α, α_2, *and* α_3 are given by [9]

$$\alpha=\frac{G}{L}\left(\frac{1}{h_1}+\frac{1}{h_2}\right),\ \alpha_2=\frac{1}{h_2}-\frac{1}{h_1}\ \ \alpha_3=\frac{h_1^3+h_2^3}{h_1^2h_2^2(h_1+h_2)}, \qquad (2.9)$$

where G is the equilibrium heat flux and L is the latent heat of transformation from the fluid of density ρ_1 to the fluid of density ρ_2. The coefficients α, α_2, and α_3 are all of order O(1).

We briefly explain the meaning of equations (2.4) - (2.8). Equations (2.4) and (2.5) express the continuity of the tangential component of the magnetic field and the normal component of the magnetic induction vector across the interface, respectively [1]. Equations (2.6), (2.7) and (2.8) express the conservation of mass, momentum and energy, respectively, by taking into account mass transfer across the interface [9]. In deriving condition (2.7), we have used the expression for the Maxwell stress tensor which includes the effects of nonlinear relation between magnetization and the magnetic induction.

Equations (2.1), (2.2) and conditions (2.3) - (2.8) consitute the governing equations of the problem. These equations can be solved by using the multiple scale expansion method {Nayfeh [15]} to investigate the nonlinear effect on the stability of the system.

3. The expansion near the critical magnetic field

Introducing $\in$ as a small paprameter, we assume the following expansion of the variables:

$$\eta(x,t) = \sum_{n=1}^{3} \in^n \eta_n(x,t_0,t_1,t_2) + O(\in^4), \tag{3.1}$$

$$\varphi_j\,(x,z,t) = \sum_{n=1}^{3} \in^n \varphi_{jn}(x,z,t_0,t_1,t_2) + O(\in^4), \tag{3.2}$$

and

$$\psi_j\,(x,z,t) = \sum_{n=1}^{3} \in^n \psi_{jn}(x,z,t_0,t_1,t_2) + O(\in^4), \tag{3.3}$$

where

$$t_n = \in^n t \quad , \qquad (n=0,1,2) \tag{3.4}$$

and

$$\eta_1 = A(t_1,t_2)\exp[i(kx-\omega t_0)] + c.c \tag{3.5}$$

In the latter equation (3.5), we have used the notation *c.c.* to denote the complex conjugate of the preceding terms, ω to the frequency and k to the wave number.

To evaluate boundary-conditions (2.3)-(2.8), we use the Maclaurin series expansions at $z=0$ for the quantities involved. Then, on substituting equations (3.1)-(3.4) into equations (2.1)-(2.8) and equating the coefficients of like powers in $\in$, we obtain the linear as well as successive higher - order perturbation equations. The hierarchy of the equations for any order can be solved with the knowledge of the solutions of the previous orders. The procedure is straightforward but lengthy and will not be included here. The details are available from the author and is outlined by Refs. [9,13].

For $n=1$ (the first-order problem), the frequency ω and the wave number k satisfy the dispersion relation

$$a_0\omega^2 + (a_1 + ib_1)\omega + a_2 + ib_2 = 0 \tag{3.6}$$

where

$$a_0 = \rho_1 \coth kh_1 + \rho_2 \coth kh_2$$
$$a_1 = -2k(\rho_1 u_1 \coth kh_1 + \rho_2 u_2 \coth kh_2)$$
$$b_1 = \alpha(\coth kh_1 + \coth kh_2)$$
$$a_2 = gk(\rho_2 - \rho_1) + k^2(\rho_1 u^2 \coth kh_1 + \rho_2 u_2^2 \coth kh_2) -$$
$$-k^2 H_0^2(\mu_2 - \mu_1)^2(\mu_1 \coth kh_1 + \mu_2 \coth kh_2)^{-1} - \sigma k^3$$
$$b_2 = -\alpha k(u_1 \coth kh_1 + u_2 \coth kh_2)$$

The study of the properties of the roots of equation (3.6) can determine the stability and the instability conditions of the given problem.

From the Routh-Hurwitz criterion, we can write the condition that a root ω of (3.6) will have a positive imaginary part as

$$a_0 b_2^2 - a_1 b_1 b_2 + a_2 b_1^2 > 0 \tag{3.7}$$

When this condition is satisfied we have instability. When $H_0 = H_C$, the critical magnetic field at which instability first sets in, we have

$$a_0 b_2^2 - a_1 b_1 b_2 + a_2 b_1^2 = 0 \tag{3.8}$$

Putting the values of a_0, a_1, a_2, b_1 *and* b_2 from (3.6) into (3.8) we can easily show that

$$H_C^2 = H_*^2 + \frac{(\rho_1 - \rho_2)^2\ (u_1 - u_2)^2\ (\mu_1 \coth kh_1 + \mu_2 \coth kh_2)}{(\mu_1 - \mu_2)^2\ (\rho_1 \coth kh_1 + \rho_2 \coth kh_2)\ (\tanh kh_1 + \tanh kh_2)^2} \tag{3.9}$$

where (H_* is the critical magnetic field in absence of mass and heat transfer)

$$H_*^2 = \left[g(\rho_2 - \rho_1) - \sigma k^2 + k\rho_1\rho_2(u_1 - u_2)^2(\rho_1 \tanh kh_2 + \rho_2 \tanh kh_1)^{-1}\right]\times$$
$$\times(\mu_1 \coth kh_1 + \mu_2 \coth kh_2)/[k(\mu_1 - \mu_2)^2] \tag{3.10}$$

It is clear that the equation (3.9) is independent of α (α is the coefficient of mass and heat transfer). Therefore, the critical magnetic field in the presence of mass and heat transfer H_C is independent of mass and heat transfer coefficient. But it differs from that of the critical field without mass and heat transfer H_* by the additional last term in (3.9).

On the other hand, we notice that the last term in (3.9) is always positive. It means that the stable regions will be decreased as in figures (1). Therefore , however α is small or large , the mass and heat transfer has a linearly destabilizing influence on the Kelvin- Helmholtz problem.

When $u_1 = u_2$ (the Rayleigh-Taylor problem) or $\rho_1 = \rho_2$ we find that the last term in (3.9) disappears. It means that $H_C = H_*$ and so that the mass and heat transfer has no effect.

4. The evolution equation and stability analysis

It is obvious from (3.6)-(3.10) that the system is stable for all $H_0 > H_C$. When the nonlinear effects are included, it is expected that the stability characteristics around the critical magnetic field H_C will be changed. However, we shall assume that the critical magnetic field, because of the nonlinear effects, will shift to

$$H_0 = H_C + \epsilon^2 \Delta , \tag{4.1}$$

with $\Delta = O(1)$

Thus the shift of the critical magnetic field is of order $O(\epsilon^2)$.

The solutions of the second-order problem have nonhomogeneous expressions in terms of the first-order solutions. In order to avoid the inconsistency that η_2 / η_1 may become unbounded as x goes to infinity, we obtain

$$\frac{\partial A}{\partial t_1} = 0 \tag{4.2}$$

which implies that the amplitude A is independent of the faster variable t_1.

Proceeding to the third-order problem, we obtain after some straight forward reductions the evolution equation for the amplitude.

$$\begin{aligned}&\left[\frac{\alpha}{k}(\coth kh_1 + \coth kh_2) + \frac{2i(\rho_1 - \rho_2)(u_1 - u_2)}{(\tanh kh_1 + \tanh kh_2)}\right]\frac{\partial A}{\partial t_2}\\ &+ \frac{2\Delta k H_C (\mu_1 - \mu_2)^2}{(\mu_2 \coth kh_2 + \mu_1 \coth kh_1)} A + (\nu - \frac{3}{2}\sigma k^4)|A|^2 A = 0\end{aligned} \tag{4.3}$$

where

$$\begin{aligned}
\nu = Nk\{&2k^2[\rho_1(u_0-u_1)^2(\coth^2 kh_1+\coth kh_1\coth 2kh_1-1)\\
&-\rho_2(u_0-u_2)^2(\coth^2 kh_2+\coth kh_2\coth 2kh_2-1)]+\\
&\alpha^2[\frac{1}{\rho_1}(\coth kh_1\coth 2kh_1-1)-\frac{1}{\rho_2}(\coth kh_2\coth 2kh_2-1)]\\
&+i\alpha k[2(u_1-u_2)-(u_0-u_1)\coth kh_1(\coth kh_1+2\alpha_2/k)\\
&+(u_0-u_2)\coth kh_2(\coth kh_2-2\alpha_2/k)]+\delta_1 H_c^2k^2\}+\\
&+2k^3[\rho_1\coth kh_1(u_0-u_1)^2(1-\mathrm{cosech}^2 kh_1)+\rho_2\coth kh_2(u_0-u_2)^2\\
&(1-\mathrm{cosech}^2 kh_2)]-\alpha^2\{2k[\frac{1}{\rho_1}\coth kh_1(\coth kh_1\coth 2kh_1-1)+\\
&\frac{1}{\rho_2}\coth kh_2(\coth kh_2\coth 2kh_2-1)]-\alpha_2[\frac{1}{\rho_1}(\coth kh_1\coth 2kh_1-1)\\
&-\frac{1}{\rho_2}(\coth kh_2\coth 2kh_2-1)]\}-2\delta_2 H_c^2k^3
\end{aligned}\tag{4.4}$$

with

$$\begin{aligned}
N = \{\ &k^2[\rho_1(u_0-u_1)^2(\coth^2 kh_1+\coth kh_1\coth 2kh_1-1)-\\
&-\rho_2(u_0-u_2)^2(\coth^2 kh_2+\coth kh_2\coth 2kh_1-1)]\\
&+\frac{1}{2}\alpha^2[\frac{1}{\rho_1}(1+\coth^2 kh_1)-\frac{1}{\rho_2}(1+\coth^2 kh_2)]\\
&2i\alpha k[(u_0-u_1)\coth 2kh_1(2\coth kh_1-\tanh 2kh_1-\alpha_2/2k)\\
&-(u_0-u_2)\coth 2kh_2(2\coth kh_2-\tanh 2kh_2+\alpha_2/2k)]+\delta_1 H_c^2k^2/2\ \}/\\
&\{g(\rho_1-\rho_2)+4\sigma k^2+2kH_c^2\delta_0(2k)-2k[\rho_1\coth 2kh_1(u_0-u_1)^2+\rho_2\coth 2kh_2(u_0-u_2)^2]\\
&-i\alpha[\coth 2kh_1(u_0-u_1)+\coth 2kh_2(u_0-u_2)]\ \}
\end{aligned}\tag{4.5}$$

$$\begin{aligned}
\delta_1 = &4(\mu_1-\mu_2)+2\delta_0(k)[2\delta_0(2k)+\delta_0(k)/k]/(\mu_2-\mu_1)+\\
&+\delta_0^2(k)(\mu_2\coth^2 kh_2-\mu_1\coth^2 kh_1)/(\mu_2-\mu_1)^2+\\
&+4\mu_1\mu_2\delta_0(k)\delta_0(2k)(\coth kh_1+\coth kh_2)(\coth 2kh_1+\coth 2kh_2)/(\mu_2-\mu_1)^3
\end{aligned}\tag{4.6}$$

$$\delta_2 = 2\delta_0(k) - 2\delta_0^2(k)\delta_0(2k)/(\mu_2-\mu_1)^2 + \delta_0^2(k)\delta_0(2k)$$
$$(\mu_2 \coth kh_2 \coth 2kh_2 - \mu_1 \coth kh_1 \coth 2kh_1)/(\mu_2-\mu_1)^3 +$$
$$\mu_1\mu_2\delta_0(2k)\delta_0^2(k)(\coth kh_1 + \coth 2kh_1)[\coth 2kh_1 + \coth 2kh_2) -$$
$$2\coth 2kh_2 \coth 2kh_1(\coth kh_1 + \coth kh_2)]/(\mu_2-\mu_1)^4 \tag{4.7}$$

$$\delta_0(k) = (\mu_2-\mu_1)^2/(\mu_2 \coth kh_2 + \mu_1 \coth kh_1) \tag{4.8}$$

$$u_0 = (u_1 \coth kh_1 + u_2 \coth kh_2)/(\coth kh_1 + \coth kh_2) \tag{4.9}$$

Equation (4.3) can be rewritten in the form [16]

$$\frac{dA}{dt_2} + (P_r + iP_i)A + (Q_r + iQ_i)|A|^2 A = 0 \tag{4.10}$$

where P_r, P_i, Q_r, *and* Q_i can be easily obtained from (4.3)-(4.9)

Equation (4.10) is the well-known Landau equation[18,19]. In the special case $u_1 = u_2 = 0$, equation (4.10) becomes

$$\frac{dA}{dt_2} + (a_1 + a_2|A|^2)A = 0 \tag{4.11}$$

where

$$a_1 = 2\Delta\, k^2 H_C(\mu_2-\mu_1)^2/[\alpha(\coth kh_1 + \coth kh_2)(\mu_1 \coth kh_1 + \mu_2 \coth kh_2)]$$

$$a_2 = k(\nu - 1.5\sigma k^4)/[\alpha(\coth kh_1 + \coth kh_2)],$$

with

$$\nu = Nk\{\ \alpha^2[\frac{1}{\rho_1}(\coth kh_1 \coth 2kh_1 - 1) - \frac{1}{\rho_2}(\coth kh_2 \coth 2kh_2 - 1)]$$
$$+\delta_1 H_C^2 k^2\ \} - \alpha^2[2k[\frac{1}{\rho_1}\coth kh_1(\coth kh_1 \coth 2kh_1 - 1) +$$
$$+\frac{1}{\rho_2}\coth kh_2(\coth kh_2 \coth 2kh_2 - 1)] - \alpha_2\{\frac{1}{\rho_1}(\coth kh_1 \coth 2kh_1 - 1)$$
$$-\frac{1}{\rho_2}(\coth kh_2 \coth 2kh_2 - 1)]\ \} - 2\delta_2 H_C^2 k^3,$$

$$N = \frac{1}{2}\{\ \alpha^2[\frac{1}{\rho_1}(1+\coth^2 kh_1) - \frac{1}{\rho_2}(1+\coth^2 kh_2)] + \delta_1 H_C^2 k^2\ \}/$$
$$[g(\rho_1-\rho_2) + 4\sigma k^2 + 2kH_C^2\delta_0(2k)].$$

There is no loss of generality in treating A as real in (4.11) since the phase associated with A remains constant. Thus, denoting $A\ (0)$ by A_0 ,we obtain

$$A^2(t_2) = a_1 A_0^2 \exp(-2a_1 t)[a_1 + a_2 A_0^2 - a_2 A_0^2 \exp(-2a_1 t)]^{-1}. \qquad (4.12)$$

A is thus asymptotically bounded except when the denominator in the last equation vanishes.The stability situation can be summarized as follows:

(*I*) $a_2 > 0$:*stable*

(*i*) $a_1 > 0:\ A^2 \to 0,\ \ as \quad t_2 \to \infty$

(*ii*) $a_1 < 0: A^2 \to -a_1/a_2 \quad , \quad as \ \ t_2 \to \infty$

(*II*) $a_2 < 0$:

(*i*) $a_1 < 0$: *unstable*

(*ii*)$a_1 > 0, and\ A_0^2 > -(a_1/a_2)$:*unstable*

(*iii*)$a_1 > 0, and\ A_0^2, < -(a_1/a_2)$:*unstable and* $A^2 \to 0\ as\ t_2 \to \infty$.

When $a_2 = 0$, the system is stable if $a_1 > 0$ and vice versa. This is the result which can be obtained from linear stability analysis. With nonlinear effects. the controlling factor shifts from a_1 *to* a_2 and a sufficient condition for stability is $a_2 > 0$. When $a_2 < 0$ even if $a_1 > 0$, the system can be unstable if the initial amplitude is large enough. The stability condition $a_2 > 0$ is equivalent to

$$\nu > \frac{3}{2}\sigma k^4 \qquad (4.13)$$

When the system is linearly unstable, i.e. ,when $\Delta < 0$,the asymptotic value of A for larger times will be given by

$$|A|^2 = -2\Delta\, k\, H_C(\mu_2-\mu_1)^2/[\nu - \frac{3}{2}\sigma k^4)(\mu_1 \coth kh_1 + \mu_2 \coth kh_2)] \qquad (4.14)$$

Therefore, the range of spectra of the stable wave length is enlarged by the finite amplitude effect. From practical considerations, we would expect that as the amplitude exceeds half the wave length, or the thickness of the fluid layer, then there is a tendency for bubbless to form and detach from the interface, or to cause the rupture of the fluid layers. Thus, more stringent practical stability criteria (4.13) are required. These means that the quantity ν plays a critical role in the nonlinear stability of the problem. Now, we shall consider some limiting cases. For problems relating to boiling heat transfer, we shall take $\rho_1 << \rho_2$. Therefore

$$\nu = \alpha^2 \kappa (\coth kh_1 \coth 2kh_1 - 1) (N - 2 \coth kh_1 \cdot \alpha_2/ \kappa)/ \rho_1 + H_C^2 k^3 (N \delta_1 - 2\delta_2) , \tag{4.15}$$

where

$$N = [\alpha^2 (1 - \coth^2 kh_1) / \rho_1 + H_C^2 k^2 \delta_1] / [6\sigma k^2 + 2kH_C^2(2\delta_o (2k) - \delta_o (k))] \tag{4.16}$$

Some simplification can be achieved , when (kh_1) and (kh_2) become extremely large or small . Let us consider two special cases:

(i) $kh_1.>>1$ and $kh_2>>1$. This is the case of two semi-infinite fluid layers we have

$$\nu = H_C^2k^3 [N (\mu_2 - \mu_1) - 2(\mu_2 + \mu_1)] (\mu_2 - \mu_1)^2 /(\mu_2 + \mu_1)^2, \tag{4.17}$$

where

$$N = [2\alpha^2 /\rho_1 + H_C^2 k^2(\mu_2 - \mu_1)^3 /(\mu_2 - \mu_1)^2] [6\sigma k^2 - 2kH_C^2 (\mu_2 - \mu_1)^2 /(\mu_2 + \mu_1)]. \tag{4.18}$$

(ii) $kh_1<< 1$ and $kh_2>> 1$ (i.e. $\coth kh_1 \simeq 1/kh_1$ and $\coth kh_2 \simeq 1$). This is the case of a heap liquid layer on top of a thin vapor layers. It is a fairly realistic approximation of the real situation .we obtain

$$\nu = \alpha^2 \kappa (1/2 k^2h^2_1 - 1) (N - 2 / kh_1 + \alpha_2/ \kappa)/ \rho_1 + H_C^2 k^3 (N \ddot{\delta}_1 - 2\ddot{\delta}_2) , \tag{4.19}$$

where

$$N = [\alpha^2 (1 + 1/ k^2h^2_1) / \rho_1 + H_C^2 k^2 \tilde{\delta}_1] / [6\sigma k^2 + 2kH_C^2(2\tilde{\delta}_o (2k) - \tilde{\delta}_o (k))] , \tag{4.20}$$

$$\tilde{\delta} i = \delta i \text{ at } kh_1 << 1 \text{ and } kh_2 >> 1.$$

Thus we show that thinner the vapor layer, easier it is to make the system stable, while with the same heat flux it is almost impossible to stabilize the system for large values of kh_1. These results appear to be in conformity with the physical fact that the effect of heat and mass transfer will

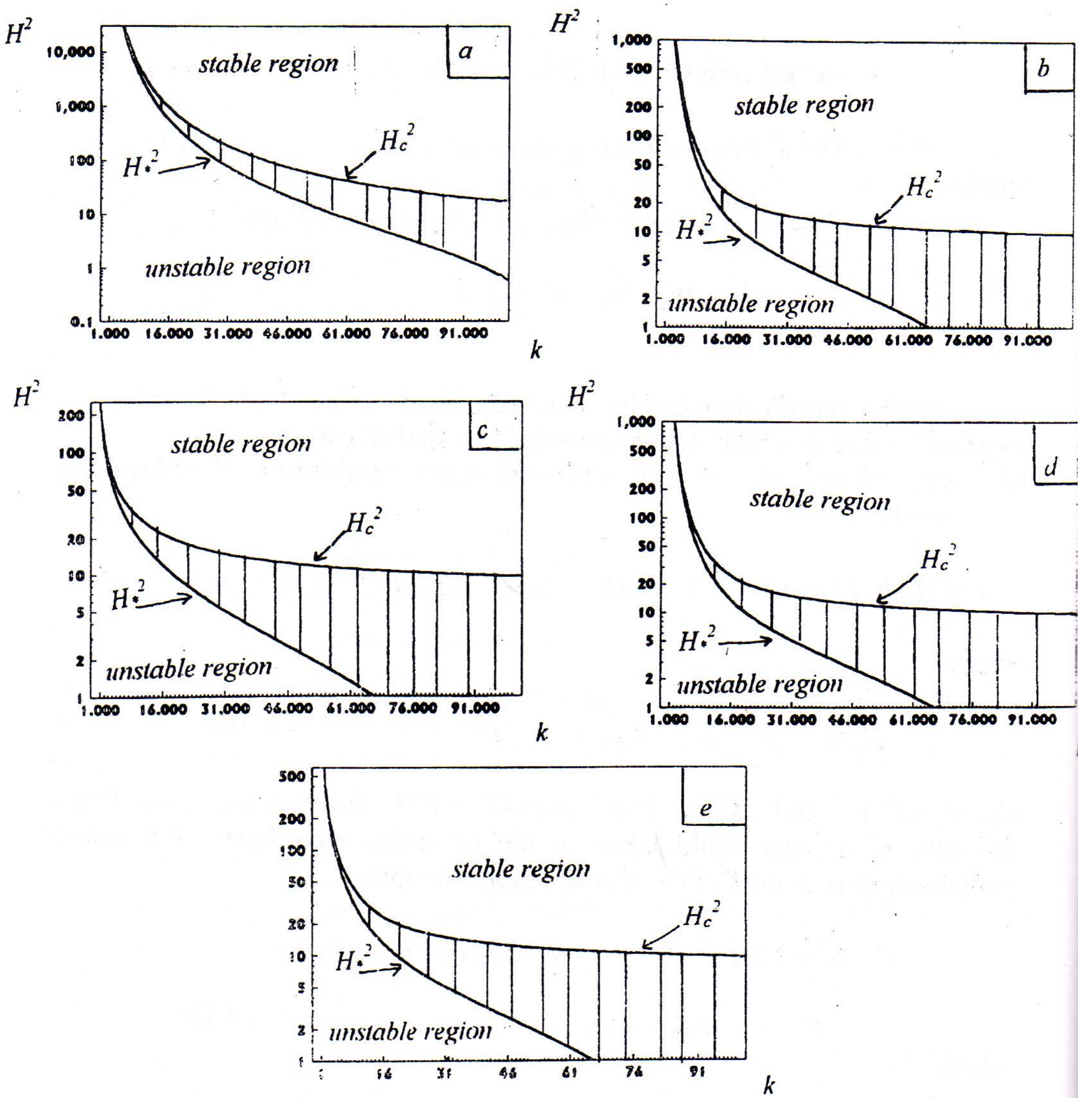

Figure 1: Stability diagram on the H^2-k Plane according to equations (3.9) and (3.10) for a system having ρ_1=0.99709 gm. /cm^3., ρ_2 = 0.66472 gm./cm^3., u_1= 110.0 cm./sec., u_2= 10.0 cm/sec., μ_1 =34.82, μ_2=78.54, σ = 43.0 dyn/cm. g = 981.0 cm/sec^2., and for different values of h_1 and h_2: [a] refers to h_1= 0.01 cm., h_2=0.01 cm., [b] to h_1 = 0.1 cm., h_2=0.1 cm., [c] to h_1 = 2.0 cm., h_2=2.0 cm. ,[d] to h_1= 2.0 cm., h_2=0.1 cm ., and [e] to h_1= 0.1 cm., h_2=2.0 cm . The shaded region is a newly formed unstable region due to the effect of mass and heat transfer across the interface.

clearly be more pronounced for layers of small thickness; this effect tends to become negligible for layers of very large thickness.

5. Conclusions

We have investigated the evolution of the amplitude of the progressive waves in superposed magnetic streaming fluids with mass and heat transfer across the interface is governed by two partial differential equations , based on the method of multiple scales. These equations were combined to yield the Landau equation. It is used to investigate the necessary conditions for stability and instability . we have shown that the mass and heat transfer coefficient plays an important role in the nonlinear stability of the system. For two semi infinite fluid layers, we have also shown that ν depends only on the magnetic field . Thus , in the absence of the magnetic field , the system cannot be stabilized by the finite amplitude effects up to this order .

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