MECHANICS AND MECHANICAL ENGINEERING
International Journal
Vol. 5 No. 2 2001

HALL EFFECT ON MAGNETOHYDRODYNAMIC VISCOELASTIC FREE CONVECTION FLOW WITH MASS TRANSFER THROUGH A POROUS MEDIUM NEAR AN INFINITE VERTICAL POROUS PLATE

Nabil T. M. El-dabe, A. Y. Ghaly, G. Saddeek and M. A. Hassan.
Math. Department, Faculty of Education, Ain Shams University,
Heliopolis, Cairo, Egypt

Abstract

The present study is devoted to investigate the influences of hall current on unsteady free convection flow of magnetohydrodynamic non-Newtonian viscoelastic incompressible fluid with mass transfer over an infinite vertical porous plate.

The system is stressed by uniform magnetic field acting in a plane, which makes an angle α with the plane transverse to the plate over an infinite vertical porous plate. The Walter's model is used to characterize the non-Newtonian fluid behavior. Similarity solution for the transformed governing equations is obtained with prescribed variable suction velocity. Numerical results for the details of the velocity, temperature and concentration profiles are shown on graphs. Excess surface temperature as well as concentration gradient at the wall have been presented for different values of the elasticity parameter n_0, magnetic parameter M, Schmidt number S_c, Grashof number G_r, modified Grashof number G_c, Hall parameter m, Dufour number D_f, Soret number S_r and permeability parameter k^*.

Introduction:

Non-Newtonian fluids are of increasing importance in modern technology. This was probably caused by the growing use of non-Newtonian fluids in many activities such as molten plastics, paints, drilling of petroleum, and polymer solutions.

The boundary layer concept in the theory of non-Newtonian fluids is relevant to a number of engineering activities, among which may be cited the possibility of reducing frictional drag on bearings and on immersed bodies such as ship hulls and submarines.

The range of free-convection flows that can occur in nature and in engineering practice is very large and has been extensively considered by Jaluria [1]. On the other hand, many flows are subjected to a combination of free and forced convection and are known as combined free-forced convection flows. Heated jets or diffusion flames created by blowing combustible gas from a vertical pipe are controlled by forced convection in the initial region and by buoyancy forces far from the jet or pipe exist. Industrial smokes takes usually have a significant imposed momentum flux to assist the initial rise of the contaminant plume [2]. The simplest physical model of such a flow is the two-dimensional laminar flow along a vertical flat plate and extensive studied have been conducted on this type of flow [3-8]. Recent results of application of this model can be found in the area of reactor safety, combustion flames and solar collectors, as well as building energy conservation [9].

Because of the application to nuclear reactors or in the study of the magnetohydrodynamic (MHD) properties of stars and planets, considerable interest has grown up regarding the effect of an applied magnetic filed on unsteady free convection flow along a vertical surface [10], and when the strength of the magnetic filed is very strong, there appears the well known phenomena of the hall effect, which has important engineering applications to flows in channels and ducts and to problems of MHD generators and hall accelerators. In recent years, the effect of hall currents on hydromagnetic Newtonian flow along a vertical surface and in the presence of a transverse magnetic filed with or without heat and mass transfer have been studied by a number of authors [11-13]. The above type of flow through a porous medium has, however, been studied by, among others [14-15].

Hence, the objective of this work is to investigate the thermal diffusion and diffusion thermal effects as well as the hall effect on the unsteady MHD non-Newtonian viscoelastic free convection flow with mass transfer through a porous medium, along an infinite vertical porous plate.

Formulation of the problem and similarity analysis:

We consider an unsteady free convection and mass transfer of a viscoelastic incompressible and electrically conducting fluid, through a porous medium, over an infinite vertical porous plate subjected to time dependent suction velocity. The flow is assumed to be in the x-direction, which is taken along the vertical plate in the upward direction, and the y-axis is taken to be normal to the plate. The surface of the plate is maintained a uniform constant temperature T_w and a uniform constant concentration c_w of a foreign fluid, which are higher than the corresponding values T_0 and c_0, respectively, sufficiently far away from the surface. Here, the fluid is permeated by a strong magnetic filed

$\bar{B}$ such that $\bar{B}=\left(0,\lambda\, B_0,\sqrt{1-\lambda^2 B_0}\right)$, where $\lambda(=Cos\,\alpha)$ is the angle made by $\bar{B}$ with the normal to the plate. In addition we shall neglect the induced magnetic field which is possible for a very small magnetic Reynolds umber. Under these assumptions the unsteady laminar free convection flow and mass transfer with hall current of a non-Newtonian fluid, is described by the following equations (1)-(5) and boundary conditions (6).

The continuity equation:

$$\frac{\partial v}{\partial y}=0\ , \tag{1}$$

The momentum equations:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=g\beta(T-T_0)+g\beta^{*}(c-c_0)-\frac{\upsilon}{k}u+\upsilon\frac{\partial^2 u}{\partial y^2}$$
$$-\frac{\eta_0}{\rho}\left(\frac{\partial^3 u}{\partial t\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right)-\frac{\sigma\mu_e^2 B_0^2\lambda}{\rho\left(1+m^2\lambda^2\right)}(u+m\lambda w), \tag{2}$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}=\upsilon\frac{\partial^2 w}{\partial y^2}-\frac{\eta_0}{\rho}\left(\frac{\partial^3 w}{\partial t\partial y^2}+v\frac{\partial^3 w}{\partial y^3}\right)+\frac{\sigma\mu_e^2 B_0^2\lambda}{\rho\left(1+m^2\lambda^2\right)}(m\lambda\, u-w), \tag{3}$$

The energy equation:

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{k_0}{\rho\, c_p}\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s\, c_p}\frac{\partial^2 c}{\partial y^2}, \tag{4}$$

The diffusion equation:

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial y}=D_m\frac{\partial^2 c}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

$$\left.\begin{array}{llll} u=0, \quad w=0, \quad \dfrac{\partial T}{\partial y}=-\dfrac{q}{k_0}, \quad c=c_w & at & y=0 \\ u\to 0, \quad w\to 0, \quad T\to T_0, \quad c\to c_0 & as & y\to\infty \end{array}\right\}, \tag{6}$$

In the above equation u, w are the barycentric fluid velocity components along and perpendicular to the surface of the flat plate, respectively, v is the suction velocity, T and c are the temperature and concentration, respectively, of the fluid, υ is the kinematic viscosity, k is the permeability of the porous medium, η_0 is the elasticity coefficient, ρ is the density of the fluid, σ is the electric conductivity, μ_e is the magnetic permeability, k_0 is the thermal conductivity, c_p is the specific heat at constant pressure, β is the volumetric coefficient of thermal expansion, β^{*} is the volumetric coefficient of expansion with concentration, g is the acceleration due to gravity, D_m is the coefficient of mass diffusivity, k_T is the thermal diffusion ratio, T_m is the mean fluid temperature, c_s is the concentration susceptibility and $m(=w_e\tau_e)$ is the hall cyclotron frequency of an electron and the electron collision time, respectively. The dimensionless variables chosen are:

$$\left.\begin{aligned} &\eta = y/h, \quad u = u_0 f(\eta), \quad w = u_0 G(\eta), \\ &T = T_0 + \left(\frac{qh}{k_0}\right)\theta(\eta), \quad c = c_0 + (c_w - c_0)\phi(\eta) \end{aligned}\right\}, \tag{7}$$

where, $h\left(=2\sqrt{\upsilon\, t}\right)$ is a length scale [13] and u_0 is the free steam velocity.

In terms of $h(t)$, a convenient solution of continuity equation (1) can be given as:

$$v = -v_0(\upsilon/h), \tag{8}$$

where v_0 is a non-dimensional transpiration parameter.

Introducing (7) and (8) into equations (2)-(6) results in

$$\begin{aligned} &n_0 v_0 f''' + f'' + v_0 f' - \frac{f}{k^*} + 2[n_0\eta f''' + 2n_0 f'' + \eta f'] \\ &\qquad = -G_r\theta - G_c\phi + \frac{M\lambda}{1+m^2\lambda^2}(f + m\lambda G), \end{aligned} \tag{9}$$

$$\begin{aligned} &n_0 v_0 G''' + G'' + v_0 G' - \frac{f}{k^*} + 2[n_0\eta G''' + 2n_0 G'' + \eta G'] \\ &\qquad = -\frac{M\lambda}{1+m^2\lambda^2}(m\lambda f - G), \end{aligned} \tag{10}$$

$$\theta'' + P_r[2\eta + v_0]\theta' = -P_r D_f \phi'', \tag{11}$$

$$\phi'' + S_c[2\eta + v_0]\phi' = -S_c S_r \theta'', \tag{12}$$

$$\left.\begin{aligned} &f = 0, \quad G = 0, \quad \theta' = -1, \quad \phi = 1, \quad \eta = 0 \\ &f \to 0, \quad G \to 0, \quad \theta \to 0, \quad \phi \to 0, \quad \eta \to \infty \end{aligned}\right\}, \tag{13}$$

where primes denote partial differentiation with respect to variable η and the dimensionless parameter are defined as:

$n_0 = \dfrac{\eta_0}{\rho h^2}$ is the elastic parameter,

$D_f = \dfrac{D_m k_T (c_w - c_0) k_0 u_0}{c_s c_p q}$ is the Dufour number,

$S_r = \dfrac{D_m k_T q}{\upsilon^2 (c_w - c_0) k_0 u_0 T_m}$ is the Soret number,

$G_r = \dfrac{g\beta q h^2}{\upsilon^2 k_0 u_0^2}$ is the Grashof number,

$G_c = \dfrac{g\beta^* (c_w - c_0) h^2}{\upsilon\, u_0}$ is the modified Grashof number,

$M = \dfrac{\sigma\mu_e^2 B_0^2 h^2}{\rho\upsilon}$ is the magnetic parameter,

$k^* = \dfrac{k}{h^2}$ is the permeability parameter,

$P_r = \dfrac{\rho\upsilon c_p}{k_0}$ is the Prandtl number,

$S_c = \dfrac{\upsilon}{D_m}$ is the Schmidt number.

Equation (9)-(12), with boundary conditions (13), describe the unsteady motion of the MHD viscoelastic free convection flow with mass transfer through a porous medium over an infinite vertical porous plate.

For Newtonian fluid $(n_0 = 0)$ and when D_f and S_r equal to zero, our system of equations reduce to the system of equation suggested by [13].

Now to solve the system of equation (9)-(12) with boundary condition (13) we shall use the perturbation technique for small n_0, where

$$\left.\begin{aligned} f &= f_0 + n_0 f_1 + O\left(n_0^2\right) \\ G &= G_0 + n_0 G_1 + O\left(n_0^2\right) \\ \theta &= \theta_0 + n_0\theta_1 + O\left(n_0^2\right) \\ \phi &= \phi_0 + n_0\phi_1 + O\left(n_0^2\right) \end{aligned}\right\}. \tag{14}$$

The coefficients of higher power of $n_0 (n_0 \geq 2)$ have negligible contribution, thus we neglect them. By this method we can obtain, theoretically, the approximate solutions. Substitution (14) in (9)-(13) and equating like powers of n_0 we have:

$$\begin{aligned} f_0'' + (2\eta + v_0) f_0' - \frac{f_0}{k^*} = \\ -G_r\theta_0 - G_c\phi_0 + \frac{M\lambda}{1 + m^2\lambda^2}\left(f_0 + m\lambda G_0\right), \end{aligned} \tag{15}$$

$$G_0'' + (2\eta + v_0) G_0' = -\frac{M\lambda}{1 + m^2\lambda^2}\left(m\lambda f_0 - G_0\right), \tag{16}$$

$$\theta_0'' + P_r(2\eta + v_0)\theta_0' = -P_r D_f \phi_0'', \tag{17}$$

$$\phi_0'' + S_c(2\eta + v_0)\phi_0' = -S_c S_r \theta_0'', \tag{18}$$

$$\begin{aligned} f_1'' + (2\eta + v_0) f_1' - \frac{f_1}{k^*} = \\ -G_r\theta_1 - G_c\phi_1 + \frac{M\lambda}{1 + m^2\lambda^2}\left(f_1 + m\lambda G_1\right) - (2\eta + v_0) f_0''' - 4 f_0'', \end{aligned} \tag{19}$$

$$G_1'' + (2\eta + v_0)G_1' = -\frac{M\lambda}{1+m^2\lambda^2}(m\lambda f_1 - G_1) - (2\eta + v_0)G_0''' - 4G_0'', \quad (20)$$

$$\theta_1'' + P_r(2\eta + v_0)\theta_1' = -P_r D_f \phi_1'', \quad (21)$$

$$\phi_1'' + S_c(2\eta + v_0)\phi_1' = -S_c S_r \theta_1'', \quad (22)$$

$$\left.\begin{array}{l} f_0 = 0,\quad f_1 = 0,\quad G_0 = 0,\quad G_1 = 0,\quad \theta_0' = -1, \\ \theta_1' = 0,\quad \varphi_0 = 1,\quad \phi_1 = 0,\quad \eta = 0 \\ f_0,\ f_1,\ G_0,\ G_1,\ \theta_0,\ \theta_1,\ \phi_0,\ \phi_1 \to 0,\quad \eta \leftarrow \infty \end{array}\right\}. \quad (23)$$

By applying the numerical explicit method [17] we can obtain the solutions of our system of equations (15-22) subjected to the boundary conditions (23). Hence, the velocity distribution, temperature distribution and concentration distribution of the fluid motion as well as the skin-friction, heat and mass transfer are illustrated and discussed graphically with the various parameter of the problem.

Results and discussion:

In this problem we have used perturbation method with elasticity as a parameter to obtain the numerical solutions of the momentum, energy and concentration equations. The formulation of the velocity, temperature, concentration, heat transfer, and mass transfer of the non-Newtonian fluid flowing through a porous medium over an infinite vertical porous plate in the presence of the uniform magnetic filed are presented and shown graphically for different values of the problem parameters.

Figure (1) and (2) show the relation between the velocity components of the fluid motion with the elasticity parameter n_0. It is clear that the velocities increase (or decrease) with increasing the elasticity parameter in the regions illustrated in the figures.

In figures (3-6), the relation between the longitudinal and transverse velocity with the Grashof number G_r and modified Grashof number G_c is illustrated. It is clear that the two components of velocity increase with G_r, while they are increases or decreases with increasing of the modified Grashof number G_c.

Figure (7) and (8) illustrate the effect of the magnetic parameter M on the velocity components. It is clear that the longitudinal component of the velocity decreases with the parameter M, while the transverse component increases with the parameter M, respectively. The effect of the hall parameter m is shown in figure (9) and (10). It is clear that the longitudinal velocity increases with m, while the transverse component

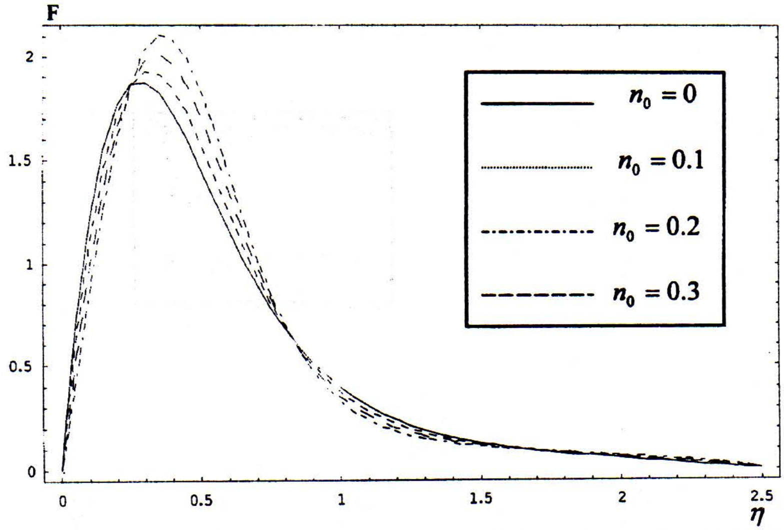

Figure (1) The velocity component F plotted versus Position with the effect of elasticity Parameter n_0.

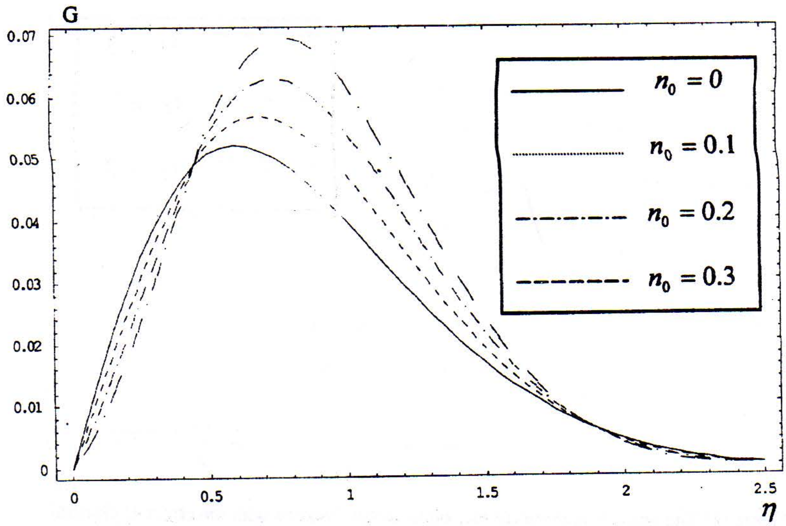

Figure (2) The velocity component G plotted versus Position with the effect of elasticity Parameter n_0.

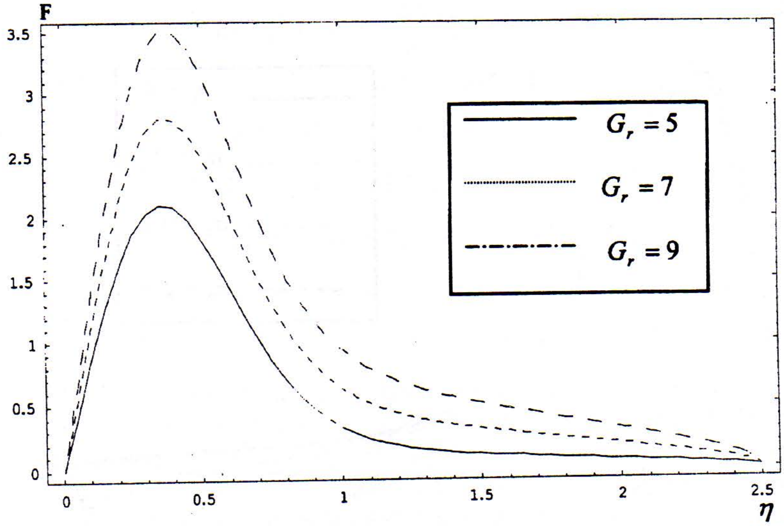

Figure (3) The velocity component F plotted versus Position with the effect of Grashof number.

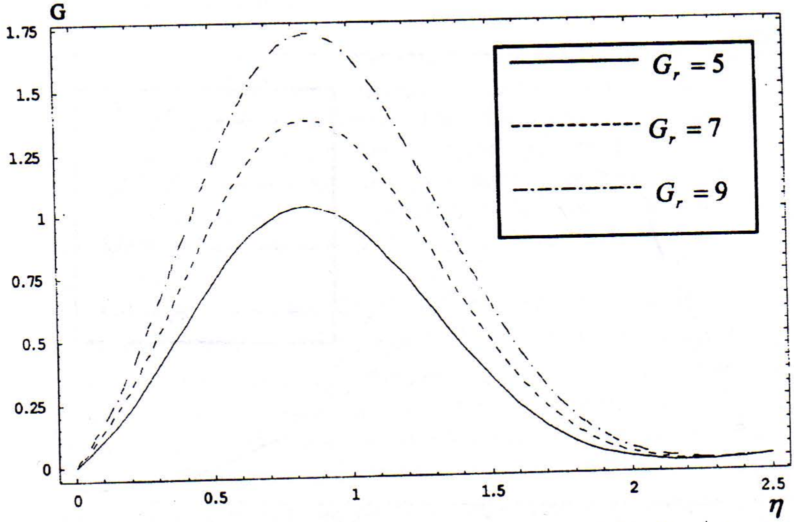

Figure (4) The velocity component G plotted versus Position with the effect of Grashof number.

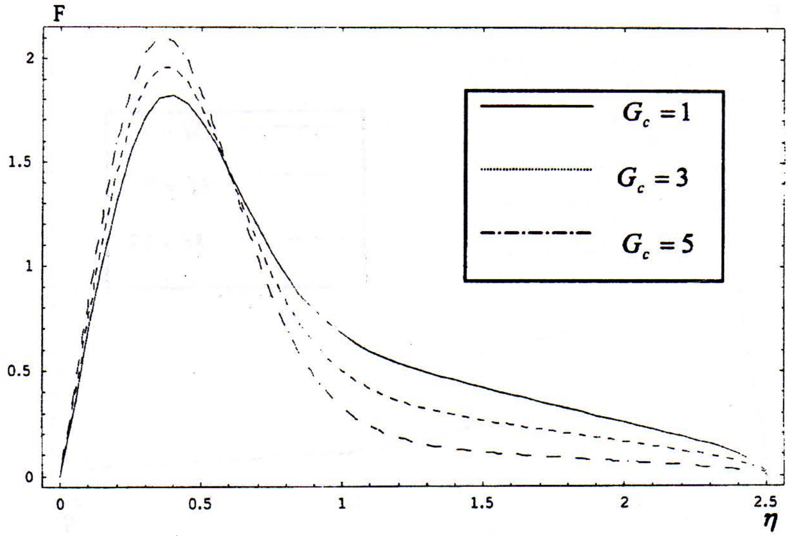

Figure (5) The velocity component F plotted versus Position with the effect of modified Grashof number.

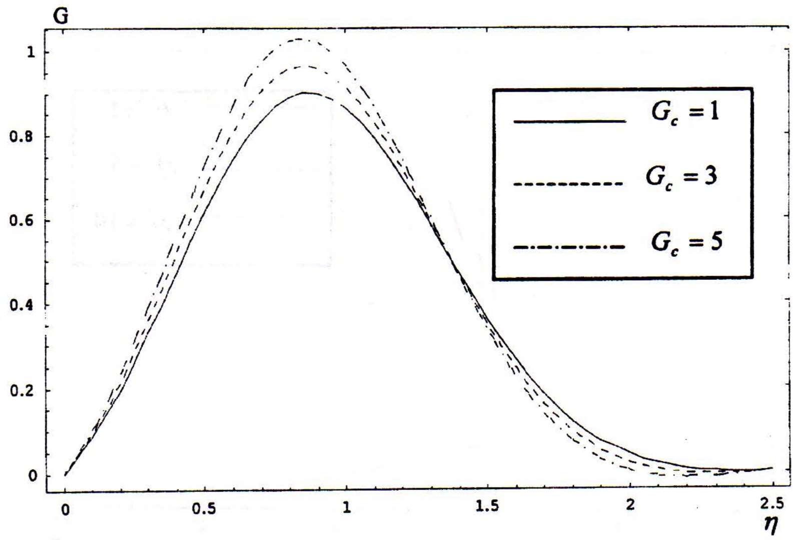

Figure (6) The velocity component G plotted versus Position with the effect of modified Grashof number.

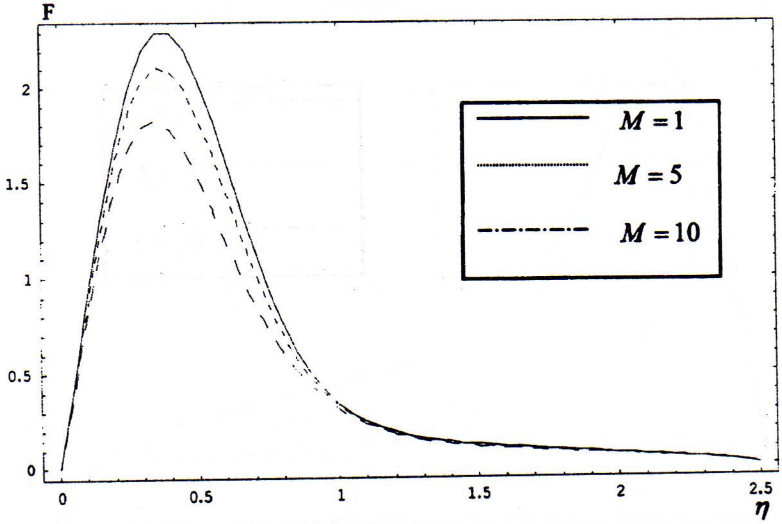

Figure (7) The velocity component F plotted versus Position with the effect of magnetic Parameter.

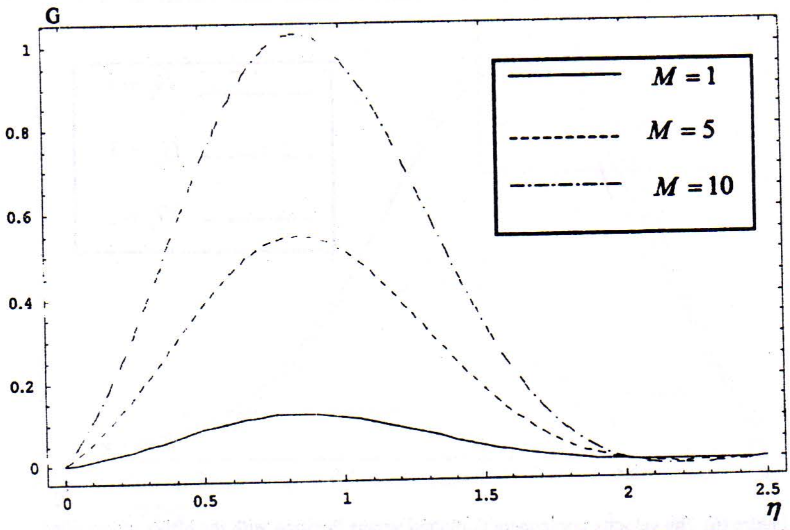

Figure (8) The velocity component G plotted versus Position with the effect of magnetic Parameter.

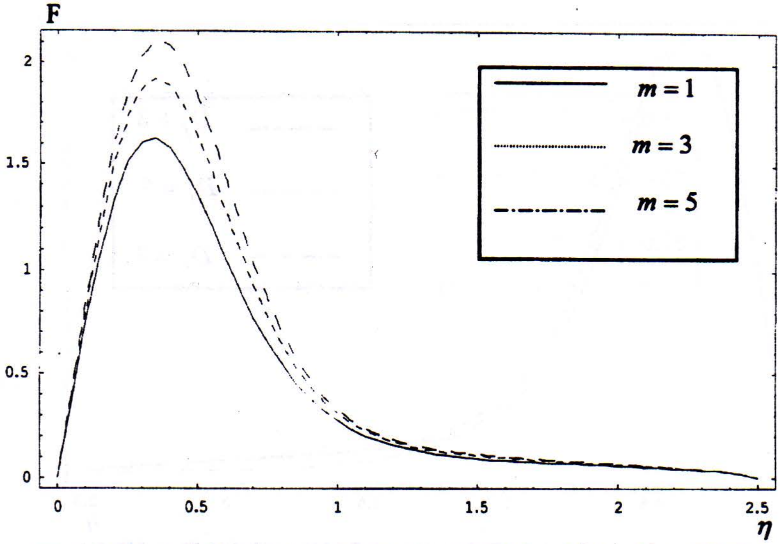

Figure (9) The velocity component F plotted versus Position with the effect of Hall Parameter.

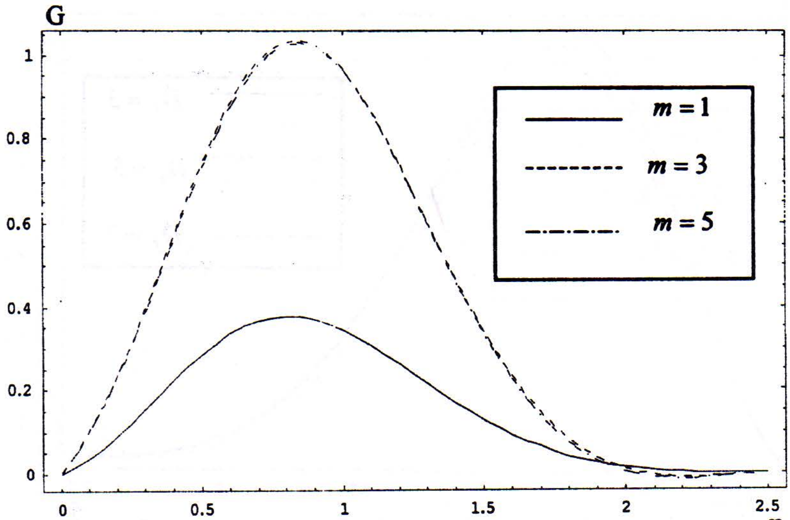

Figure (10) The velocity component G plotted versus Position with the effect of Hall Parameter.

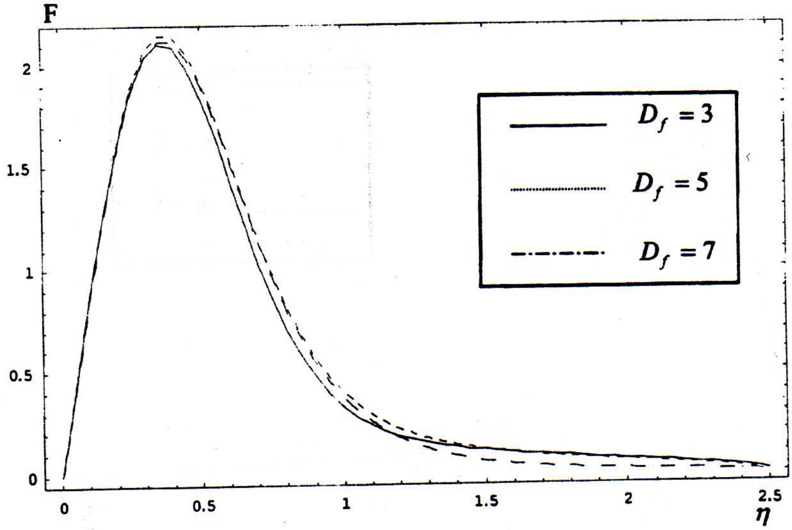

Figure (11) The velocity component F plotted versus Position with the effect of Dufour number.

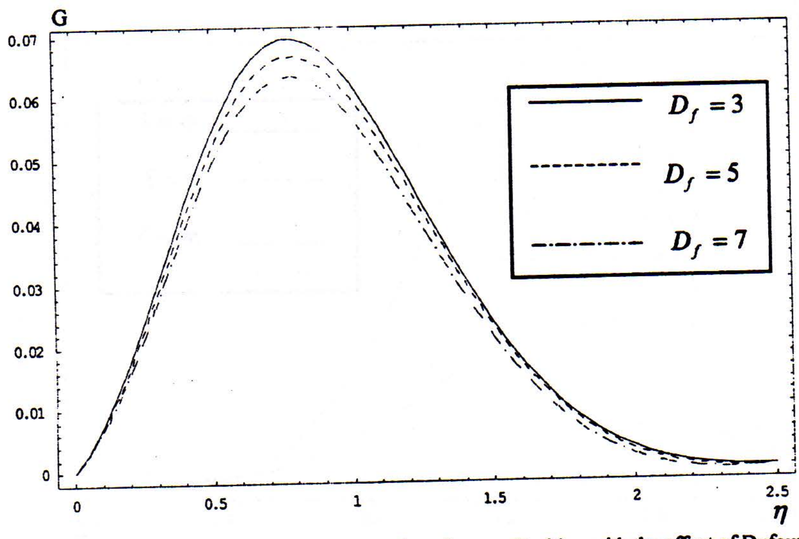

Figure (12) The velocity component G plotted versus Position with the effect of Dufour number.

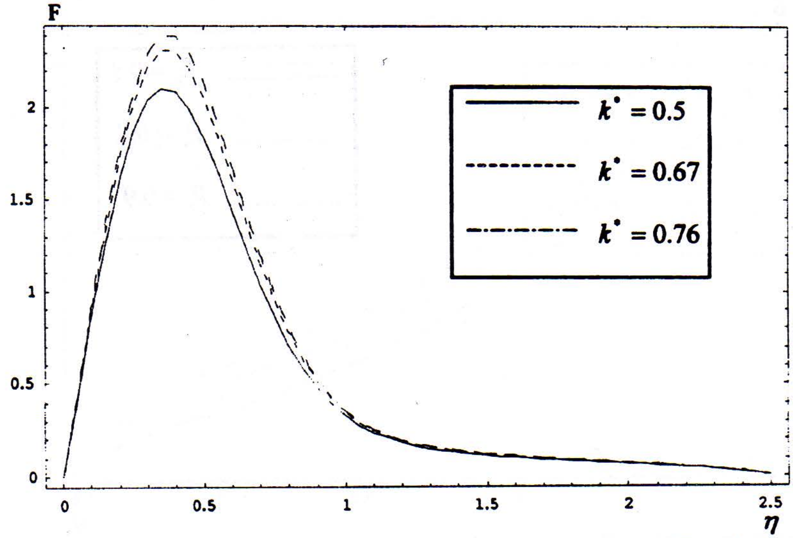

Figure (13) The velocity component F plotted versus Position with the effect of permeability parameter.

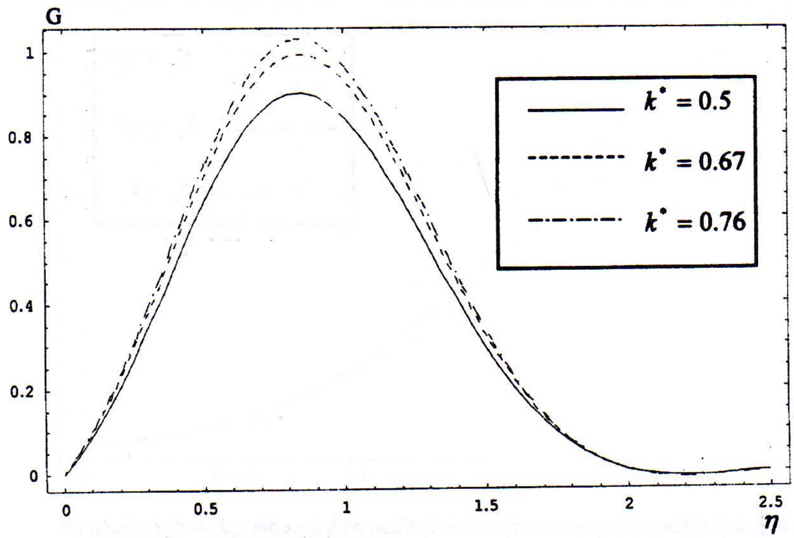

Figure (14) The velocity component G plotted versus Position with the effect of permeability parameter.

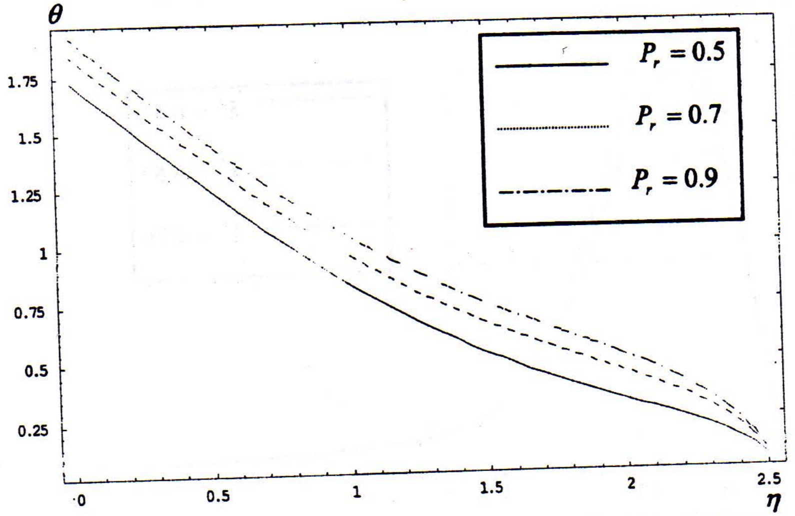

Figure (15) Temperature plotted versus Position with the effect of elasticity Prandtl number

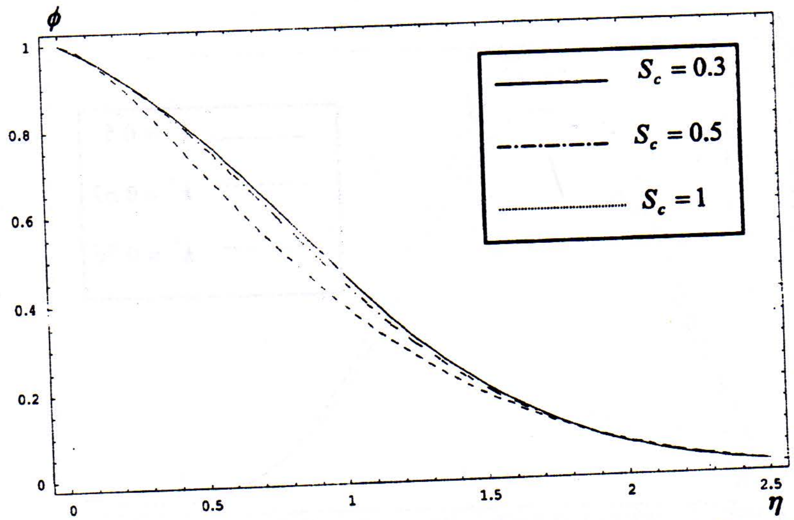

Figure (16) Concentration plotted versus Position with the effect of elasticity Schmidt number

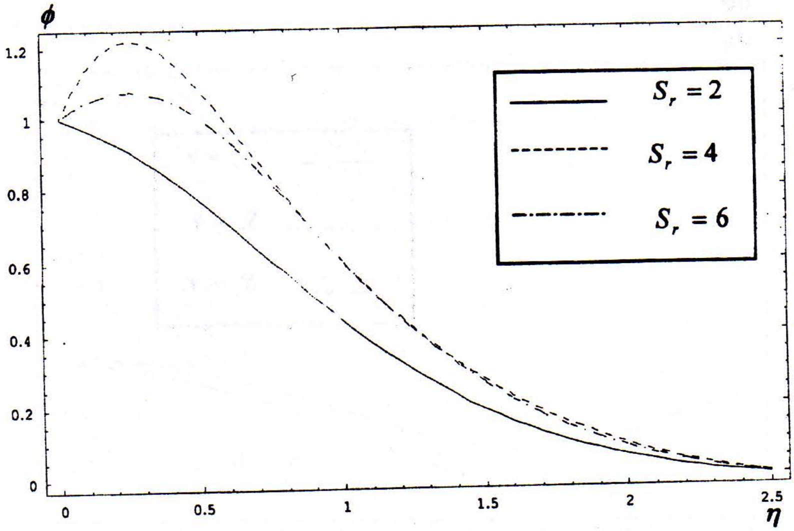

Figure (17) Concentration plotted versus Position with the effect of Soret number.

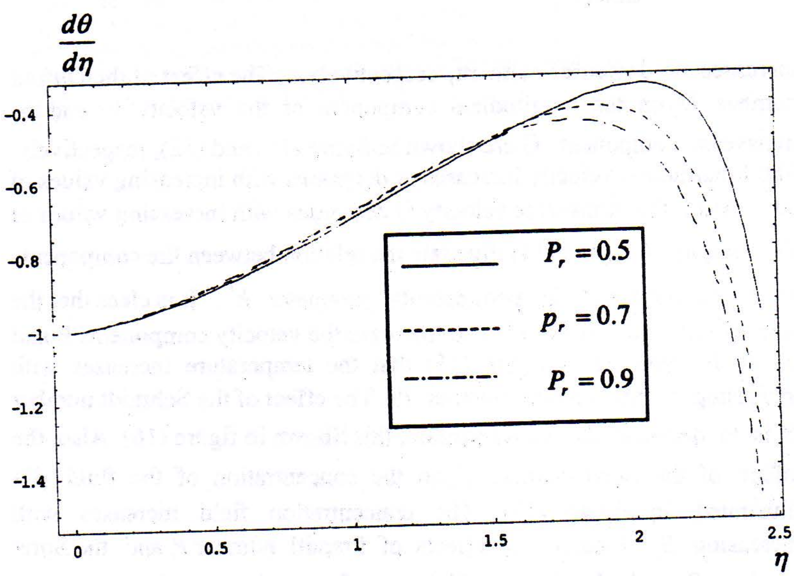

Figure (18) The rate of heat transfer plotted versus Position with the effect of Prandtl number.

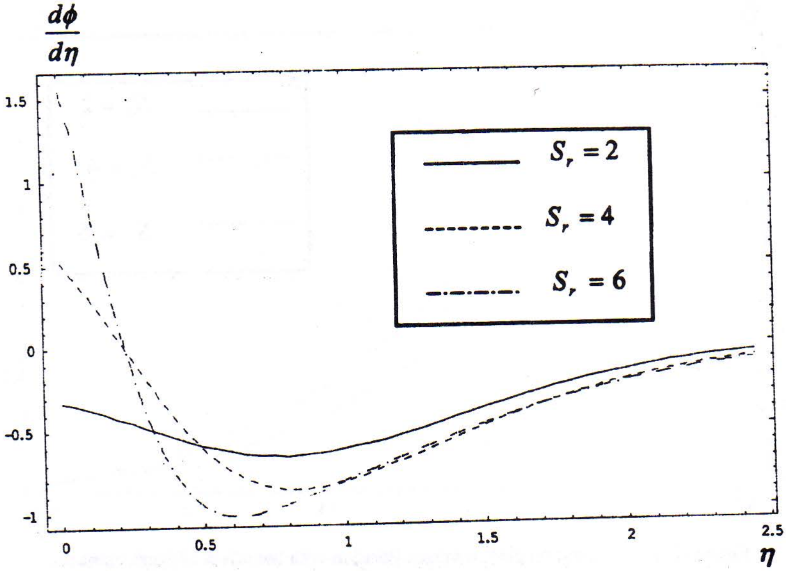

Figure (19) The rate of mass transfer plotted versus Position with the effect of Soret number.

increases or decreases with m, respectively. The effect of the Dufour number D_f on the longitudinal component of the velocity F and the transverse component G are shown in figure (11) and (12), respectively. The longitudinal velocity increases or decreases with increasing values of D_f. Also, the transverse velocity G decreases with increasing values of D_f. Figure (13) and (14) illustrate the relation between the components of the velocity and the permeability parameter k^*. It is clear that the permeability parameter k^* is to increase the velocity components F and G. It is seen from figure (15) that the temperature increases with increasing of the Prandtl number P_r. The effect of the Schmidt number S_c is to decrease the concentration, this Shown in figure (16). Also, the effect of the Soret number S_r on the concentration of the fluid ϕ is illustrated in figure (17). The concentration field increases with increasing S_r. Finally, the effects of Prandtl number P_r and the Soret number S_r on both the rate of heat transfer and the rate of mass transfer are illustrate through figure (18) and (19). It is clear that the rate of heat transfer and the rate of mass transfer increases or decreases with increasing of P_r and S_r, respectively.

References:

[1] Y. Jaluria, Natural convection heat and mass transfer. Pergamon press. Oxford [1980].

[2] T. Cebect and P. Bradshaw, Physical and computational aspects of convective heat transfer. Springer, Berlin [1988].

[3] E. M. Sparrow and J. L. Greg, J. Appl. Mech. 81, 133 [1959].

[4] J. H. Merkin, J. Fluid Mech. 35, 439 [1969].

[5] J. R. Lloyd and E. M. Sparrow, Int. J. Heat Mass Transfer 13, 434 [1970].

[6] G. Whks, Int. J. Heat Mass Transfer 16, 1958 [1973].

[7] V. M. Soundalgekar, N. V. Vighnesam and I. Pop. Int. J. Energy Res. 5, 213 [1981].

[8] M. D. Jahagirdar and R. M. Lahurika, Indian J. Pure Appl. Math. 20, 711 [1989].

[9] M. A. Hosain and M. Ahmed, Int. J. Heat Mass Transfer, 33, 571 [1990].

[10] A. S. Gupta, Acta Mechanica 22, 281 [1979].

[11] M. A. Hossain and R. I. M. A. Rashid, J. Phys Soc. Jpn, 56 (1), 97 [1987].

[12] M. A. Hossain and K. Mohammed, Jpn. J. Appl. Phys. 27 (8), 1531 [1988].

[13] MD. Addussattar, Int. J. Energy Res. 18, 771 [1994].

[14] A. Raptis and N. Kafousias, Can. J. Phys. 60, 1275 [1982].

[15] N. T. M. Eldabe, and S. N. Sallam, Can. J. Phys. 76, [1998].

[16] H. Schilchting. Boundary layer theory, Mcgraw Hill, [1968].

[17] G. D. smith, numerical solution of partial differential equations, Clarendon press, Oxford [1985].