HEAT AND MASS TRANSFER IN A HYDROMAGNETIC FLOW BETWEEN TWO PARALLEL PERMEABLE PLATES

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Abstract

An analytical study of magnetohydrodynamic free-convective and mass-transfer flow between two parallel permeable plates, is presented, tacking into account the Eckert number effects. The solution of the problem is obtained using perturbation method with Eckert number $E(<< 1)$ as perturbation parameter. Analytical expressions are given for the velocity distribution, temperature field, skin-friction, heat transfer and mass transfer. The detailed study has been made to know the effects of the various parameters entering into the problem.

Introduction

Effects of magnetic field on doubly diffusive fluid systems find application in various branches of modern science like biochemistry, oceanography, stellar convection, etc. Eldabe [1] have made an interesting analysis of the magnetohydrodynamic unsteady free convective flow through a porous medium bounded by an infinite vertical porous plate. Also, magnetohydrodynamic flow and heat transfer in a viscoelastic incompressible fluid confined between a horizontal stretching sheet and a parallel porous wall was presented by Eldabe and Sallam [2]. In a similar manner, Singh [3], Graham and Hunt [4] and Srivastava and Sharma [5] have studied some problems in this field.

However, all of the above research dealing with the Newtonian and non-Newtonian fluids concentrates only on the heat transfer problems. None of them deals with the much more complicated problem which involves both the heat and the mass transfer in the electrically conducting fluid. As a result, this research attempts to solve this much more complicated problem, taking into account the thermal-diffusion effect. For the governing equations of the problem, closed form analytic solutions are obtained and evaluated numerically for different parameters, such as the Prandtl number P, the magnetic parameter M, the modified Schmidt number S_c, Soret number, and the Eckert number. The contributions of these parameters to the heat and mass transfer characteristics are shown to be quite significant.

Analysis

Consider a steady two dimensional hydromagnetic flow of a viscous, incompressible and electrically-conducting fluid confined between two parallel plates. The lower plate fixed, and the upper moves with a uniform velocity in its own plane. The temperature and salinities at the lower y=0 are T_0 and C_0 and at the upper y=h are T_1 and C_1, respectively, y-axis being taken as vertical. A uniform vertical magnetic field of strength B_0 is applied on the system. The induced magnetic field is assumed to be negligible. The equations governing the motion of a fluid flowing between two parallel plates, in the usual notation, are:

$$v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\lambda(T - \bar{T}) + g\lambda^*(C - \bar{C}) - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2}$$

$$v\frac{\partial C}{\partial y} = \beta\frac{\partial^2 T}{\partial y^2} + \gamma\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial v}{\partial y} = 0 \tag{4}$$

where (u,v) are the velocity components of the fluid in x and y directions respectively, ρ is the density of the fluid, $v = \frac{\mu}{\rho}$, is the kinematic viscosity, μ is the coefficient of viscosity, T is the temperature, $\overset{*}{C}$ is the solute concentration, g is the acceleration due to gravity, $\bar{T}$ is the mean of T_0 and T_1, $\bar{C}$ is the mean of C_0 and C_1, λ is the coefficient of volume expansion, λ^* is the coefficient of soluts mass expansion, α, β are the thermal diffusivity of the fluid medium, γ is the molecular diffusivity, σ is the electrical conductivity of the fluid and C_p is the specific heat.

Equations (1)-(4) express the conservation of momentum, heat, solute and mass. Also, in equation (2), due to negligible induced magnetic field, the heat due to Joule dissipation is also assumed to be negligible. Equation (4) shows that v is either a constant or a function of time and as we assume a steady state the suction velocity v will takes the constant value v_0. Also the two plates are infinite in extent, hence all quantities are functions of y only. Equations (1)-(3) reduce to

$$v_0\frac{du}{dy} = v\frac{d^2u}{dy^2} + g\lambda(T - \bar{T}) - g\lambda^*(C - \bar{C}) - \frac{\sigma B_0^2}{\rho}u \tag{5}$$

$$v_0\frac{dT}{dy} = \alpha\frac{d^2T}{dy^2} + \frac{v}{C_p}\left(\frac{du}{dy}\right)^2 \tag{6}$$

$$v_0\frac{dC}{dy} = \beta\frac{d^2T}{dy^2} + \gamma\frac{d^2C}{dy^2} \tag{7}$$

The appropriate boundary conditions for the problem are:

$$u = 0 \,, \ T = T_0 \,, \ C = C_0 \,, \ at \ y = 0$$
$$u = u_0 \,, \ T = T_1 \,, \ C = C_1 \,, \ at \ y = h$$

(8)

where u_0 is the uniform velocity of the upper plate in its own plane.
Introducing the following non-dimensional quantities

$u' = \frac{u}{u_0}, \ y' = \frac{yv_0}{\nu}, \ \theta = \frac{T - \bar{T}}{T_0 - \bar{T}}, \ C' = \frac{C - \bar{C}}{C_0 - \bar{C}}, \ n = \frac{T_1 - \bar{T}}{T_0 - \bar{T}}, \ m = \frac{C_1 - \bar{C}}{C_0 - \bar{C}}, \ S = \frac{hv_0}{\nu}, \ G =$
$\frac{g\nu\lambda(T_0 - \bar{T})}{v_0^2 u_0}$, the Grashof number, $G_c = \frac{g\nu\lambda^*(C_0 - \bar{C})}{v_0^2 u_0}$, the modified Grash of number, $P = \frac{\nu}{\alpha}$
the Prandtl number, $S_c = \frac{\nu}{\gamma}$ the schmidt number, $S_0 = \frac{\beta(T_0 - \bar{T})}{\nu(C_0 - \bar{C})}$ the soret number,
$E = \frac{u_0^2}{C_p(T_0 - \bar{T})}$ the Eckert number, $M = \frac{\sigma B_0^2}{\rho v_0^2}$ the magnetic field parameter.
Equations [5]-[8] after substituting from equation (9) may be written in dimensionless
form after dropping the dash mark.

$$\frac{d^2 u}{dy^2} - \frac{du}{dy} = Mu - G\theta - G_c C,$$

(10)

$$\frac{d^2 \theta}{dy^2} - P\frac{d\theta}{dy} = -PE\left(\frac{du}{dy}\right)^2,$$

(11)

$$\frac{d^2 C}{dy^2} - S_c \frac{dC}{dy} = -S_0 S_c \frac{d^2 \theta}{dy^2},$$

(12)

with the boundary conditions:

$$u = 0, \ \theta = 1, \ C = 1, \ at \ y = 0$$
$$u = 1, \ \theta = n, \ C = m, \ at \ y = s$$

(13)

These equations are coupled and non-linear and hence very difficult to solve. To solve
them, u, θ and C are expanded in powers of E, the Eckert number, the Eckert number
for all incompressible fluids is always $<< 1$ [6]. Hence we take

$$u(y) = u_0(y) + Eu_1(y) + O(E^2)$$
$$\theta(y) = \theta_0(y) + E\theta_1(y) + O(E^2)$$
$$C(y) = C_0(y) + E\theta_1(y) + O(E^2)$$

(14)

Substituting (14) in (10)-(13), equating the coefficients of different powers of E, we
have the set of the following coupled linear equations:

$$(D^2 - D - M)u_0 = -G\theta_0 - G_c C_0$$

(15)

$$(D^2 - D - M)u_1 = -G\theta_1 - G_c C_0$$

(16)

$$(D^2 - PD)\theta_0 = 0$$

(17)

$$(D^2 - PD)\theta_1 = -P(Du_0)^2 \tag{18}$$

$$(D^2 - S_cD)C_0 = -S_0S_cD^2\theta_0 \tag{19}$$

$$(D^2 - S_cD)C_1 = -S_0S_cD^2\theta_1 \tag{20}$$

with the corresponding boundary conditions

$$u_0 = 0, \ \theta_0 = 1, \ C_0 = 1, \ at \ y = 0 \tag{21}$$
$$u_0 = 1, \ \theta_0 = n, \ C_0 = m, \ at \ y = s$$

$$u_1 = 0, \ \theta_1 = 0, \ C_1 = 0, \ at \ y = 0 \tag{22}$$
$$u_1 = 0, \ \theta_1 = 0, \ C_1 = 0, \ at \ y = s$$

where D means $\frac{d}{dy}$.

The solutions of (15)-(20) in virtue of the boundary conditions (21) and (22) are respectively given by:

$$\theta_0 = b_3 e^{Py} + b_4 \tag{23}$$

$$C_0 = \left(b_8 + \frac{S_0S_cb_0b_3P}{P - S_c}\right)e^{S_cy} - \frac{S_0S_cb_3P}{P - S_c}e^{Py} + b_9 - \frac{S_0S_cb_3P(b_0 - b_5)}{b_5(P - S_c)} \tag{24}$$

$$u_0 = b_{19}e^{yb_{16}} + b_{20}e^{yb_{17}} - b_{10}e^{Py} - b_{11}e^{S_cy} + b_{12}, \tag{25}$$

$$\theta_1 = b_{59}e^{Py} + b_{60} - P[b_{38}e^{2yb_{16}} + b_{40}e^{2yb_{17}} + b_{41}e^{2Py} + b_{43}e^{2S_cy} \tag{26}$$
$$+ b_{44}e^{yb_{26}} - b_{45}e^{yb_{28}} + b_{46}e^{yb_{30}} - b_{47}e^{yb_{32}} - b_{48}e^{yb_{34}} - b_{49}e^{yb_{36}}]$$

$$C_1 = b_{79}e^{yS_c} + b_{80} - b_{62}e^{Py} + b_{63}e^{2yb_{16}} + b_{64}e^{2yb_{17}} + b_{65}e^{2Py} + b_{66}e^{2yS_c} + \tag{27}$$
$$b_{67}e^{yb_{26}} - b_{68}e^{yb_{28}} + b_{69}e^{yb_{30}} - b_{70}e^{yb_{32}} - b_{71}e^{yb_{34}} - b_{72}e^{yb_{36}},$$

$$u_1 = b_{106}e^{yb_{16}} - b_{107}e^{yb_{17}} + b_{84}e^{Py} - b_{85}e^{yS_c} + b_{86}e^{2P} + b_{87}e^{2yS_c} \tag{28}$$
$$+ b_{88}e^{2yb_{16}} + b_{89}e^{2yb_{17}} + b_{90}e^{yb_{26}} + b_{91}e^{yb_{28}} +$$
$$+ b_{92}e^{yb_{30}} + b_{93}e^{yb_{32}} + b_{94}e^{yb_{34}} + b_{95}e^{yb_{36}} + b_{96}$$

Hence, the expressions for u, θ and C can be derived from (23)-(28) and (14) as follows:

$$u = b_{108}e^{yb_{16}} + b_{109}e^{yb_{17}} + b_{110}e^{Py} - b_{111}e^{yS_c} + E\left\{b_{86}e^{2Py} + b_{87}e^{2yS_c} + b_{88}e^{2yb_{16}}\right.$$
$$\left. + b_{89}e^{2yb_{17}} + b_{90}e^{yb_{26}} + b_{91}e^{yb_{28}} + b_{92}e^{yb_{30}} + b_{93}e^{yb_{32}} + b_{94}e^{yb_{34}} + b_{95}e^{yb_{36}} + b_{112}, \right.$$
$$\tag{29}$$

$$\theta = (b_3 + Eb_{59})e^{Py} - PE\left\{b_{38}e^{2yb_{16}} + b_{40}e^{2yb_{17}} + b_{41}e^{2Py} + b_{43}e^{2yS_c} + b_{44}e^{yb_{26}}\right.$$
$$\left. -b_{45}e^{yb_{28}} + b_{46}e^{yb_{30}} - b_{47}e^{yb_{32}} - b_{48}e^{yb_{34}} - b_{49}e^{yb_{36}} + (b_4 + Eb_{60}),\right. \tag{30}$$

$$C = b_{81}e^{yS_c} - b_{82}e^{Py} + E\left\{b_{63}e^{2yb_{16}} + b_{64}e^{2yb_{17}} + b_{65}e^{2Py} + b_{66}e^{2yS_c} + b_{67}e^{yb_{26}}\right.$$
$$\left. -b_{68}e^{yb_{28}} + b_{69}e^{yb_{30}} - b_{70}e^{yb_{32}} - b_{71}e^{yb_{34}} - b_{72}e^{yb_{36}} + b_{83}\right. \tag{31}$$

where $b_0 - b_{112}$ are defined in the Appendix.

From the knowledge of the velocity, we can calculate the skin friction which is given in dimensionless form by

$$\tau_{xy} = \frac{du}{dy}, \tag{32}$$

At the lower plate

$$\tau_{xy}|_{y=0} = b_{117}, \tag{33}$$

and at the upper plate

$$\tau_{xy}|_{y=5} = b_{122}, \tag{34}$$

Knowing the temperature field, it is important to know the Nusselt number and is given in non-dimensional form by

$$N_u = \frac{d\theta}{dy}, \tag{35}$$

At the lower plate

$$N_u|_{y=0} = b_{126}, \tag{36}$$

and at the upper plate

$$N_u|_{y=5} = b_{132}, \tag{37}$$

Knowing the concentration field, it is important to know the rate of concentration and is given in dimensionless form by

$$M_c = \frac{dC}{dy}, \tag{38}$$

At the lower plate

$$M_c|_{y=0} = b_{137}, \tag{39}$$

and at the upper plate

$$M_c|_{y=s} = b_{142}, \tag{40}$$

where, $b_{117}, b_{122}, b_{126}, b_{132}, b_{137}$ and b_{142} are defined in the Appendix.

Results and Discussion

Heat and mass transfer in a Hydromagnetic flow between two parallel permeable plates represents a very important class of fluid mechanics problems. That flow is governed by five parameters, namely, the Prandtl number P, the magnetic parameter M, the Schmidt number Sc, the Soret number S0, the Eckert number Ec. An insight into the effect of these parameters on the flow can be obtain by the study of the velocity distribution, temperature field and concentration. In figs.1-3, for M=1, Sc=0.3, So=0.5, Ec=0.1, the velocity, the temperature and the concentration are plotted for several values of the prandtl number P, it is found that all of them increase with the increase of P.
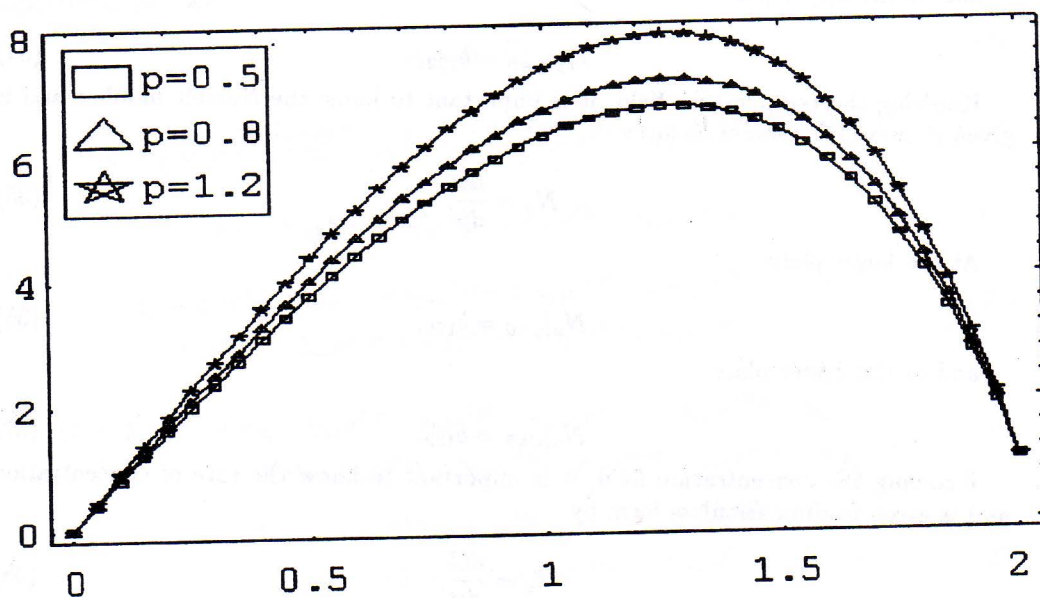
fig. 1: Velocity distribution plotted against position in the case M=1, Sc=0.3, S0=0.3, G=5, Gc=5, Ec=0.1
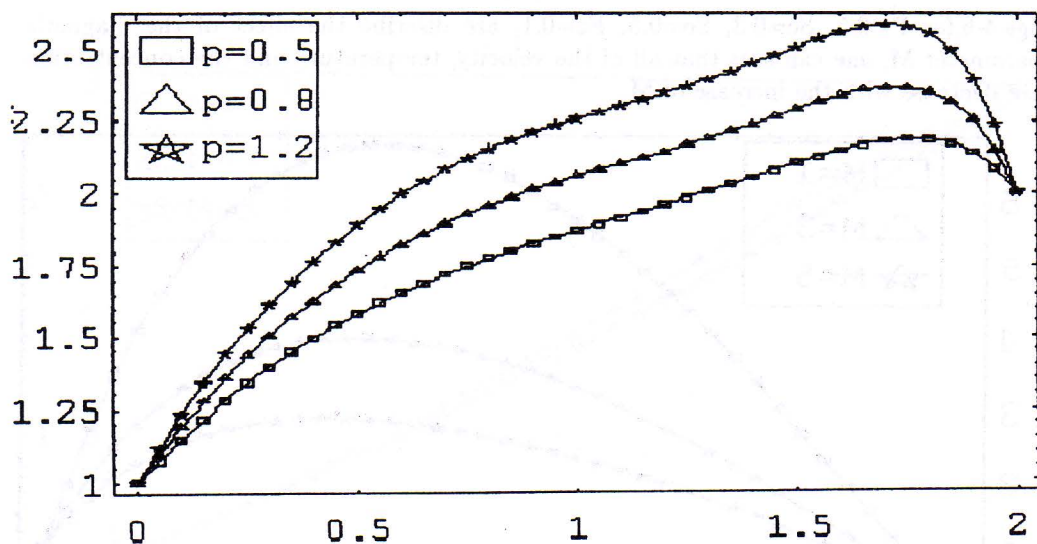
fig. 2: Temperature distribution plotted against position in the case M=1, Sc=0.3, S0=0.3, G=5,Gc=5, Ec=0.1
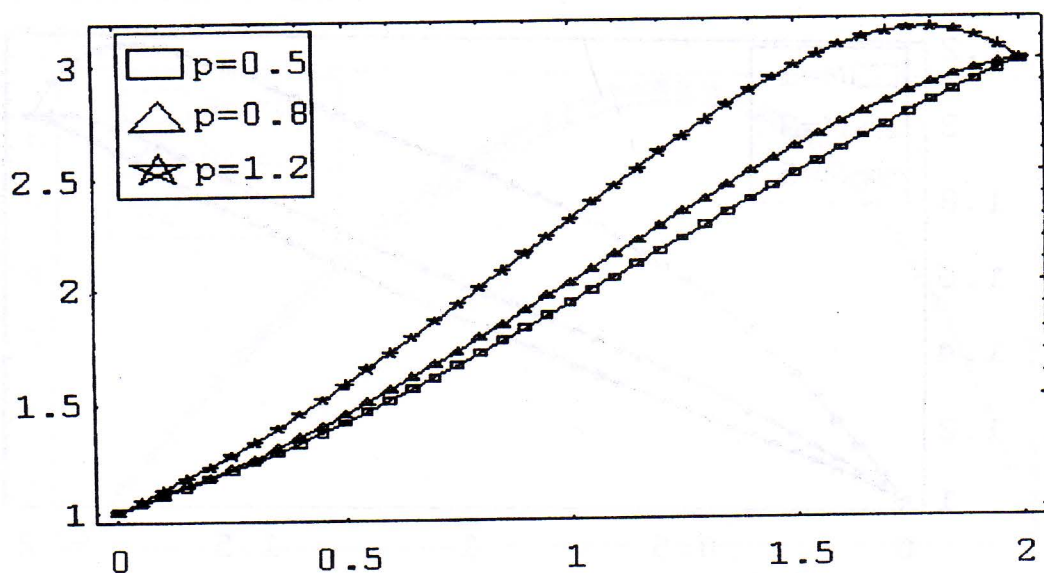

fig. 3: salinity distribution plotted against position in the case M=1, Sc=0.3, S0=0.3, G=5, Gc=5,Ec=0.1

figs.4-6,for P=0.5, Sc=0.3, So=0.5, Ec=0.1, are discribe the effect of the magnetic parameter M, one can sees that all of the velocity, temperature and the concentration are decrease with the increase of M.
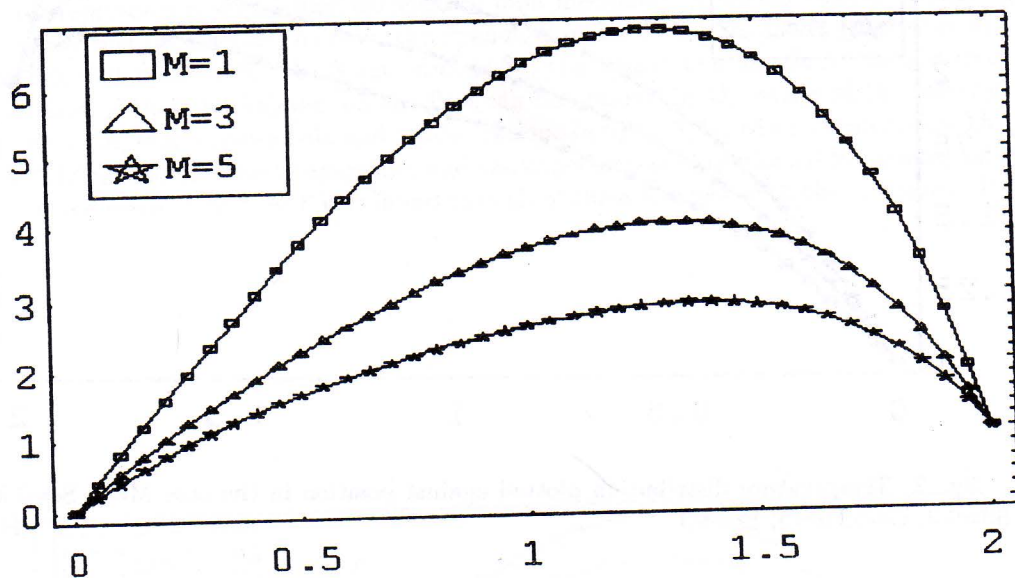
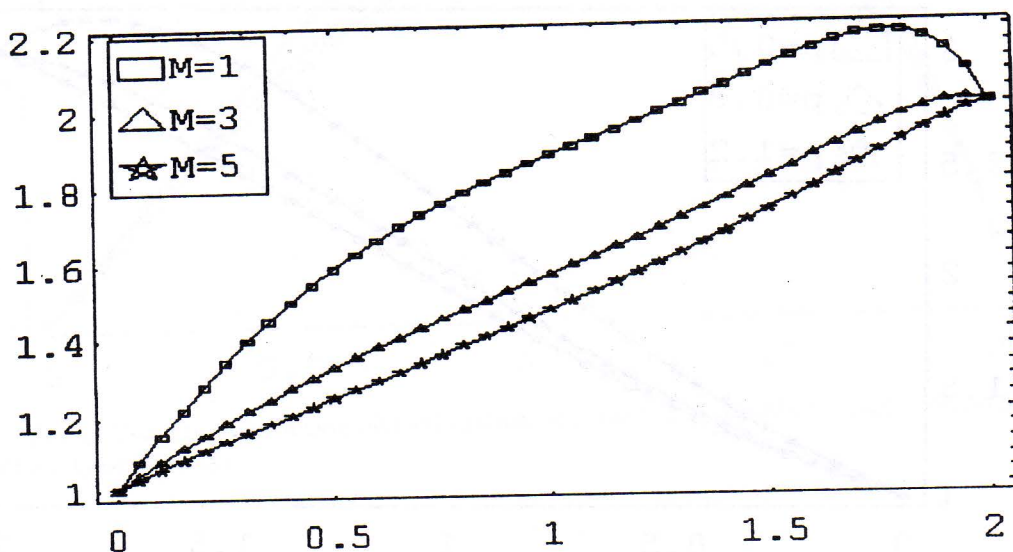fig. 4: Velocity distribution plotted against position in the case P=0.5, Sc=0.3, S0=0.3, G=5, Gc=5,Ec=0.1

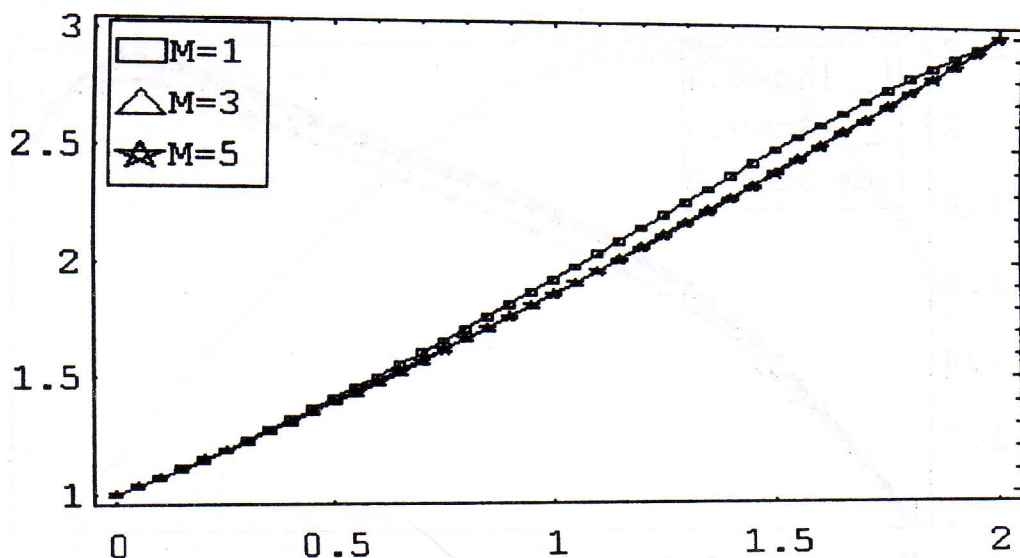fig. 5: Temperature distribution plotted against position in the case P=0.5, Sc=0.3, S0=0.3, G=5,Gc=5, Ec=0.1

fig. 6: Salinity distribution plotted against position in the case P=0.5, Sc=0.3, S0=0.3, G=5, Gc=5, Ec=0.1

From figs.7-9,for P=0.5, M=1, So=0.5, Ec=0.1,it is found that both the velocity and the temperature are decrease with the increase of the schmidt number Sc but the concentration decreases in the rangefrom y=0 to y=1.4 while it increases in the rangefrom y¿1.4 to y=2
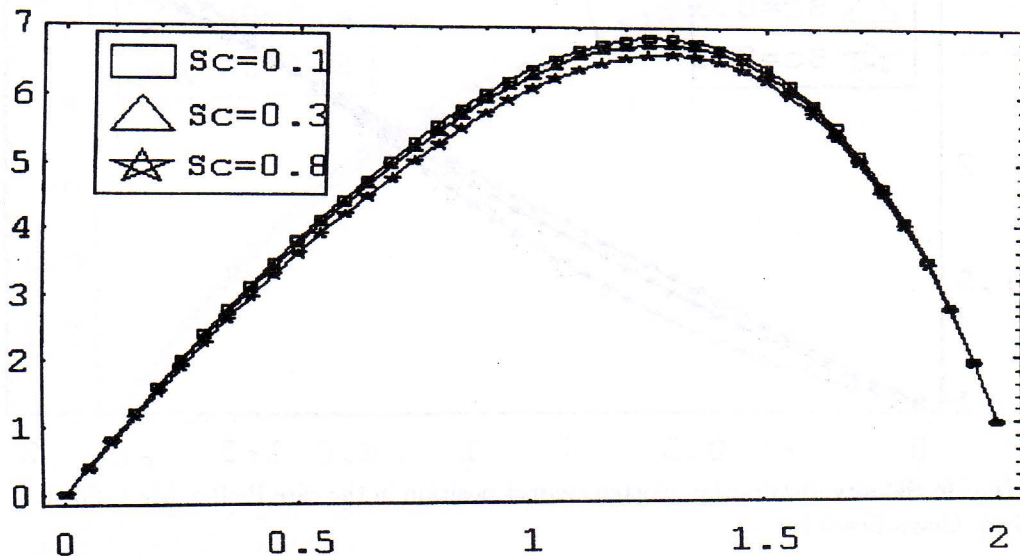

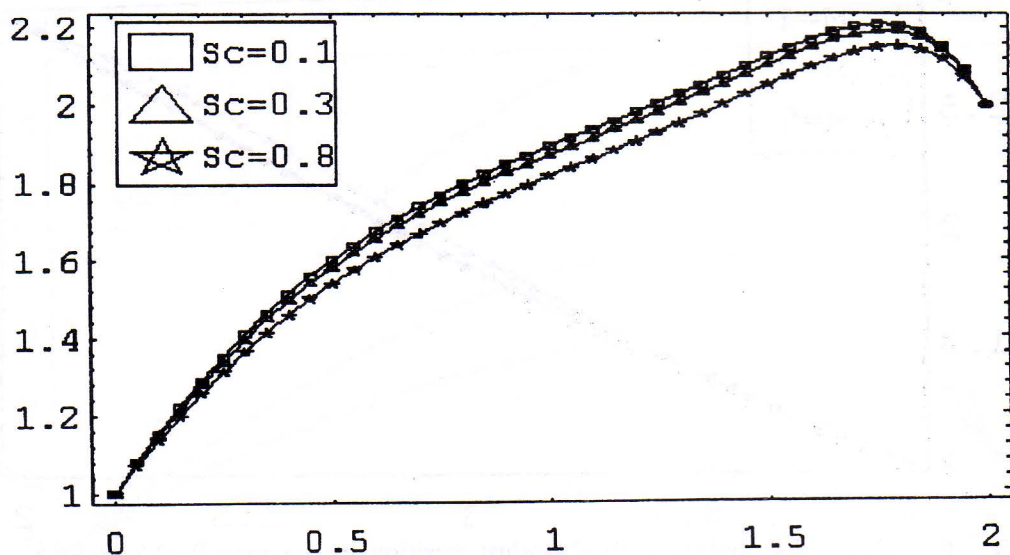fig. 7: Velocity distribution plotted against position in the case P=0.5, M=1, S0=0.3, G=5, Gc=5,Ec=0.1

fig. 8: Temperature distribution plotted against position in the case P=0.5, M=1, S0=0.3, G=5, Gc=5,Ec=0.1

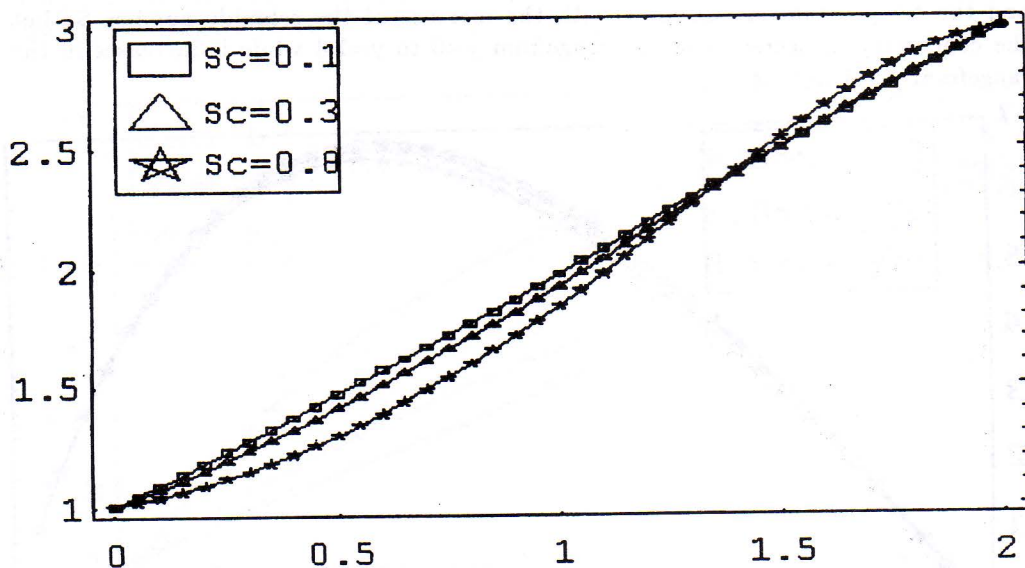fig. 9: Salinity distribution plotted against position in the case P=0.5, M=1, S0=0.3, G=5, Gc=5,Ec=0.1

Figs.10-12, for P=0.5, M=1, Sc=0.3, Ec=0.1,discribe the effect of the Soret number So on the velocity, the temperature and the concentration, it is found that all of them

increase with the increase of So

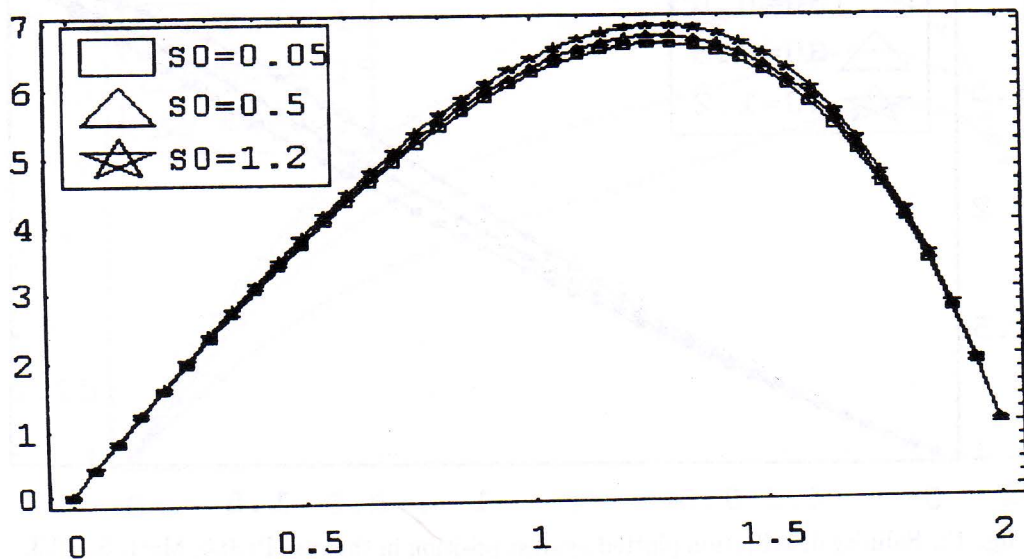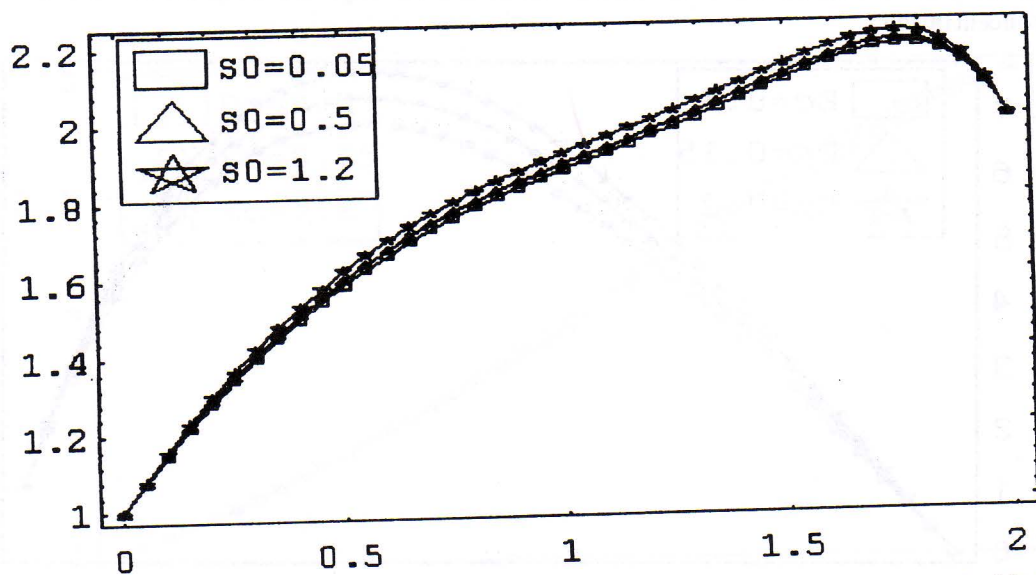fig. 10: Velocity distribution plotted against position in the case P=0.5, M=1, Sc=0.3, G=5, Gc=5,Ec=0.1

fig. 11: Temperature distribution plotted against position in the case P=0.5, M=1, Sc=0.3, G=5,Gc=5,Ec=0.1
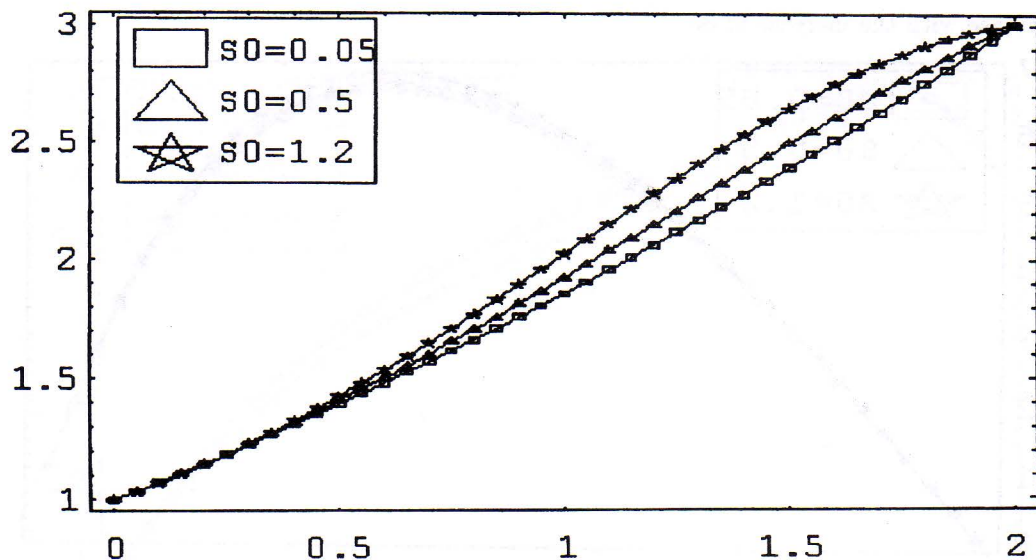
fig. 12: Salinity distribution plotted against position in the case P=0.5, M=1, Sc=0.3, G=5, Gc=5, Ec=0.1

Finally, from figs.13-15, for P=0.5, M=1, So=0.5, Sc=0.3, it is found that the effect of the Eckert number Ec is to increase all of the velocity, the temperature and the concentration.

fig. 13: Velocity distribution plotted against position in the case P=0.5, M=1, Sc=0.3, S0=0.3, G=5, Gc=5,

fig. 14: Temperature distribution plotted against position in the case P=0.5, M=1, Sc=0.3, S0=0.3, G=5, Gc=5,
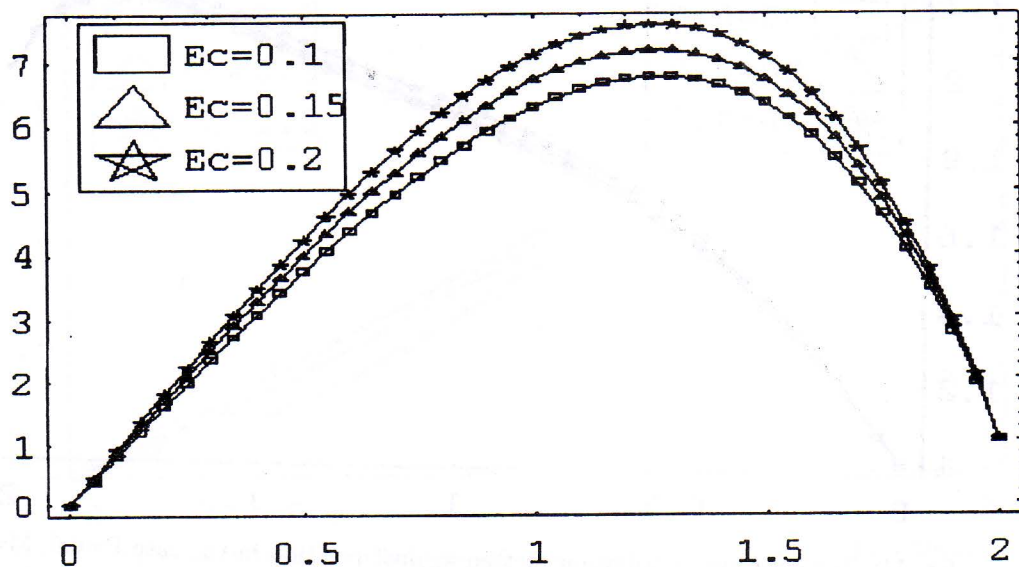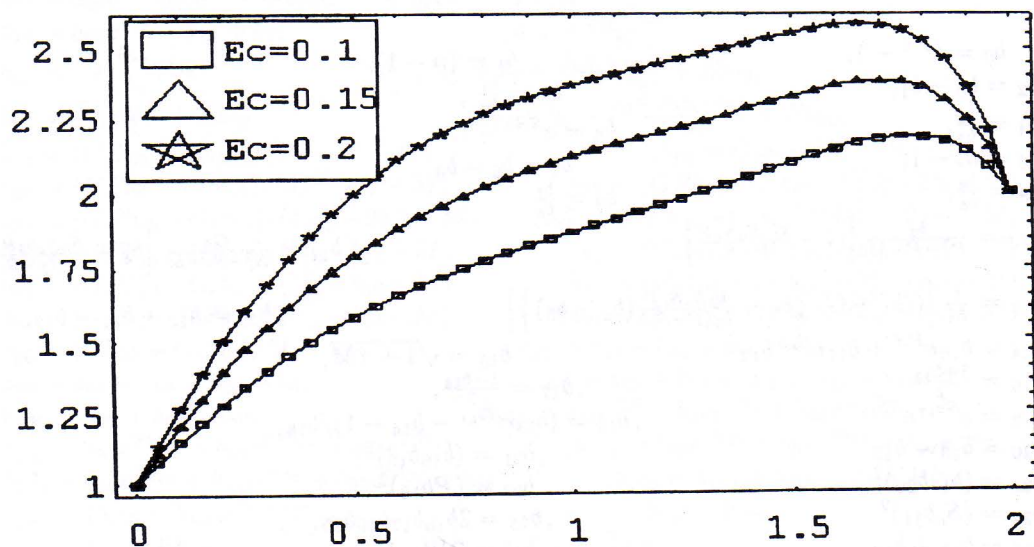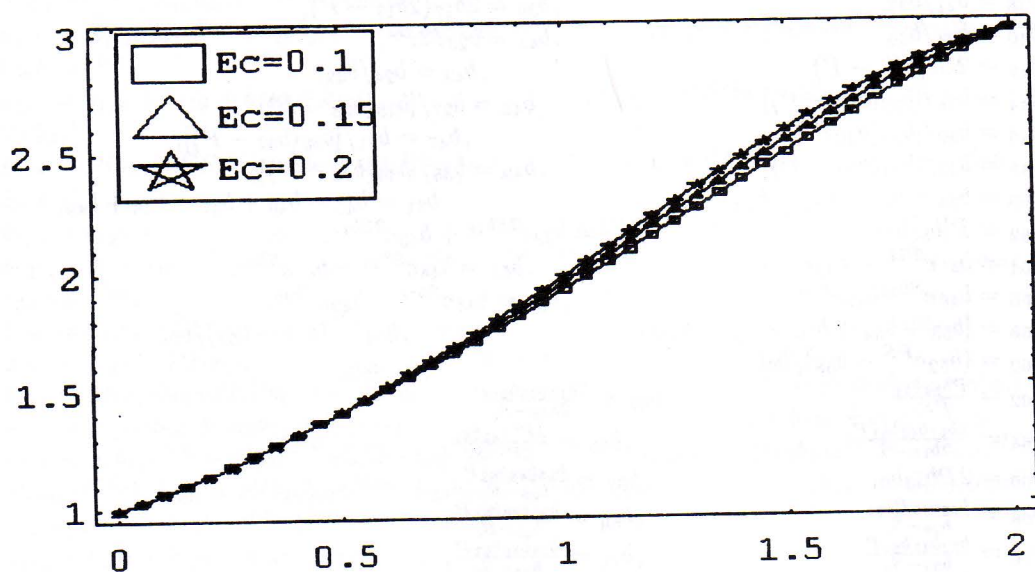

fig. 15: Salinity distribution plotted against position in the case P=0.5, M=1, Sc=0.3, S0=0.3, G=5, Gc=5,

Appendix

$$b_0 = e^{PS} - 1, \qquad\qquad\qquad b_1 = (n-1),$$

$$b_2 = b_0 - b_1, \qquad\qquad\qquad b_3 = \frac{b_1}{b_0},$$

$$b_4 = \frac{b_2}{b_0}, \qquad\qquad\qquad b_5 = e^{S S_c} - 1,$$

$$b_6 = m - 1, \qquad\qquad\qquad b_7 = b_5 - b_6,$$

$$b_8 = \frac{b_6}{b_5}, \qquad\qquad\qquad b_9 = \frac{b_7}{b_5},$$

$$b_{10} = \frac{b_3}{P^2 - P - M}\left[G - \frac{G_c S_o S_c P}{P - S_c}\right] \qquad , b_{11} = \frac{Gc}{S_c^2 - S_c - M}\left[b_8 + \frac{S_o S_c b_3 P b_o}{b_5 (p - S_c)}\right],$$

$$b_{12} = \frac{1}{M}\left[G b_4 + G_c \left\{ b_9 - \frac{b_3 S_o S_c P}{b_5 (P - S_c)}(b_o - b_5)\right\}\right] \qquad , b_{13} = b_{10} + b_{11} - b_{12},$$

$$b_{14} = b_{10} e^{PS} + b_{11} e^{S_c S} b_{12} \qquad\qquad , b_{15} = \sqrt{1 + 4M},$$

$$b_{16} = \frac{1 + b_{15}}{2} \qquad\qquad , b_{17} = \frac{1 - b_{15}}{2},$$

$$b_{18} = e^{S b_{17}} e^{S b_{16}} \qquad\qquad , b_{19} = (b_{13} e^{S b_{17}} - b_{14} - 1)/b_{18},$$

$$b_{20} = b_{13} - b_{19} \qquad\qquad , b_{21} = (b_{16} b_{19})^2,$$

$$b_{22} = (b_{17} b_{20})^2 \qquad\qquad , b_{23} = (P b_{10})^2,$$

$$b_{24} = (S_c b_{11})^2 \qquad\qquad , b_{25} = 2 b_{16} b_{17} b_{19} b_{20},$$

$$b_{26} = b_{16} + b_{17} \qquad\qquad , b_{27} = 2 P b_{10} b_{17} b_{20},$$

$$b_{28} = P + b_{17} \qquad\qquad , b_{29} = 2 P S_c b_{10} b_{11},$$

$$b_{30} = P + S_c \qquad\qquad , b_{31} = 2 P b_{10} b_{16} b_{19},$$

$$b_{32} = P + b_{16} \qquad\qquad , b_{33} = 2 S_c b_{11} b_{16} b_{19},$$

$$b_{34} = S_c + b_{16} \qquad\qquad , b_{35} = 2 S_c b_{11} b_{17} b_{20},$$

$$b_{36} = S_c + b_{17} \qquad\qquad , b_{37} = 2 b_{16}(2 b_{16} - P),$$

$$b_{38} = b_{21}/b_{37} \qquad\qquad , b_{39} = 2 b_{17}(2 b_{17} - P),$$

$$b_{40} = b_{22}/b_{39} \qquad\qquad , b_{41} = b_{23}/2 P^2,$$

$$b_{42} = 2 S_c (2 S_c - P) \qquad\qquad , b_{43} = b_{24}/b_{42},$$

$$b_{44} = b_{25}/[b_{26}(b_{26} - P)] \qquad\qquad , b_{45} = b_{27}/[b_{28}(b_{28} - P)],$$

$$b_{46} = b_{29}/[b_{30}(b_{30} P)] \qquad\qquad , b_{47} = b_{31}/[b_{32}(b_{32} - P)],$$

$$b_{48} = b_{33}/[b_{34}(b_{34} - P)] \qquad\qquad , b_{49} = b_{35}/[b_{36}(b_{36} - P)],$$

$$b_{50} = b_{38} + b_{40} + b_{41} + b_{43} + b_{44} \qquad , b_{51} = b_{45} - b_{46} + b_{47} + b_{48} + b_{49},$$

$$b_{52} = P[b_{50} b_{51}] \qquad\qquad , b_{53} = b_{38} e^{2 S b_{16}} + b_{40} e^{2 S b_{17}},$$

$$b_{54} = b_{41} e^{2 S P} + b_{43} e^{2 S S_c} \qquad , b_{55} = b_{44} e^{S b_{26}} - b_{45} e^{S b_{28}},$$

$$b_{56} = b_{46} e^{S b_{30}} b_{47} e^{S b_{32}} \qquad , b_{57} = b_{48} e^{S b_{34}} + b_{49} e^{S b_{36}},$$

$$b_{58} = [b_{53} + b_{54} + b_{55} + b_{56} - b_{57}] P \qquad , b_{59} = (b_{58} - b_{52})/b_0,$$

$$b_{60} = (b_{52} e^{PS} - b_{58})/b_0 b \qquad , 61 = S_0 S_c,$$

$$b_{62} = \frac{P b_{59} b_{61}}{P S_c} \qquad\qquad , b_{63} = \frac{2 b_{16} b_{38} b_{61} P}{2 b_{16} - S_c},$$

$$b_{64} = \frac{2 b_{17} b_{40} b_{61} P}{2 b_{17} - S_c} \qquad\qquad , b_{65} = \frac{2 P^2 b_{41} b_{61}}{2 P - S_c},$$

$$b_{66} = 2 P b_{43} b_{61} \qquad\qquad , b_{67} = \frac{b_{26} b_{44} b_{61} P}{b_{26} - S_c},$$

$$b_{68} = \frac{b_{28} b_{45} P b_{61}}{b_{28} - S_c} \qquad\qquad , b_{69} = \frac{b_{30} b_{46} b_{61} P}{b_{30} - S_c},$$

$$b_{70} = \frac{b_{32} b_{47} b_{61} P}{b_{32} - S_c} \qquad\qquad , b_{71} = \frac{b_{34} b_{48} b_{61} P}{b_{34} - S_c},$$

$$b_{72} = \frac{b_{36} b_{49} b_{61} P}{b_{36} - S_c} \qquad\qquad , b_{73} = b_{63} + b_{64} + b_{65} + b_{66} + b_{67},$$

$$b_{74} = b_{68} + b_{69} + b_{70} + b_{71} + b_{72} \qquad , b_{75} = b_{62} - b_{73} + b_{74},$$

$$b_{76} = b_{63} e^{2 S b_{16}} + b_{64} e^{2 S b_{17}} + b_{65} e^{2 P S} + b_{66} e^{2 S S_c} + b_{67} e^{S b_{26}},$$

$b_{77} = b_{68}e^{2b_{28}}b_{69}e^{Sb_{30}} + b_{70}e^{Sb_{32}} + b_{71}e^{Sb_{34}} + b_{72}e^{Sb_{36}},$

$b_{78} = b_{62}e^{PS} - b_{76} + b_{77}$, $b_{79} = \frac{b_{78}-b_{75}}{b_5},$

$b_{80} = \frac{b_{75}e^{SS_c}-b_{78}}{b_5}$, $b_{81} = b_8 + \frac{S_0 S_c b_3 b_0 P}{b_5(P-S_c)} + Eb_{79},$

$b_{82} = \frac{S_0 S_c b_3 P}{P-S_c} + Eb_{62}$, $b_{83} = b_9 - \frac{S_0 S_c P(b_0-b_3)}{P-S_c} + Eb_{80},$

$b_{84} = (G_c b_{62} - G b_{59})/(P^2 - P - M)$, $b_{85} = G_c b_{79}/(S_c^2 - S_c - M),$

$b_{86} = (GPb_{41} - G_c b_{65})/(4P^2 - 2P - M)$, $b_{87} = (GPb_{43} - G_c b_{66})/(4S_c^2 - 2S_c - M),$

$b_{88} = (GPb_{38} - G_c b_{63})/(4b_{16}^2 - 2b_{16} - M)$, $b_{89} = (GPb_{40} - G_c b_{64})/(4b_{17}^2 - 2b_{17} - M),$

$b_{90} = (GPb_{44} - G_c b_{67})/(b_{26}^2 - b_{26} - M)$, $b_{91} = (G_c b_{68} - GPb_{45})/(b_{28}^2 - b_{28} - M),$

$b_{92} = (GPb_{46} - G_c b_{69})/(b_{30}^2 - b_{30} - M)$, $b_{93} = (G_c b_{70} - GPb_{47})/(b_{32}^2 - b_{32} - M),$

$b_{94} = (G_c b_{71} - GPb_{48})/(b_{34}^2 - b_{34} - M)$, $b_{95} = (G_c b_{72} - GPb_{49})/(b_{36}^2 - b_{36} - M),$

$b_{96} = (G b_{60} + G_c b_{80})/M$, $b_{97} = b_{84} - b_{85} + b_{86} + b_{87},$

$b_{98} = b_{88} + b_{89} + b_{90} + b_{91}$, $b_{99} = b_{92} + b_{93} + b_{94} + b_{95} + b_{96},$

$b_{100} = b_{97} + b_{98} + b_{99}$, $b_{101} = b_{84}e^{PS} - b_{85}e^{SS_c} + b_{86}e^{2SP},$

$b_{102} = b_{87}e^{2SS_c} + b_{88}e^{2Sb_{16}} + b_{89}e^{2Sb_{17}}$, $b_{103} = b_{90}e^{Sb_{26}} + b_{91}e^{Sb_{28}} + b_{92}e^{Sb_{30}},$

$b_{104} = b_{93}e^{Sb_{32}} + b_{94}e^{Sb_{34}} + b_{95}e^{Sb_{36}} + b_{96}$, $b_{105} = b_{101} + b_{102} + b_{103} + b_{104},$

$b_{106} = (b_{105} - b_{100}e^{Sb_{17}})/b_{18}$, $b_{107} = b_{100} + b_{106},$

$b_{108} = b_{19} + Eb_{106}$, $b_{109} = b_{20} - Eb_{107},$

$b_{110} = -b_{10} + Eb_{84}$, $b_{111} = b_{11} + Eb_{85},$

$b_{112} = b_{12} + Eb_{96}$, $b_{113} = b_{16}b_{108} + b_{17}b_{109} + Pb_{110} - S_c b_{111},$

$b_{114} = 2E[Pb_{86} + S_c b_{87} + b_{16}b_{88} + b_{17}b_{89}]$, $b_{115} = E[b_{26}b_{90} + b_{28}b_{91} + b_{30}b_{92}],$

$b_{116} = E[b_{32}b_{93} + b_{34}b_{94} + b_{36}b_{95}]$, $b_{117} = b_{113} + b_{114} + b_{115} + b_{116},$

$b_{118} = b_{16}b_{108}e^{Sb_{16}} + b_{17}b_{109}e^{Sb_{17}} + Pb_{110}e^{SP} - S_c b_{111}e^{SS_c}$, $b_{119} = 2E[Pb_{86}e^{2PS} + S_c b_{87}e^{2SS_c} + b_{16}b_{88}e^{2Sb_{16}} + b_{17}b_{89}e^{2Sb_{17}}],$

$b_{120} = E[b_{26}b_{90}e^{Sb_{26}} + b_{28}b_{91}e^{Sb_{28}} + b_{30}b_{92}e^{Sb_{30}}]$, $b_{121} = E[b_{32}b_{93}e^{Sb_{32}} + b_{34}b_{94}e^{Sb_{34}} + b_{36}b_{95}e^{Sb_{36}}],$

$b_{122} = b_{118} + b_{119} + b_{120} + b_{121}$, $b_{123} = PE(b_{59} - 2b_{16}b_{38} - 2b_{17}b_{40} - 2Pb_{41}),$

$b_{124} = P(b_3 - 2ES_c b_{43} - Eb_{26}b_{44} + Eb_{28}b_{45})$, $b_{125} = PE(b_{32}b_{47} + b_{34}b_{48} + b_{36}b_{49} - b_{30}b_{46}),$

$b_{126} = b_{123} + b_{124} + b_{125}$, $b_{127} = P(b_3 + Eb_{59})e^{PS},$

$b_{128} = 2[b_{16}b_{38}e^{2Sb_{16}} + b_{17}b_{40}e^{2Sb_{17}} + Pb_{41}e^{2PS} + S_c b_{43}e^{2SS_c}]$, $b_{129} = b_{26}b_{44}e^{Sb_{26}}b_{28}b_{45}e^{Sb_{28}} + b_{30}b_{46}e^{Sb_{30}},$

$b_{130} = b_{32}b_{47}e^{Sb_{32}} + b_{34}b_{48}e^{Sb_{34}} + b_{36}b_{49}e^{Sb_{36}}$, $b_{131} = b_{128} + b_{129} - b_{130},$

$b_{132} = b_{127} - PEb_{131}$, $b_{133} = S_c b_{81} - Pb_{82} + 2Eb_{16}b_{63},$

$b_{134} = 2Eb_{17}b_{64} + 2EPb_{65} + 2ES_c b_{66}$, $b_{135} = E(b_{26}b_{67} - b_{28}b_{68} + b_{30}b_{69}),$

$b_{136} = E(b_{32}b_{70} + b_{34}b_{71} + b_{36}b_{72})$, $b_{137} = b_{133} + b_{134} + b_{135} - b_{136},$

$b_{138} = S_c b_{81}e^{SS_c} - Pb_{82}e^{PS} + 2Eb_{16}b_{63}e^{2Sb_{16}}$, $b_{139} = 2E(b_{17}b_{64}e^{2Sb_{17}} + Pb_{65}e^{2PS} + S_c b_{66}e^{2SS_c}),$ $b_{140} = E(b_{26}b_{67}e^{Sb_{26}} - b_{28}b_{68}e^{Sb_{28}} + b_{30}b_{69}e^{Sb_{30}})$, $b_{141} = E(b_{32}b_{70}e^{Sb_{32}} + b_{34}b_{71}e^{Sb_{34}} + b_{36}b_{72}e^{Sb_{36}}),$

$b_{142} = b_{138} + b_{139} + b_{140} - b_{141}$

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