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FREE-CONVECTION MAGNETOHYDRODYNAMIC FLOW FOR NON-NEWTONIAN FLUID PAST POROUS FLAT PLATE

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Abstract

In this paper the steady motion of an electrically conducting, viscous and incompressible non-Newtonian fluid past a porous flat plate under a transverse magnetic filed is considered. Analytically expressions for the velocity, temperature, skin friction and magnetic induction have been obtained by using the perturbation technique. Our results are compared with the previous ordinary Newtonian fluid results. The results have been shown graphically, and the effect of different parameters on the velocity, the magnetic induction, coefficient of skin friction and temperature are discussed in these cases.

Introduction

Previous studies for free convection flow of electrically conducting fluids along a vertical or horizontal flat plates were restricted, in general, to Newtonian fluids only, [Gubta (1), P. C. Lue (2), Masuoka (3)]. Few pay attention to the problem of natural convection of electrically conducting non-Newtonian fluids. However, the study of electrically conducting non-Newtonian fluids is important in a number of geophysical and other engineering applications, such as petroleum drillings. Eldabe. et. al. (4-9) studied some problems of Newtonian and non-Newtonian fluids, which flowing on an infinite plate and between two parallel plates. The problem of free convection flow of an electrically conducting non-Newtonian fluid obeying along a horizontal porous flat plate is considered in this paper and we assumed that the system is stressed by a transverse magnetic filed. The governing partial differential equations for this problem can be transformed into a set of coupled ordinary differential equations, which can be solved analytically by using the perturbation technique.

Basic equations

The constitutive equation for incompressible visco-elastic fluids suggested by [10] is

$$\tau_{ij} = 2\mu \ d_{ij} - 2\lambda \ E_{ij} + 4\psi \ d_i^a d_{aj} , \tag{1}$$

where

$$d_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right), \tag{2}$$

$$E_{ij} = \frac{1}{2}\left(a_{i,j} + a_{j,i} + 2v_{,i}^m v_{m,j}\right), \tag{3}$$

$$a_i \,(\text{Acceleration vector}) = \frac{\partial v_i}{\partial t} + v^m v_{i,m} , \tag{4}$$

v_i is the velocity vector. A comma followed by an index implies covariant differentiation. $\mu, \lambda,$ and ψ are material constants representing, respectively, viscosity, elastico-viscosity, and cross-viscosity coefficients of the fluid.

The basic equations of magnetohydrodynamics (MHD) neglecting the displacement currents and free charges are [11]

$$\nabla \cdot \vec{V} = 0 , \tag{5}$$

$$\nabla \cdot \vec{H} = 0 , \tag{6}$$

$$\nabla \wedge \vec{H} = \vec{J} , \tag{7}$$

$$\vec{J} = \sigma\left(\vec{E} + \mu_e \vec{V} \wedge \vec{H}\right), \tag{8}$$

$$\left(\vec{V} \cdot \nabla\right)\vec{V} = \frac{1}{\rho} F - \frac{1}{\rho}\nabla P + \frac{1}{\rho}\nabla \cdot \tau + \frac{\mu_e}{\rho}\vec{J} \wedge \vec{H} , \tag{9}$$

$$\left(\vec{V} \cdot \nabla\right)T = \frac{k}{\rho\, c}\nabla^2 T + \frac{1}{\rho\, c}\tau_{ij}\frac{\partial v_i}{\partial x_j} + \frac{1}{\rho\, c\, \sigma}\left(\vec{J} \cdot \vec{J}\right), \tag{10}$$

Here $\vec{H}$ is the magnetic field, $\vec{E}$ the electric field, $\vec{J}$ is the current density, F is the body forces per unit mass, T is the temperature, and τ is defined by (1). Also ρ, μ_e, σ and P denote, respectively, the density, magnetic permeability, conductivity and fluid pressure.

Analysis

We consider an electrically conducting non-Newtonian fluid flowing along a horizontal porous wall, such that x and y axes are taken parallel and perpendicular to the wall, respectively. The system is stressed by a transverse uniform magnetic filed of strength H_0, we are take into account the effect of free convection when the body force per unit mass is acting in the negative x-direction. We assume that all variables, except the pressure p are functions of y only, then we have

$\vec{V} = (u, v, 0)$, and $\vec{H} = (H_x, H_y, 0)$,

from (6) we get $H_y = H_0$, is the applied field.

Equation (5) gives

$$v = \text{constant} = -v_0, \text{ (say)}, \tag{11}$$

where v_0 is the suction velocity of the fluid at the wall.

Now we can write the equations which describe our problem as following

$$-\rho\, v_0 \frac{du}{dy} = -\frac{\partial p}{\partial x} - \rho\, g + \mu \frac{d^2u}{dy^2} + \lambda\, v_0 \frac{d^3u}{dy^3} + \mu_e\, H_y \frac{dH_x}{dy} \quad , \tag{12}$$

$$0 = -\frac{\partial p}{\partial y} - \mu_e H_x \frac{dH_x}{dy} \quad , \tag{13}$$

$$0 = \frac{d}{dy}\left(u\, H_0 + v_0 H_x\right) + \frac{1}{\mu_e \sigma} \frac{d^2 H_x}{dy^2} \quad , \tag{14}$$

$$-\rho\, v_0 c \frac{dT}{dy} = k \frac{d^2T}{dy^2} + \mu \left(\frac{du}{dy}\right)^2 + \frac{1}{\sigma}\left(\frac{dH_x}{dy}\right)^2, \tag{15}$$

where g is the acceleration according to gravity.

From equation (13) we find that $\frac{\partial p}{\partial x}$ must be a function of x only, then we can write

$$\frac{\partial p}{\partial x} = \text{Constant} = -\rho_\infty g \quad , \tag{16}$$

Under the assumption of semi-incompressibility for the fluid we can write

$$\rho - \rho_\infty = -\rho\, \beta\, \theta \quad , \tag{17}$$

where β is the thermal coefficient of volumetric expansion.
The appropriate boundary conditions are

$$u(0) = 0, \qquad u(\infty) = u_\infty, \tag{18}$$

$$T(0) = T_0, \qquad T(\infty) = T_\infty, \tag{19}$$

$$H_y(0) = H_0\,, \qquad H_x(\infty) = 0. \tag{20}$$

Let us introduce the non-dimensional quantities as follows

$$\left\{\begin{array}{l}
u = v_0 u^*\,,\ H_x = H_0 h^*\,,\ y = \dfrac{\upsilon}{v_0} y^*\,, q = \dfrac{u_\infty}{v_0}, \theta = \dfrac{T - T_\infty}{T_0 - T_\infty} \\
\lambda = \dfrac{\rho\, v^2 \lambda^*}{v_0^2}\text{, is the elasticity parameter,} \\
S = \dfrac{\mu_e H_0^2}{\rho\, v_0^2}\text{, is the magnetic parameter,} \\
v = \dfrac{\mu}{\rho}\text{, is the kinematic viscosity ,} \\
\phi = \dfrac{\rho}{\mu\, \mu_e \sigma}\text{, is the reciprocal of magnetic Rynolds number,} \\
P_r = \dfrac{\mu\, c}{k}\text{, is the Prandtl number ,} \\
G = \dfrac{\mu\, \beta\, g (T_0 - T_\infty)}{\rho\, v_0^3}\text{, is the Grashof number,} \\
E_c = \dfrac{v_0^2}{c(T_\infty - T_0)}\text{, is the Eckart number.}
\end{array}\right\} \tag{21}$$

by virtue of equation (21) , equation (12-15) with boundary conditions become after dropping the star mark

$$\lambda\, u''' + u'' + u' + Sh' = -G\theta\,, \tag{22}$$

$$(u-q) + h + \phi\, h' = 0\,, \tag{23}$$

$$\theta'' + P_r\theta' = -P_r E_c (u')^2 - \lambda\, P_r E_c u' u'' - S\phi\, P_r E_c (h')^2\,, \tag{24}$$

where dash denotes the differentiation with respect to y.
And the boundary conditions

$$\left.\begin{cases} u = 0\,, \theta = 1\,, & at\ y = 0 \\ u \to q\,, \theta \to 0\,, h \to 0\ as\ y \to \infty \end{cases}\right\}. \tag{25}$$

Equation (22) reduces to that governing the flow of a Newtonian fluid if $\lambda = 0$. Also, we find that the order of the governing equation increases from two to three owing to the presence of the elastic property of the fluid.
Equation (24) for very small values of Eckart number becomes

$$\theta'' + P_r\theta' = 0\,. \tag{26}$$

The general solution of (26) using the boundary conditions (25) for the temperature is

$$\theta = e^{-p_r y}\,, \tag{27}$$

using (27) with (22) and integrating the resultant equation with respect to y we get

$$h = \frac{1}{S}\left(\frac{G}{p_r}e^{-p_r y} - \lambda\,(u-q)'' - (u-q)' - (u-q)\right)\,, \tag{28}$$

inserting (28) in (23) we get

$$\phi\,\lambda\, f''' + (\phi+\lambda)\, f'' + (\phi+1)\, f' - (S-1)\, f = W e^{-p_r y}\,, \tag{29}$$

where $f = u - q$ and $W = -G\left(\phi - \dfrac{1}{p_r}\right)$.

Now we assume the solution of equation (29) in the form

$$f = f_0 + \lambda\, f_1\,, \tag{30}$$

this assumption is satisfied in the case of liquids with short memories, see[10].
Inserting (30) in (29) and equating the coefficients of λ, we obtain

$$\phi\, f_0'' + (\phi+1)\, f_0' - (S-1)\, f_0 = W e^{-p_r y} \quad , \tag{31}$$
$$\phi\, f_1'' + (\phi+1)\, f_1' - (S-1)\, f_1 = -\phi\, f_0''' - f_0'' \quad , \tag{32}$$

with appropriate boundary conditions

$$\begin{Bmatrix} f_0 = -q\,, & f_1 = 0\,,\, at\ y = 0 \\ f_0 \to 0\,, & f_1 \to 0\,,\, as\, y \to \infty \end{Bmatrix}. \tag{33}$$

The solutions of the system of equations (31) and (32) subject to the boundary conditions (33) is obtained analytically and gives the following form of the velocity distribution of the fluid,

$$u = e^{-\gamma y}(\lambda\, W_2 y - \lambda\, W_1 - z - q) + e^{-p_r y}(\lambda\, W_1 + z) + q \quad . \tag{35}$$

Where

$$\gamma = \frac{(\phi+1) + \sqrt{(\phi+1)^2 + 4\,\phi\,(S-1)}}{2\,\phi} \quad , \ S \ge 1$$

$$z = \frac{W}{\phi\, p_r^2 - (\phi+1)p_r - (S-1)} \quad ,$$

$$W_1 = \frac{(p_r\,\phi - 1)z^2 p_r^2}{W} \quad , \qquad W_2 = \frac{(q+z)(1-\gamma\,\phi)\gamma^2}{\phi - 2\,\gamma\,\phi + 1} \quad .$$

Now to obtain the solution of the magnetic induction h, we shall assume that

$$h = h_0 + \lambda h_1 \,, \tag{36}$$

by using (35) and (36) with (28) we get

$$h = \frac{1}{S}\left[\begin{aligned} & e^{-\gamma y}\big((q+z)(\lambda\gamma^2 - \gamma + 1) + \lambda\, W_1(1-\gamma) + \lambda\, W_2(\gamma\ y - y - 1)\big) \\ & + e^{-p_r y}\left(\frac{G}{p_r} + \lambda\, W_1(p_r - 1) - z(\lambda p_r^2 - p_r + 1)\right) \end{aligned}\right] \quad . \tag{37}$$

The coefficient of skin-friction in the dimensionless form for non-Newtonian fluid at the plate takes the form

$$C_f = \left.\frac{du}{dy} + \lambda \frac{d^2 u}{dy^2}\right|_{y=0} \quad ,$$

from (35) we get

$$C_f = \gamma(q+z) - p_r z + \lambda\left(\gamma W_1 + W_2 - p_r W_1 - \gamma^2 (q+z) + p_r^2 z\right).$$

Results and discussion

The system of equations for the problem is solved analytically by using perturbation method, to determine the expressions for temperature, velocity, magnetic induction and skin friction. The effect of various parameters of the problem are discussed to illustrate the different between Newtonian ($\lambda = 0$) and non-Newtonian fluids ($\lambda \neq 0$). The results of present investigation are equivalently in agreement with the general results, although different types of flow and values are considered.

Figures (1-5) illustrate the effect of the elasticity parameter λ, Grashof number G, Prandtl number P_r, the reciprocal of magnetic Rynolds number ϕ and the magnetic parameter S on the velocity profile. It is clear that the velocity increases as λ, G, P_r and ϕ increases, while the velocity decreases as S increase. From figures (6-10) we notes that the magnetic induction increases when S, P_r and ϕ decreases, and increases with the increasing of both λ and G. Figure (11) shows that the temperature decreases with increasing of Prandtl number. Finally it seems from figures (12) and (13) that the coefficient of skin friction increases when λ decreases and both P_r and S increases.

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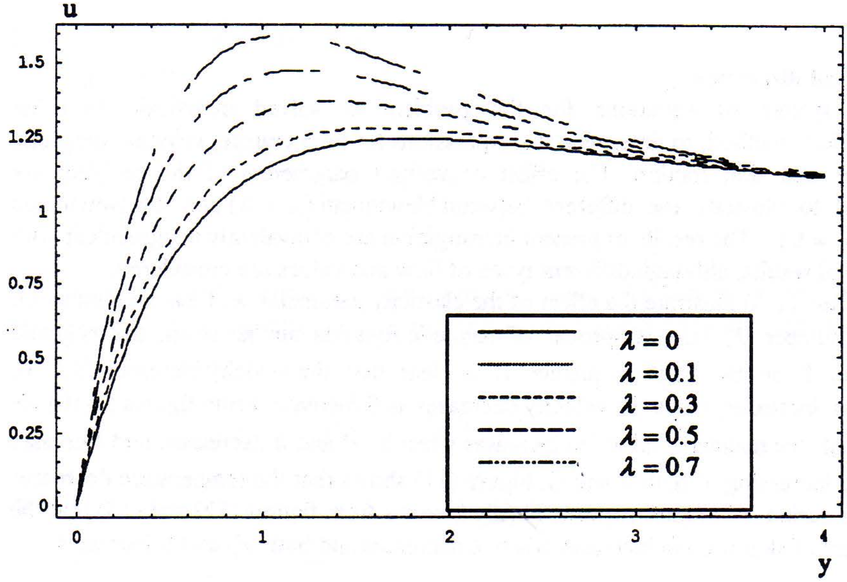

Figure (1) the velocity plotted versus position for $q = 1, \phi = 3, G = 2, p_r = 0.5, S = 2$.

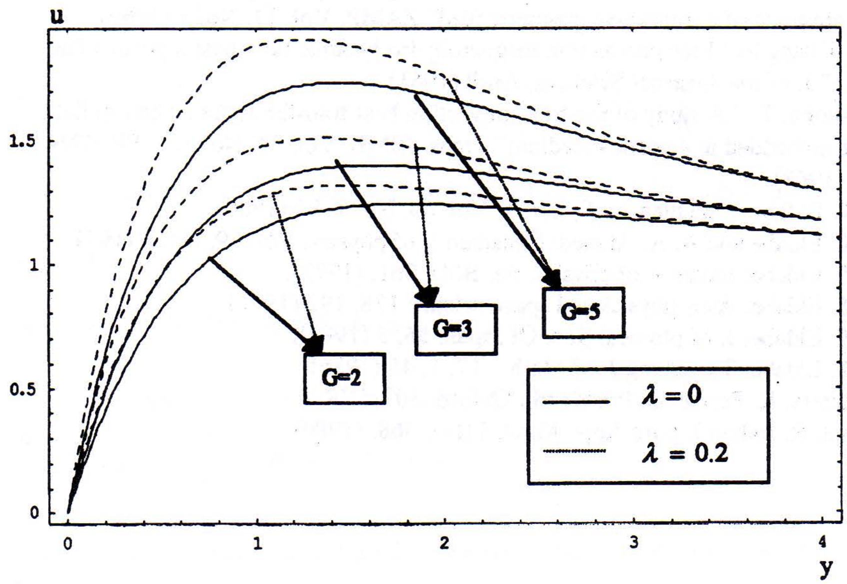

Figure (2) the velocity plotted versus position for $q = 1, \phi = 3, p_r = 0.5, S = 2$.

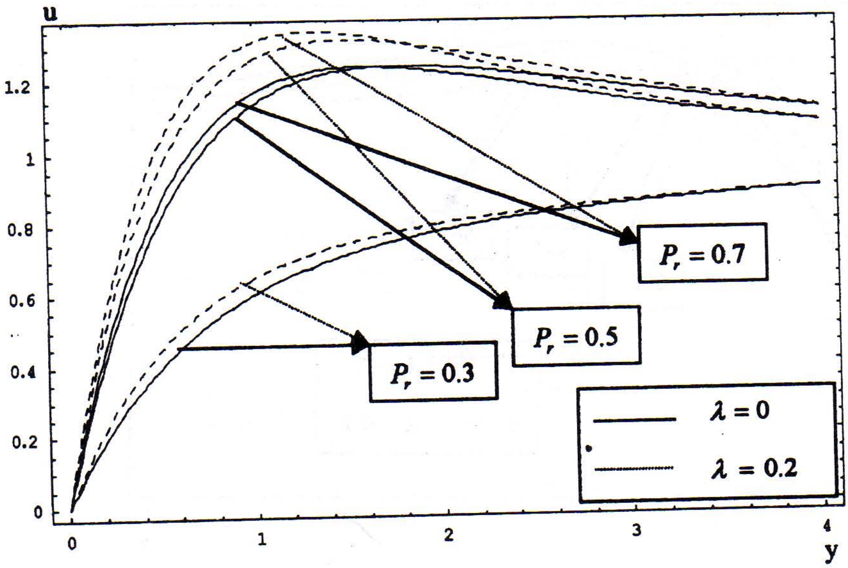

Figure (3) the velocity plotted versus position for
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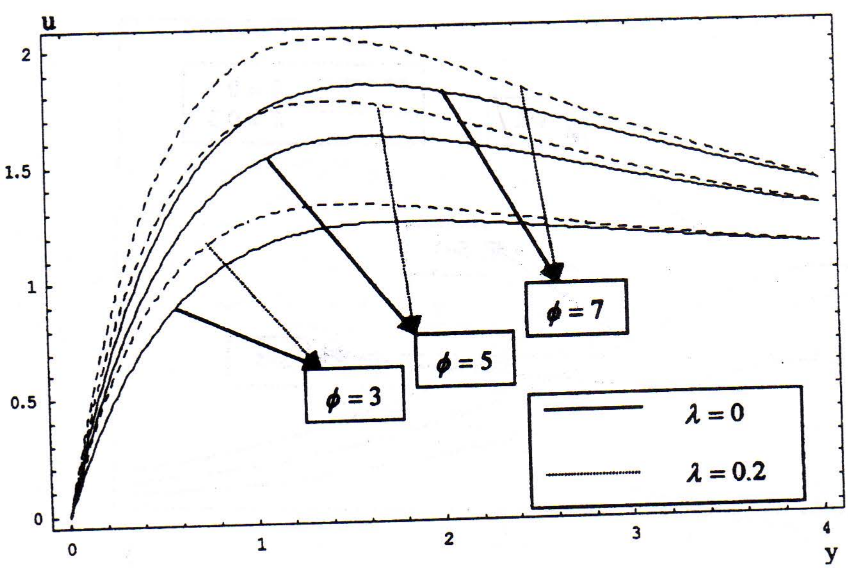

Figure (4) the velocity plotted versus position for
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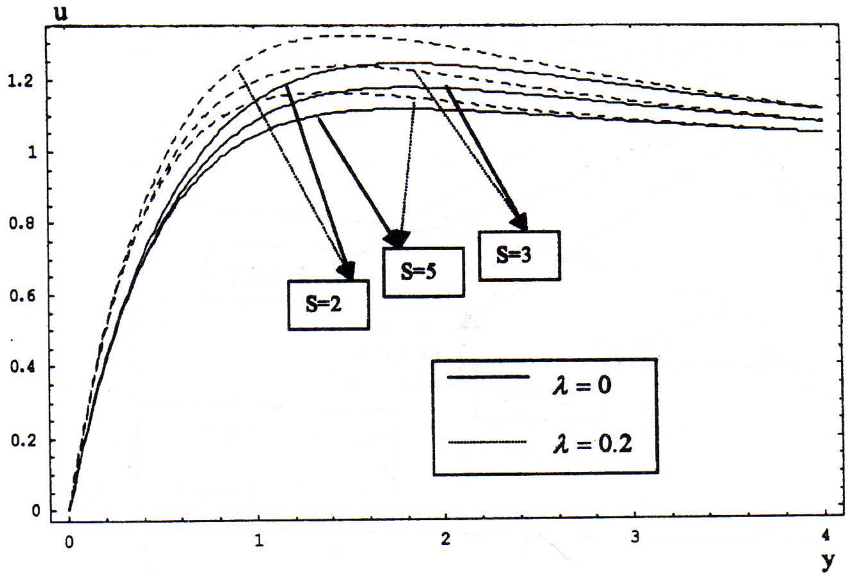

Figure (5) the velocity plotted versus position for
$q=1$, $G=2$, $p_r=0.5$, $\phi=3$.

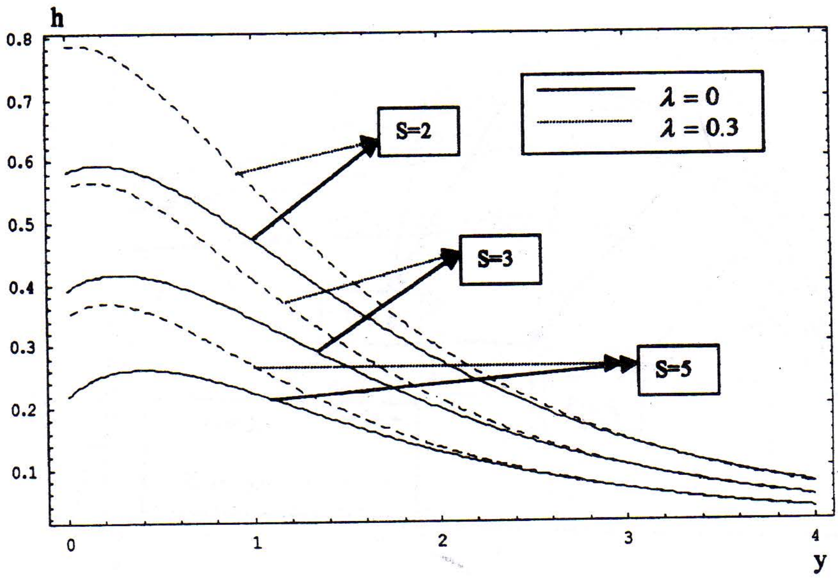

Figure (6) the magnetic induction plotted versus position for
$q=1$, $G=2$, $p_r=0.7$, $\phi=3$.

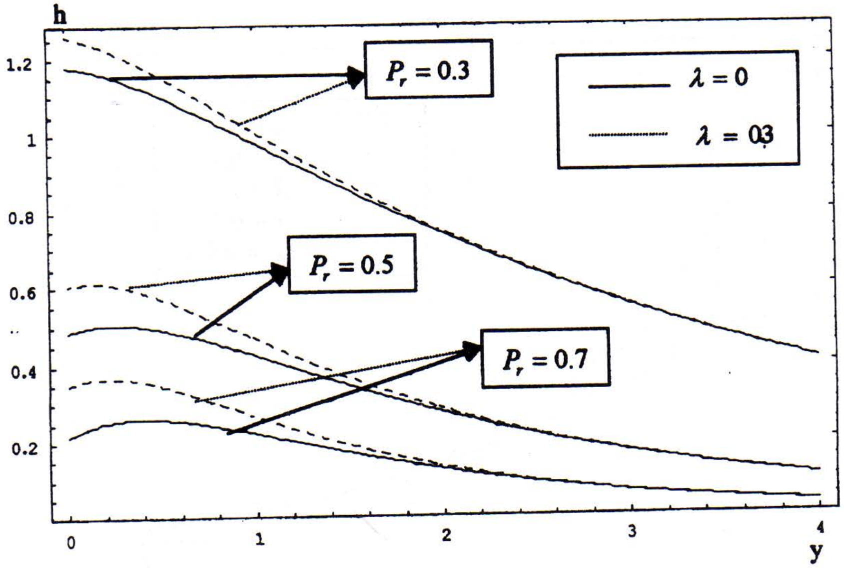

Figure (7) the magnetic induction plotted versus position for
$q = 1$, $G = 2$, $S = 5$, $\phi = 3$.

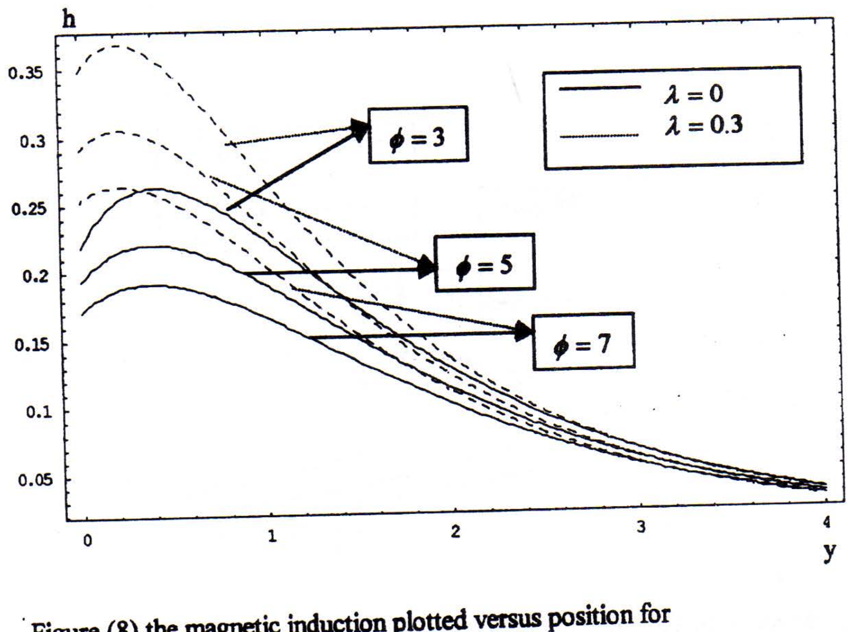

Figure (8) the magnetic induction plotted versus position for
$q = 1$, $G = 2$, $p_r = 0.7$, $S = 5$.

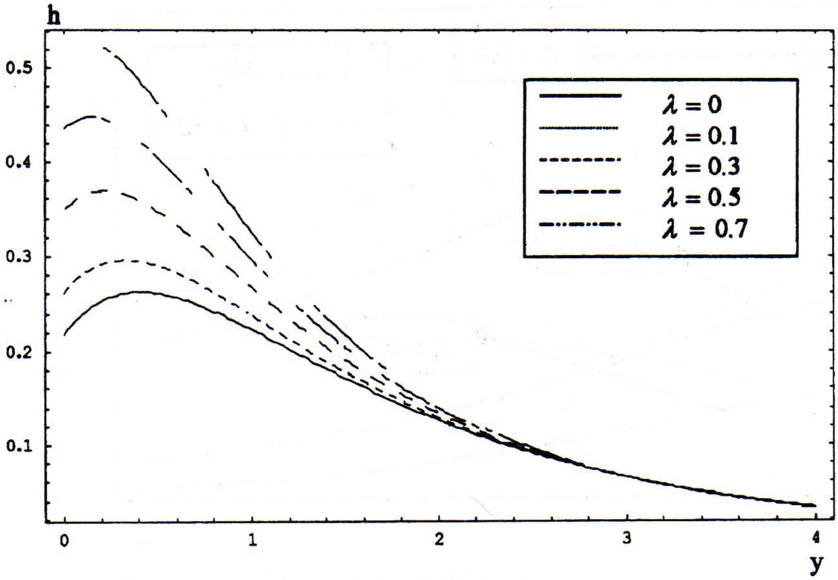

Figure (9) the magnetic induction plotted versus position for $q = 1$, $G = 2$, $p_r = 0.7$, $\phi = 3$, $S = 5$.

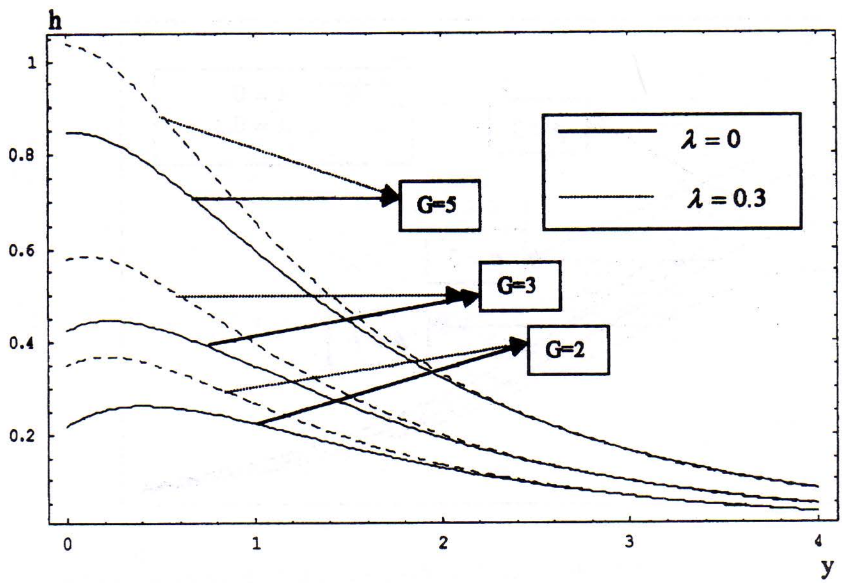

Figure (10) the magnetic induction plotted versus position for $q = 1$, $S = 5$, $p_r = 0.7$, $\phi = 3$.

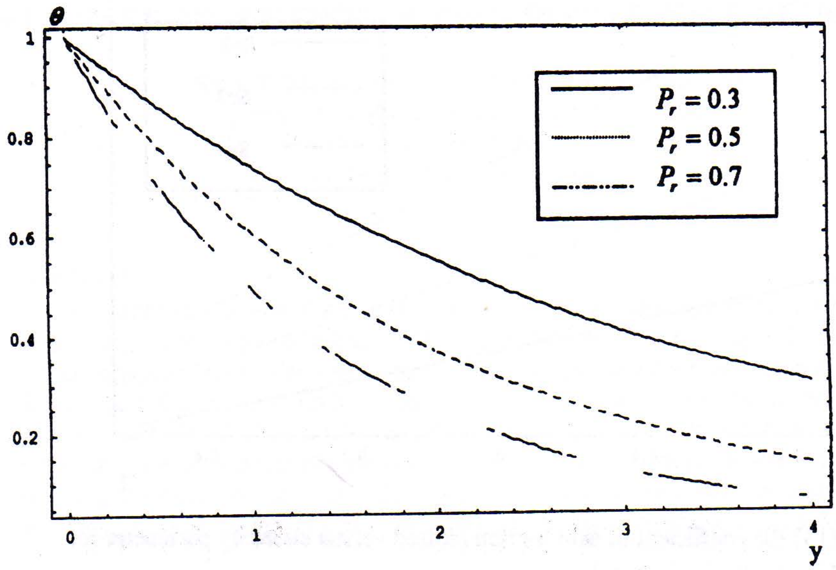

Figure (11) the temperature plotted versus position for $p_r = 0.3$, $p_r = 0.5$, $p_r = 0.7$.

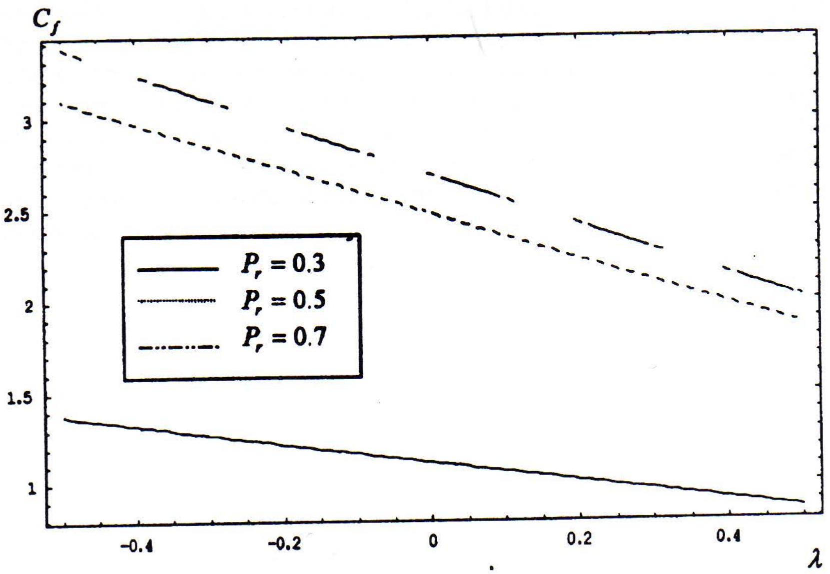

Figure (12) the coefficient of skin friction plotted versus elasticity parameter for $q = 1$, $S = 2$, $G = 2$, $\phi = 3$.

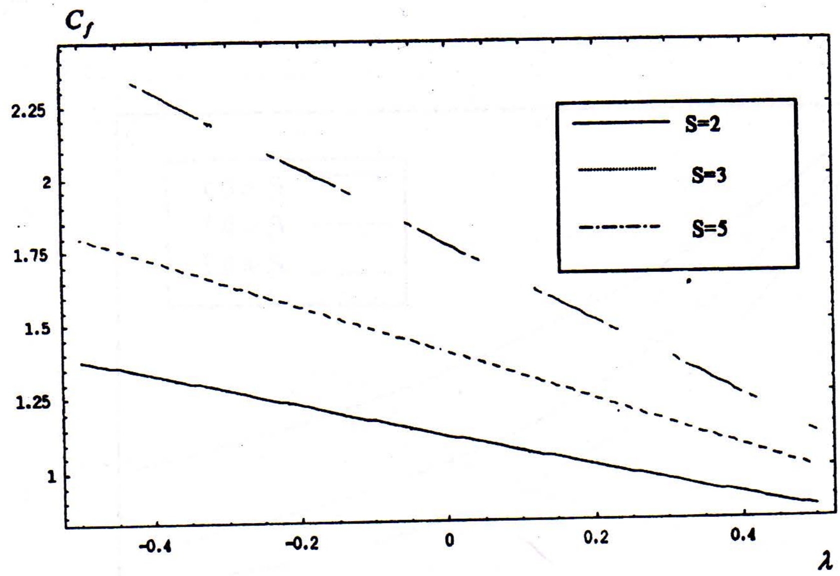

Figure (13) the coefficient of skin friction plotted versus elasticity parameter for $q = 1$, $G = 2$, $p_r = 0.3$, $\phi = 3$.