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NONSIMILARITY SOLUTIONS FOR MIXED CONVECTION FROM VERTICAL PLATE EMBEDDED IN A POROUS MEDIUM WITH VARIABLE PERMEABILITY – VARIABLE SURFACE HEAT FLUX

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Abstract

The problem of mixed convection along non-isothermal vertical flat plate embedded in a porous medium with variable permeability is analyzed. Non-similar solutions are obtained for the power- law variation of the surface heat flux in the form $q_w(x) = bx^m$. The entire mixed convection regime is covered by non-similarity parameter $\zeta = \left[1 + (Ra_x / Pe_x^{3/2})^{1/3}\right]^{-1}$, from pure forced convection $\zeta = 1$ to pure free convection $\zeta = 0.0$. A finite difference scheme was used to solve the system of transformed governing equations. Velocity and temperature profiles, and local Nusselt numbers are presented. It is found that as ζ decreases from 1 to 0, the thermal boundary thickness increases first and then decreases. But the local Nusselt number in the form $Nu_x(Pe_x^{1/2} + Ra_x^{1/3})^{-1}$ decreases first and then increases. The variation of permeability increases Nusselt number of all values of ζ.

Introduction

The problem of natural convection or mixed convection boundary layer flow along an impermeable surface embedded in fluid a saturated porous medium has received much attention in recent years. Many of studies [1-5] were based on Darcy`s law, which neglects the viscous force acting on the impermeable surface. From the work of Hong et al. [6], Darcy's model's is still acceptable, especially when the flow velocity is low and the heat transfer is of interest.

Hsieh et al. [7] reported non-similar solutions for the problem of mixed convection along a vertical flat plate embedded in porous media by dividing the entire mixed convection regime into two regions, one covers the forced convection dominated regime and the others covers the free convection regime. Two different non-similarity parameters were found to characterize the two separate regions.

Most of the published results are limited to situations in which similarity solutions exist [1,2]. A general similarity transformation for mixed convection flow in porous media was reported by Nakayama and Koyama [8] for different types of geometries. However the flow and thermal fields in mixed convection from surfaces in porous medium are non-similar in nature. Nakayama and Pop [9] proposed a unified similarity transformation to cover all possible similarity solutions for free, forced and mixed convection within Darcy and non-

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Darcy porous media, but the cases they considered for solutions were restricted to the local similarity approximations. Chamakha and Khanafer [10] formulated the problem of combined forced-free convection flow over an isothermal vertical surface embedded in a variable porosity porous medium with heat generation or absorption. Verma and Vyas [11] investigated the flow of fluid past a porous spherical shell when the permeability at any point of the shell varies as some power of its radial distance from the center. Chandrasekhara et al. [12] studied the influence of variable permeability on the basic flows in porous media, their results show that the variable permeability have an appreciable influence on the heat transfer rate. Ibrahim and Hassanien [16] repoted non-similarity solutions for the variable permeability on combined convection along a non-isothermal wedge in a saturated porous medium. The problem of Nonsimilarity solutions for mixed convection flow along non-isothermal vertical surfaces embedded in porous media with variable permeability formalited by Hassanien and Omer[17]. A nonsimilar boundary layer analysis was analyzed for the problem of non-Darcian mixed convection in power-law type non-Newtonian fluids along a vertical plate embedded in a fluid-saturated porous media by Ibrahim[18].

The objective of the present work is to investigate the influence of variable permeability $K(y)$ on mixed convection heat transfer along a vertical surface embedded in porous media, under the condition of power low variation of surface heat flux in the form $q_w(x) = bx^m$. The single parameter $\zeta = \left[1 + (Ra_x / Pe_x^{3/2})^{1/3}\right]^{-1}$, which varies from 1 for pure forced convection to 0 for pure free convection, is introduced to cover the entire mixed convection regime. Numerical results were obtained by using a finite difference scheme to solve the transformed systems of equations. Results of major interest, such as temperature profiles, velocity profiles, and the local Nusselt number are presented for some representative values of the power–law variation of surface heat flux for the cases of uniform and variable permeability.

Analysis

Consider the problem of mixed convection along an impermeable vertical flat plate embedded in a fluid saturated porous medium. The vertical plate is assumed to be heated in such away that its surface heat flux varies in the power-law form, $q_w(x) = bx^m$ where b is a constant and m is the exponent. The axial and normal coordinates are x and y. The gravitational acceleration g is acting downward in the direction opposite to the x coordinate. For the mathematical analysis of the problem, fluid properties are assumed to be constant expect for variations in density, permeability and thermal resistance are functions of vertical coordinate y, and the porous media is treated as isotropic. In addition, the flow velocity and the pores of the porous medium are assumed to be small for the Darcy`s model to be valid [6] with these assumptions and the applications of Boussinesq and boundary layer approximations, the governing system of conservation equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{K(y) g \rho_\infty \beta}{\mu} \frac{\partial T}{\partial y} + \frac{u}{K(y)} \frac{\partial K(y)}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial Y}=\alpha(y)\frac{\partial^2 T}{\partial y^2}+\frac{\partial\alpha(y)}{\partial y}\frac{\partial T}{\partial y} \tag{3}$$

In the above equations, u and v are Darcy`s velocities in the x and y directions; T is the temperature; ρ, β and μ are the density, thermal expansion coefficient of the fluid and dynamic viscosity. K (y) and α (y) are respectively, the permeability and equivalent thermal diffusivity of the porous media.

Here we assume that the porosity $\varepsilon(y)$ and the permeability $K(y)$ vary exponential from the wall [13].

$$K(y)=K_\infty(1+de^{-y/k}),\qquad \varepsilon(y)=\varepsilon_\infty(1+d^*e^{-y/k}) \tag{4}$$

Where $K_\infty, \varepsilon_\infty$ are the permeability and porosity at the edge of the boundary layer d, d^* are constants, where values taken 3.0 and 1.5, respectively [13]. Further, $\alpha(y)=\lambda_m(y)/(\rho_\infty c_p)_f$ also varies since it is related to the effective thermal conductivity of the saturated porous medium $\lambda_m(y)$. $\lambda_m(y)$ can be computed according to the following semi-analytical expression given by Nayagam et al. [14]. $\lambda_m=\lambda_f\varepsilon+(1-\varepsilon)\lambda_s$, Where λ_f and λ_s are the thermal conductivities of the fluid and solid, respectively.
Hence the expression for the thermal diffusivity has the form

$$\alpha(y)=\alpha_\infty[\varepsilon_\infty(1+d^*e^{-y/k})+\sigma(1-\varepsilon_\infty(1+d^*e^{-y/k}))], \tag{5}$$

where

$\alpha_\infty=\lambda_f/(\rho_\infty c_p)_f$ and $\sigma=\lambda_s/\lambda_f$

The boundary conditions for the present problem are

$$v=0\quad,\quad q_w(x)=bx^m\quad \text{at}\quad y=0 \tag{6}$$

$$u\to U_\infty\quad,\quad T\to T_\infty\quad \text{as}\quad y\to\infty \tag{7}$$

Where b and m are prescribed constants.
It is noted that the case of uniform surface heat flux corresponds to m=0.
To facilitate the present analysis, the system of equations (2-4) will be transformed into dimensionless form by introducing

$$\eta=\frac{y}{x}Pe_x^{1/2}\zeta^{-1} \tag{8}$$

$$\psi=\alpha Pe_x^{1/2}\zeta^{-1}f(\zeta,\eta)\quad,\qquad \theta(\zeta,\eta)=\frac{(T-T_\infty)Pe_x^{1/2}\zeta^{-1}}{q_w(x)x/K_\infty}, \tag{9}$$

$$\zeta=\left[1+\left(\frac{Ra_x}{Pe_x^{3/2}}\right)^{1/3}\right]^{-1} \tag{10}$$

Where the stream function ψ satisfies the continuity equation (1) with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, $Pe_x = U_\infty x/\alpha$ is the local Peclet number, $Ra_x = g\beta q_w(x)Kx^2/k\nu\alpha$ is the local Rayleigh number, and ζ is the non-similarity mixed convection parameter and we have choose $k = x Pe_x^{-1/2}\zeta$ such that K(y) and $\alpha(y)$ are purely function of η only.

Substituting equations (8)-(10) into equations (2)-(4) one can obtain the following system of equations: -

$$f'' = (1-\zeta)^3(1+de^{-\eta})\theta' - \left(\frac{de^{-\eta}}{1+de^{-\eta}}\right)f' \tag{11}$$

$$\begin{aligned} A\theta'' = &-\frac{1}{2}[1+\frac{1}{3}(2m+1)(1-\zeta)]f\,\theta' + \frac{2m+1}{2}[1-\frac{1}{3}(1-\zeta)]f'\theta \\ &+\frac{1}{6}(2m+1)\zeta(1-\zeta)\left[\theta'\frac{\partial f}{\partial \zeta} - f'\frac{\partial \theta}{\partial \zeta}\right] + \varepsilon_\infty d^* e^{-\eta}(\sigma-1)\theta' \end{aligned} \tag{12}$$

Where

$$A = \left|\varepsilon_\infty + \sigma(1-\varepsilon_\infty) + \varepsilon_\infty d^* e^{-\eta}(1-\sigma)\right|$$

With the boundary conditions

$$\begin{gathered} \frac{1}{2}[1+\frac{1}{3}(2m+1)(1-\zeta)]f(\zeta,0) - \frac{1}{6}(2m+1)\zeta(1-\zeta)\frac{\partial f(\zeta,0)}{\partial \zeta} = 0 \quad \text{or} \quad f(\zeta,0) = 0 \\ \theta'(\zeta,0) = -1, \qquad f'(\zeta,\infty) = \zeta^2, \qquad \theta(\zeta,\infty) = 0 \end{gathered} \tag{13}$$

The primes in equations (11)-(13) denote partial differentiation with respect to η.

The system of equations (11)-(13) for the variable heat flux can be solved using a finite differences method.

We now obtain an approximate solution to equations (11)-(13) based on the local similarity and local non-similarity methods [19]. For the first level of truncation the ζ derivatives in equations (12) and (13) can be neglected. Thus, the governing equations for the first level of the truncation are equation (11) and the following equation:-

$$A\theta'' = -\frac{1}{2}[1+\frac{1}{3}(2m+1)(1-\zeta)]f\theta' + \frac{2m+1}{2}[1-\frac{1}{3}(1-\zeta)]f'\theta + \varepsilon_\infty d^* e^{-\eta}(\sigma-1)\theta'\} \tag{14}$$

With the boundary conditions.

$$\begin{aligned} f(\zeta,0) &= 0, & \theta'(\zeta,0) &= -1 \\ f'(\zeta,\infty) &= \zeta^2, & \theta(\zeta,\infty) &= 0 \end{aligned} \tag{15}$$

For the second level of truncation, we introduce $G = \partial f/\partial\zeta$ and $\phi = \partial\theta/\partial\zeta$, and restore all of neglected terms in the first level of truncation. Thus, the governing equations are equation (11) and the following equation: -

$$A\theta'' = -\frac{1}{2}[1+\frac{1}{3}(2m+1)(1-\zeta)]f\theta' + \frac{2m+1}{2}[1-\frac{1}{3}(1-\zeta)]f'\theta$$
$$+\frac{1}{6}(2m+1)\zeta(1-\zeta)[\theta' G - f'\phi] + \varepsilon_\infty d^* e^{-\eta}(\sigma-1)\theta' \tag{16}$$

With the same boundary conditions defined in equation (15)

The introduction of the two new dependent variables G and ϕ in the problem requires two additional equations with appropriate boundary conditions. This can be obtained by differentiating equation (11), (15), and (16) with respect to ζ and neglecting the terms $\partial G/\partial\zeta$ and $\partial\phi/\partial\zeta$, which leads to

$$G'' = -3(1-\zeta)^2(1+de^{-\eta})\theta' + (1-\zeta)^3(1+de^{-\eta})\phi' - \left(\frac{de^{-\eta}}{1+de^{-\eta}}\right)G' \tag{17}$$

$$A\phi'' = -\frac{1}{2}[1+\frac{1}{3}(2m+1)(1-\zeta)](f\phi' + G\theta') + \frac{2m+1}{2}[1-\frac{1}{3}(1-\zeta)]$$
$$(f'\phi + \theta G') + \frac{2m+1}{2}f'\theta + \frac{1}{2}[1+\frac{1}{3}(2m+1)]f\theta' + \frac{1}{6}(2m+1) \tag{18}$$
$$\zeta(1-\zeta)[G\phi' - \phi G'] + \frac{1}{6}(2m+1)(1-2\zeta)[\theta' G - f'\phi] - \varepsilon_\infty d^* e^{-\eta}(1-\sigma)\phi'$$

The corresponding boundary conditions are given as

$$\phi(\zeta,0)=0,\ G(\zeta,0)=0,\ G'(\zeta,\infty)=2\zeta,\ \phi(\zeta,\infty)=0 \tag{19}$$

The physical quantities of interest include the velocity components u and v, the wall shear stress τ_w defined as $\tau_w = \mu(\partial u/\partial y)_{y=0}$ and the local Nusselt number $Nu_x = hx/\lambda_f$ where $h = q_w(x)/(T_w - T_\infty)$. They are given by:-

$$u = U_\infty \zeta^{-2} f' \tag{20}$$

$$v = -(\frac{\alpha_\infty}{x})Pe_x^{1/2}\zeta^{-1}\{[\frac{1}{2}+\frac{1}{6}(2m+1)(1-\zeta)]f - [\frac{1}{2}-\frac{1}{6}(2m+1)$$
$$(1-\zeta)]\eta f' - \frac{1}{6}(2m+1)\zeta(1-\zeta)\frac{\partial f}{\partial\zeta}\} \tag{21}$$

$$\tau_w(\frac{x^2}{\mu\alpha})(Pe_x^{1/2} + Ra_x^{1/3})^{-3} = f''(\zeta,0) \tag{22}$$

$$Nu_x(Pe_x^{1/2} + Ra_x^{1/2})^{-1} = \frac{1}{\theta(\zeta,0)} \tag{23}$$

Numerical scheme

The numerical scheme to solve the coupled nonlinear equations (11),(12),(14),(16), (17) and (18) with boundary conditions (13),(15) and (19) is based on the following concepts :

(i) The boundary conditions for $\eta \to \infty$ are replaced by
(ii)

$$f'(\zeta, \eta_{max}) = \zeta^2, \ \theta(\zeta, \eta_{max}) = 0$$
$$G'(\zeta, \eta_{max}) = 2\zeta, \ \phi(\zeta, \eta_{max}) = 0 \quad (24)$$

Where η_{max} is sufficiently value of η where the boundary conditions for velocity field is satisfied.

(ii) The two-dimensional domain of interest, (ζ, η) is discretized with an equi-spaced mesh in the ζ direction and another equi-spaced mesh in the η direction.

(iii) The partial derivatives with respect to ζ and η are all evaluated by the central difference approximations.

(iv) Two iteration loops based on the successive substitution are used because of the nonlinearly of the equations.

(v) In each inner iteration loop, the value of ζ is fixed while each of the equations is solved as linear second–order boundary value problem of ordinary differential on the η domain. The inner iteration is continued until the nonlinear solution converges for the fixed value of ζ.

(vi) In the outer iteration loop, the value of ζ is advanced from 0 to1. The derivatives with respect to ζ are updated after every outer iteration step.

More details on the numerical scheme are explained in Pereyra[20]. It is worth noting that step size of $\Delta\eta = 0.02$ and $\Delta\zeta = 0.05$, and η_∞ of 8-18 were found to be satisfactory for a convergence with relative error of 10^{-4} in nearly all cases.

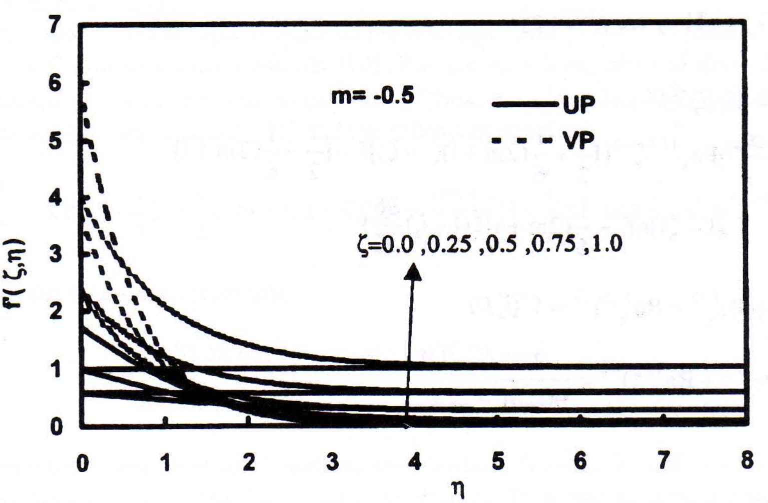

Fig.1. Velocity profile for various values of ζ with m = -0.5

Results and Discussions

Numerical results are obtained for varies values of m and ζ, for both uniform permeability (UP), i.e., d=d*=0 and variable permeability (VP), i.e., $d, d^* \neq 0$ cases. The mixed convection parameter ζ ranged between 0 and 1, increments of 0.05. For the purpose of numerical integration we have assumed d=3.0, d*=1.5 and $\varepsilon = 0.4$ (see Chandrasekara et al. [12].

To asses the accuracy of our results, we have shown a comparison of our results with those Hsieh et al. [7] in Table 1 for the flow over a flat plate, in the case of uniform permeability. A comparison of our results with those from the literature indicates that the agreement between the two good.

Figures (1-8) show the dimensionless velocity profiles $f'(\zeta,\eta)$ and temperature profiles $\theta(\zeta,\eta)$ at selected values of m and ζ for both UP and VP cases. It can be seen from figures (1-4) that at a given value of ζ the velocity gradient at the wall increases and the momentum boundary layer thickness decreases as m increases.

Also from figures (5-8) we can see that for a given value of ζ, as m increases the thermal boundary layer thickness decreases and the temperature gradient at the wall increases. This means that a higher value of the heat transfer rate is associated a higher value of m. Further, from these figures variable permeability effect increase the velocity and reduce the thermal boundary leading to an enhancement of heat transfer rate. Figure 9 shows the local Nusselt number in terms of $1/\theta(\zeta,0)$ or $Nu_x(Pe_x^{1/2} + Ra_x^{1/3})^{-1}$ at selected values of m for UP and VP cases. At a given value of ζ as m increases the Nusselt number increases. This increment is higher for VP case than UP one. It is also seen the Nusselt number curve initially decreases as ζ increases from 0 until they reach a minimum value and therefore increases as ζ increases further to 1.0. These local minima occur at certain values of ζ which depends on the values of m as well as the variation of the permeability.

Conclusions

In this paper, nonsimilarity solutions for mixed convection from vertical plate embedded in porous medium with variable permeability has been analyzed for the case of power–law variation in surface heat flux.

The entire mixed convection regime is covered by a single nonsimilarity parameter $\zeta = \left[1 + (Ra_x / Pe_x^{3/2})^{1/3}\right]^{-1}$ from pure forced convection ($\zeta = 1$) to pure free convection ($\zeta = 0$). The boundary layer equations were solved numerically by means of finite difference method. Profiles for the velocity and temperature fields as well as the variation of the local heat transfer fields at the wall with ζ are presented at selected values of the exponent m for the surface heat flux for uniform and variable permeability. The results clearly indicate that the local Nusselt number increase with increasing value of the exponent m for a given ζ for both uniform and variable permeability cases.

ζ	Hsieh et al .[7]				Present results			
	m=-0.5	m=0.0	m=0.5	m=1.0	m=-0.5	m=0.0	m=0.5	m=1.0
0.0	0.5818	0.7979	0.8998	0.9999	0.58187	0.77158	0.89987	1.00004
0.1	0.5257	0.7127	0.8154	0.9070	0.52589	0.6987	0.81592	0.9076
0.2	0.4749	0.6364	0.7448	0.8313	0.47519	0.63565	0.74558	0.83209
0.3	0.4315	0.5790	0.6930	0.7791	0.43181	0.5858	0.69335	0.77913
0.4	0.3986	0.5508	0.6623	0.7588	0.39895	0.55486	0.66728	0.75892
0.5	0.3812	0.5548	0.6765	0.7800	0.38146	0.55126	0.67855	0.78443
0.6	0.3840	0.5853	0.7248	0.8258	0.38497	0.58145	0.73173	0.8571
0.7	0.4112	0.6350	0.8058	0.9463	0.41142	0.64014	0.81412	0.95851
0.8	0.4563	0.6983	0.9064	1.0672	0.45528	0.71524	0.91142	1.07406
0.9	0.5082	0.7715	1.0159	1.1968	0.50816	0.79871	1.01731	1.19865
1.0	0.5642	0.8863	1.1284	1.3294	0.56419	0.88623	1.12838	1.32934

Table 1. Comparison values of $Nu_x/(Pe_x^{1/2}+Ra_x^{1/3})=1/\theta(\zeta,0)$ at selected values of ζ and m for uniform permeability (UP)case

	m=-0.5		m=0.0		m=0.5		m=1.0	
ζ	UP	VP	UP	VP	UP	VP	UP	VP
0.0	0.58187	0.92905	0.77158	1.12259	0.89987	1.2624	1.00004	1.3745
0.1	0.52589	0.85679	0.6987	1.03599	0.81592	1.16541	0.9076	1.26916
0.2	0.47519	0.79472	0.63565	0.96426	0.74558	1.08746	0.83209	1.18668
0.3	0.43181	0.74846	0.5858	0.91476	0.69335	1.03769	0.77913	1.1381
0.4	0.39895	0.72568	0.55486	0.89789	0.66728	1.02972	0.75892	1.14014
0.5	0.38146	0.73411	0.55126	0.9234	0.67855	1.0751	0.78443	1.20505
0.6	0.38497	0.77508	0.58145	0.99058	0.73173	1.16817	0.8571	1.32109
0.7	0.41142	0.83997	0.64014	1.08628	0.81412	1.28973	0.95851	1.46447
0.8	0.45528	0.91709	0.71524	1.19714	0.91142	1.42643	1.07406	1.62291
0.9	0.50816	0.99839	0.79871	1.31511	1.01731	1.57169	1.19865	1.79108
1.0	0.56419	1.07987	0.88623	1.43591	1.12838	1.72174	1.32934	1.9655

Table 2. Results for the local Nusselt number $Nu_x/(Pe_x^{1/2}+Ra_x^{1/3})=1/\theta(\zeta,0)$ at selected values of ζ and m for uniform permeability (UP) and variable permeability (VP) cases.

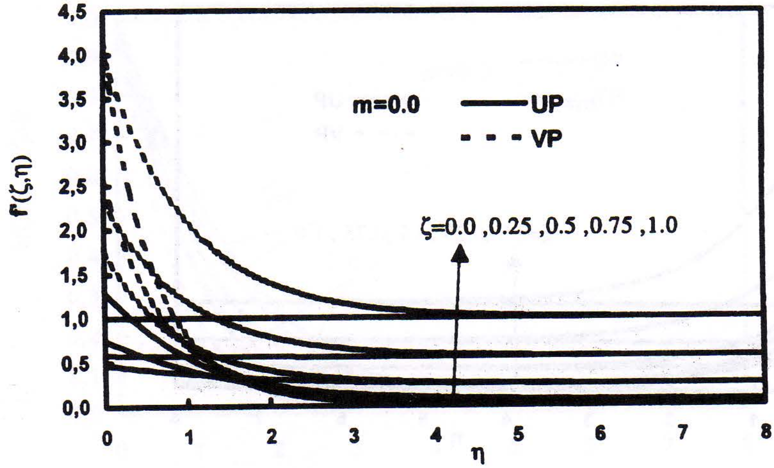

Fig.2. Velocity profile for various values of ζ with m = 0.0

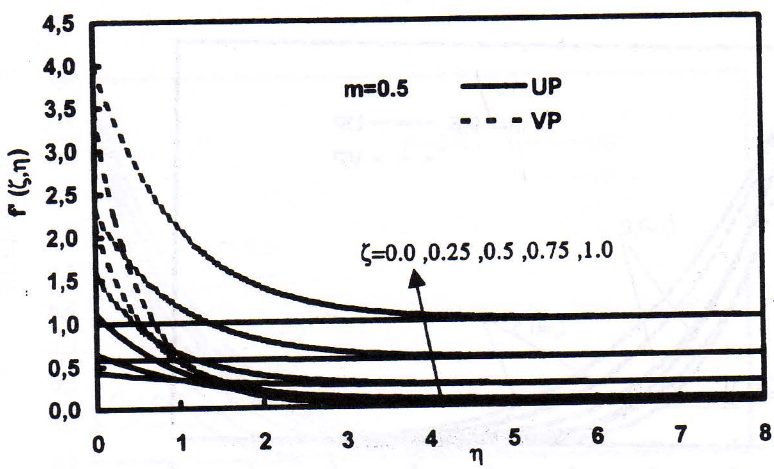

Fig.3. Velocity profile for various values of ζ with m = 0.5

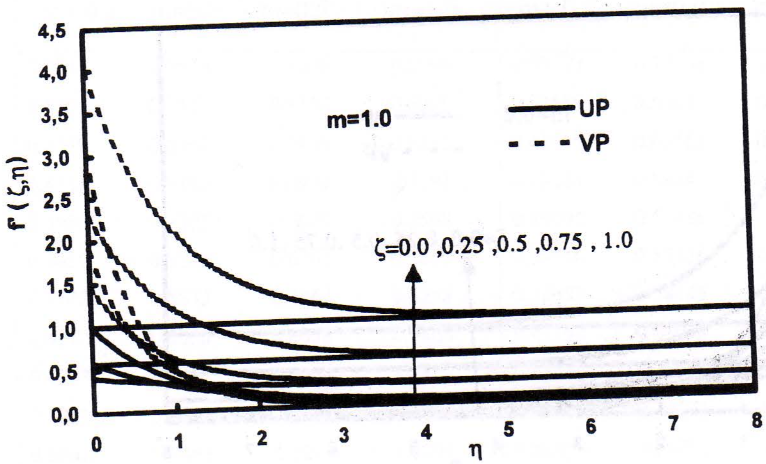

Fig.4. Velocity profile for various values of ζ with m = 1.0

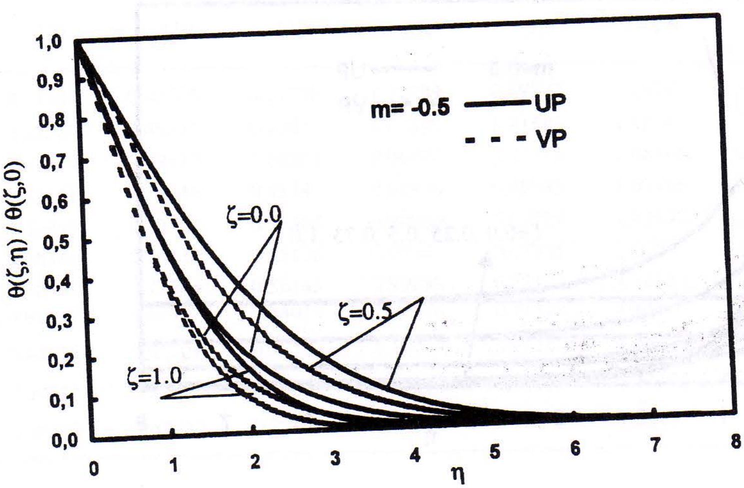

Fig.5. Temperature profile for various values of ζ with m = -0.5

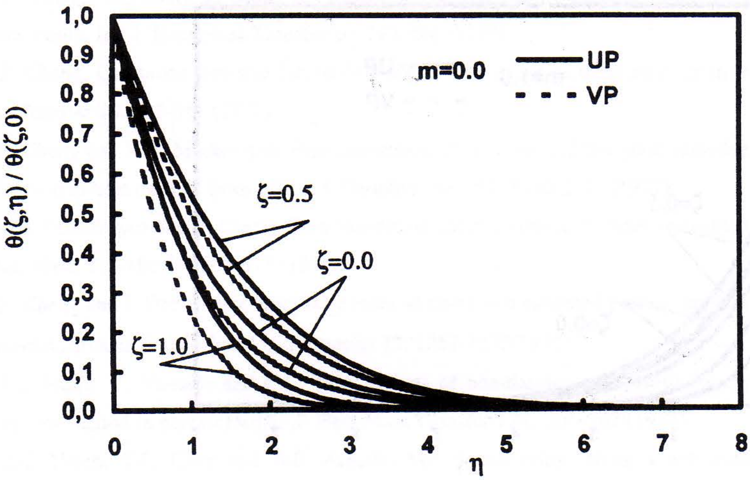

Fig.6. Temperature profile for various values of ζ with m = 0.0

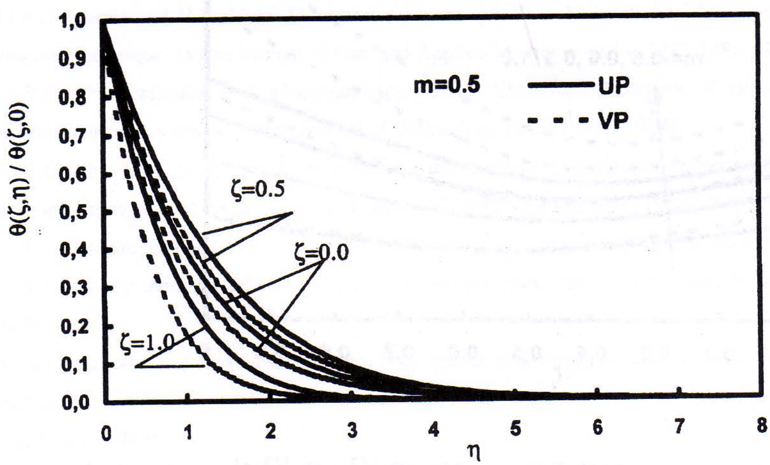

Fig.7. Temperature profile for various values of ζ with m = 0.5

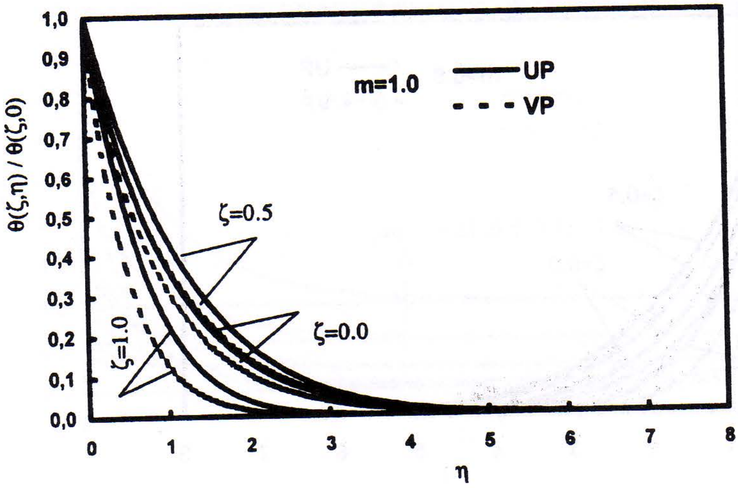

Fig.8. Temperature profile for various values of with m = 1.0

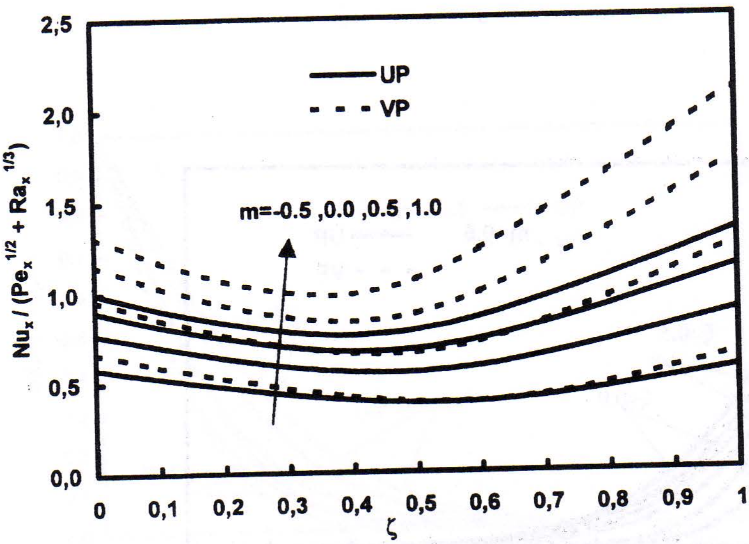

Fig.9. local Nusselt number in terms of $1/\theta(\zeta,0)$ or $Nu_x(Pe_x^{1/2} + Ra_x^{1/3})^{-1}$ at selected values of m

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Notation

b , m	real constants in equation(6)
d	constant defined in equation (4)
d^*	constant defined in equation (4)
f	dimensionless stream function
G	dimensionless normal velocity $\partial f/\partial \zeta$
g	gravitational acceleration
h	local heat transfer coefficient
k	Coefficient defined by equation (4)
K(y)	permeability of porous medium
K_∞	permeability of the porous medium at the edge of the boundary
$\alpha(y)$	thermal conductivity of the saturated porous medium
Nu_x	local Nusselt number
Pe_x	local Peclet number
Ra_x	local Rayliegh number
T	fluid temperature
T_w	wall temperature
T_∞	free stream temperature
u	streamwise velocity component
U_∞	free stream velocity
v	normal velocity component
x	axial coordinate
y	normal coordinate
	Greek symbols
$\alpha(y)$	thermal diffusivity
α_∞	thermal diffusivity at the edge of the boundary layer
β	volumetric coefficient of thermal expansion
$\varepsilon(y)$	porosity of the saturated porous medium
ε_∞	porosity of the saturated porous medium at the edge of the boundary layer
η	pseudo-similarity variable
θ	dimensionless temperature
λ_f	Thermal diffusivity of the fluid
λ_s	Thermal diffusivity of the solid
λ_m	effective thermal conductivity of the saturated porous medium
μ	dynamic viscosity of the fluid
ν	kinematic viscosity of the fluid
ζ	mixed convection parameter
ρ	fluid density
σ	ratio of thermal conductivity of the solid to the fluid
ϕ	dimensionless temperature gradient with $(\partial\theta/\partial\zeta)$
ψ	stream function
	Subscripts
w	conditions at the wall
∞	conditions at the free stream
f	fluid
s	sold