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STABILITY OF A JOURNAL BEARING SYSTEM WITH A DIVIDED AND PIEZOELECTRICALLY CONTROLLED BEARING SHELL

Piotr M. Przybyłowicz
Warsaw University of Technology
Institute of Machine Design Fundamentals
ul. Narbutta 84, 02-524 Warszawa, Poland
tel. +48 22 660-8258, fax. +48 22 660-8622, e-mail: piotrp@ipbm.simr.pw.edu.pl

Abstract

In the paper the problem of stability of a journal bearing system with divided and movable shell actively controlled by piezoelectric elements is considered. The shell consists of three segments – two fixed and the middle one connected with a piezoelectric actuator. The actuator shifts the movable part in the direction normal to the bearing axis. Hence, the whole support loses its co-axiality. A slightest displacement of the middle segment yields in fact a change in the oil gap – the most influential factor deciding about the stability. The paper presents the concept and provides one with the theoretical fundamentals. The equations of motion are derived and examined in terms of the stability of the non-trivial equilibrium, so that the critical rotation speed, at which self-excited vibration appears, could be determined. The plane model based on the simplified Reynolds Equation is employed in the investigations. Results of numerical simulations prove that the method is quite effective and brings considerable growth of the critical speed.

1. Introduction

Recently, much attention has been paid to geometry of journal bearings as it highly influences dynamics of the systems. Dynamic properties of bearings having non-circular contours were studied by Kaniewski and Stasiak [1] who introduced pericycloidal contours approximated by a combination of circular arcs. Problems related to multi-lobe bearings were also considered by Flack and Allaire [2], Osiński and Starczewski [3]. Non-circular bushes employed in the bearings (pericycloids with different multiplication coefficients) were found to exhibit greater critical speed than it was observed in bearings with circular bushes, however load capacity of the non-circular bearings considerably dropped.

In the last decade, a very interesting concept of stabilising journal bearing system was proposed by Kurnik [4], [5]. The idea consisted in adding magnetic actuators to classical bearings. The actuators generated attractive force that, when appropriately driven, placed the journal in regions in the oil space characterised by safer operating conditions. It was shown that the stabilisation efficiency has a maximum corresponding to the more than a double increase of the critical rotation speed with respect to the supplied voltage. The distinct maximum was brought about by effect of the magnetic saturation in the actuator core. The author examined parameter- and state-based control strategies. The latter approach, unlike the first method, did not affect frequency of the free vibration.

Monitoring and controlling size of the oil gap – one of the most important factors responsible for stability threshold of journal bearings has been a subject of study of numerous researchers. Santos and Ulbrich [6] used an active tilting pad journal bearing to control damping behaviour. They could change the oil film thickness between the rotor and the bush, and that way affect stiffness and damping coefficients of the lubricating medium. Experimental results showed a significant drop of vibration amplitude at the critical speed. At the same time, Bonneau et al. [7] employed piezoelectric elements to a journal bearing system. They studied an adaptive bearing that consisted of mobile housing mounted on piezoelectric actuators. There were four piezoelectric jacks made of lead zirconate titanate (PZT) or lead magnesium niobate (PMN) that possessed strain ability of order of the oil gap thickness under shaft loading of 30 N and operating frequency 8 kHz.

Another concept of applying piezoelectric elements to the journal bearing system was introduced by Przybyłowicz [8], [9]. The idea assumed making use of a piezoelectric ring made of PZT forced in between the bearing housing and the bush. The piezoceramics were incorporated for changing the radial dimension under application of electric field. The field was developed by a voltage applied to the inner (connected with the bush) and outer (connected with the housing) surface of the piezoceramic ring. A change of the radial dimension of the bearing bush (via radial contraction or expansion of the PZT ring) was coupled with the angular velocity of the rotor. Assumption of the simplest control strategy, which realised generation of the voltage proportional to the rotation speed, resulted in an apparent change of the dynamic properties of the journal bearing system. The critical threshold was shifted upwards and the stability domain was enlarged.

In this paper another way of making use of piezoelectric elements is proposed. The concept of stabilisation itself is based on affecting the oil gap by dividing the bearing shell into three close but separate cylinders, the middle one of which can be moved perpendicularly with respect to the axis of the bearing (jacked up by a piezoelectric actuator). Admittedly, the journal becomes supported on two bearings with different but controllable geometrical parameters.

2. Equations of motion

Consider a single journal bearing support with divided bearing shell as shown in Fig. 1. The edge shells are seated in rings fixed to the housing. They are, this way, the unmoveable elements of the system.

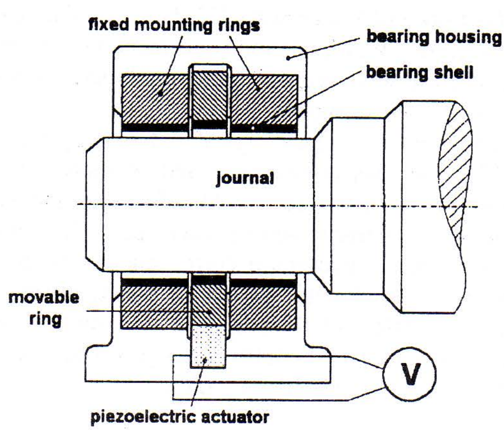

Fig. 1. Journal bearing with divided shell

The middle part can slide vertically when jacked up or down by the piezoelectric actuator put in between the housing and the movable bearing shell.
The idea is to force, via oil film wedge, the journal to move in the area of safer properties as far as the stability is concerned. The movable part is made just made for this purpose.

A piezoelectric element is capable of changing its size under application of an electric field. That change can be quantitatively expressed by the constitutive law of piezoelectric materials, describing the so-called converse piezoelectric effect [10], [11], see Fig. 2.

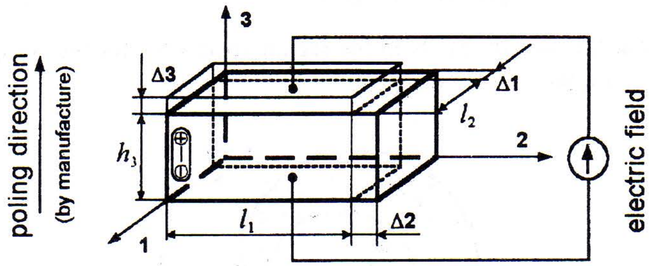

Fig. 2. Converse piezoelectric effect

Regarding the symmetry of the stress and strain tensors, the constitutive law can be written down as in the following matrix form:

$$\varepsilon_i = s_{ij}^{(E)}\sigma_j + d_{ij}^{\mathrm{T}}E_j \tag{1}$$

By applying the voltage V without any mechanical stress the piezoelectric actuator undergoes changes of its edge sizes as follows:

$$\begin{aligned} \Delta 1 &= d_{31}\frac{l_1}{h_3}V \\ \Delta 2 &= d_{31}\frac{l_2}{h_3}V \\ \Delta 3 &= d_{33}V \end{aligned} \tag{2}$$

It should be mentioned that the mechanical strain can be achieved only if the electric field is applied along the poling direction 3 – see Fig. 2. Any of the displacements in Eq. 2 can be used for jacking up or down the movable bearing shell.

Find now the equations of motion of the rigid rotor supported on the divided bearing. Explicit formulas for the hydrodynamic forces can be found from the simplified Reynolds's equation for the plane distribution of the oil pressure [12]:

$$\frac{\partial}{\partial\varphi}\left(H^3\frac{\partial p}{\partial\varphi}\right) - 6\mu\delta^2\left(\omega\frac{\partial H}{\partial\varphi} + 2\frac{\partial H}{\partial t}\right) = 0 \tag{3}$$

The forces can be determined by double integration of Eq. (3) and taking into account the boundary conditions with respect to the oil film. Assuming, that the inertia forces of the lubricating medium are negligible and the flow is isothermic, and then taking into consideration all components of the journal bearing motion, i.e. rotation around its own axis (ω), precession around the bearing axis $(\dot{\alpha})$ - corresponding to the wedge effect, and radial motion ($\dot{\beta}$) - corresponding to the squeeze effect one can determine the hydrodynamic forces acting in the radial (β) and circumferential (α) direction, see Fig. 3. They are as follows:

$$F_\beta = -\frac{12\mu R\left(L-\tilde{L}\right)}{c^2}\left[\frac{\beta^2(\omega-2\dot{\alpha})}{(1-\beta^2)(2+\beta^2)}+\frac{\beta\dot{\beta}}{1-\beta^2}+\frac{2\dot{\beta}}{\sqrt{(1-\beta^2)^3}}\operatorname{arctg}\sqrt{\frac{1+\beta}{1-\beta}}\right]$$
$$F_\alpha = \frac{6\pi\mu R\left(L-\tilde{L}\right)}{c^2}\frac{\beta(\omega-2\dot{\alpha})}{\sqrt{(1-\beta^2)(2+\beta^2)}} \tag{4}$$

if it is assumed that the oil film exists in the region $(\alpha,\alpha+\pi)$ for the wedge effect, and in $(\alpha-\pi/2,\alpha+\pi/2)$ for the squeeze one [12].

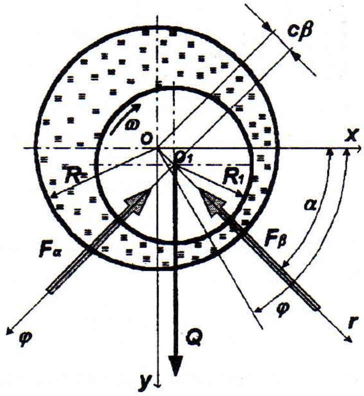

Fig. 3. Hydrodynamic uplift forces and assumed coordinate system

Admittedly, the support is divided, thus Eq. (4) is applicable only to the fixed part of the bearing. In fact, the middle part develops analogous components of the uplift forces, however in different conditions (different eccentricity and circumferential position). By denoting these components by $\tilde{F}_\alpha$ and $\tilde{F}_\beta$, respectively, one writes down:

$$\tilde{F}_\beta = -\frac{12\mu R\tilde{L}}{c^2}\left[\frac{\tilde{\beta}^2\left(\omega-2\dot{\tilde{\alpha}}\right)}{(1-\tilde{\beta}^2)(2+\tilde{\beta}^2)}+\frac{\tilde{\beta}\dot{\tilde{\beta}}}{1-\tilde{\beta}^2}+\frac{2\dot{\tilde{\beta}}}{\sqrt{(1-\tilde{\beta}^2)^3}}\arctan\sqrt{\frac{1+\tilde{\beta}}{1-\tilde{\beta}}}\right]$$
$$\tilde{F}_\alpha = \frac{6\pi\mu R\left(L-\tilde{L}\right)}{c^2}\frac{\tilde{\beta}\left(\omega-2\dot{\tilde{\alpha}}\right)}{\sqrt{(1-\tilde{\beta}^2)(2+\tilde{\beta}^2)}} \tag{5}$$

The eccentricity $\tilde{\beta}$ as well as the angular position of the journal $\tilde{\alpha}$ in the movable part of the support strictly depends on β and α in the fixed reference shell and current displacement s of the movable element. It is graphically shown in Fig. 4.

The relationships between the eccentricities and angular positions of the journal in the fixed and movable parts of the bearing are as follows:

$$\tilde{\beta}=\sqrt{\beta^2+\zeta^2+2\beta\zeta\sin\alpha}\ , \quad \tilde{\alpha}=\arccos\left\{\frac{\beta\cos\alpha}{\tilde{\beta}}\right\}, \quad \cos\gamma=\sqrt{1-\frac{\zeta^2\cos^2\alpha}{\tilde{\beta}^2}} \tag{6}$$

where $\zeta = s/c$.

Finally, one can set forth the equations of motion, which in the polar co-ordinate system have the form:

$$\begin{aligned} m(\ddot{\beta}-\beta\dot{\alpha}^2)c &= F_\beta(\beta,\dot{\beta},\alpha,\dot{\alpha})+\tilde{F}_\beta(\beta,\dot{\beta},\alpha,\dot{\alpha})\cos\gamma+Q\sin\alpha \\ m(\beta\ddot{\alpha}+2\dot{\beta}\dot{\alpha})c &= F_\alpha(\beta,\dot{\beta},\alpha,\dot{\alpha})+\tilde{F}_\alpha(\beta,\dot{\beta},\alpha,\dot{\alpha})\cos\gamma+Q\cos\alpha \end{aligned} \tag{7}$$

where the explicit expressions for F_β, F_α are given in Eq. (4), and $\tilde{F}_\beta, \tilde{F}_\alpha$ can be found from Eq. (5) after substituting (6).

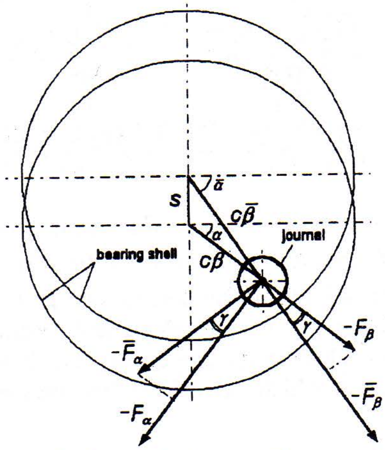

Fig. 4. Hydrodynamic forces in both parts of the journal bearing system

3. Stability of the system

Investigation of the stability requires knowledge of the equilibrium position of the journal and its evolution with growing rotation speed. The equilibrium position can be directly found from Eq. (7) by assuming zero velocity and acceleration components of the journal motion: $\dot{\alpha}=\dot{\beta}=0$ and $\ddot{\alpha}=\ddot{\beta}=0$. The equilibrium eccentricity and angular position will be denoted as β_0 and α_0, respectively (in the fixed reference part of the support). Successive calculation of β_0 and α_0 for increasing ω enables one to draw a line called the static equilibrium curve. It is shown in Fig. 5 for some exemplary jack-ups of the middle bearing. It can be clearly seen that the displacement of the movable part of the bearing shell places the journal centre in a completely different position, certainly characterised by different damping and stiffness properties. Obviously, this affects the stability of the journal equilibrium position.

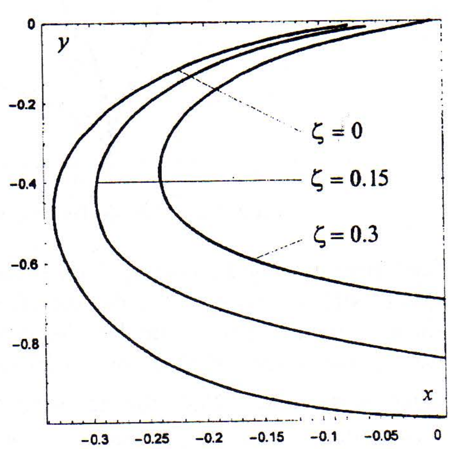

Fig. 5. Static equilibrium curves

In order to examine stability of a given equilibrium position transform, at first, the equations of motion so that they could be written down as:

$$\begin{aligned} \ddot{\beta} &= f_\beta(\beta,\dot{\beta},\alpha,\dot{\alpha}) \ , \ f_\beta = \frac{1}{mc}\left\{F_\beta + \tilde{F}_\beta \cos\gamma + Q\cos\alpha\right\} + \beta\dot{\alpha}^2 \\ \ddot{\alpha} &= f_\alpha(\beta,\dot{\beta},\alpha,\dot{\alpha}) \ , \ f_\alpha = \frac{1}{mc\beta}\left\{F_\alpha + \tilde{F}_\alpha \cos\gamma + Q\sin\alpha\right\} - 2\frac{\dot{\beta}\dot{\alpha}}{\beta} \end{aligned} \tag{8}$$

Introduce then new variables: $u_1 = \beta - \beta_0$, $u_2 = \dot{\beta}$, $u_3 = \alpha - a_0$, $u_4 = \dot{\alpha}$, which enable investigation of the stability around the trivial equilibrium $\mathbf{u} = [0,0,0,0]$, and transform the equations into a set of four ordinary differential equations of the first order. Thus, one obtains:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \\ \dot{u}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{\partial f_\beta(\beta_0,0,\alpha_0,0)}{\partial \beta} & \dfrac{\partial f_\beta(\beta_0,0,\alpha_0,0)}{\partial \dot{\beta}} & \dfrac{\partial f_\beta(\beta_0,0,\alpha_0,0)}{\partial \alpha} & \dfrac{\partial f_\beta(\beta_0,0,\alpha_0,0)}{\partial \dot{\alpha}} \\ 0 & 0 & 0 & 1 \\ \dfrac{\partial f_\alpha(\beta_0,0,\alpha_0,0)}{\partial \beta} & \dfrac{\partial f_\alpha(\beta_0,0,\alpha_0,0)}{\partial \dot{\beta}} & \dfrac{\partial f_\alpha(\beta_0,0,\alpha_0,0)}{\partial \alpha} & \dfrac{\partial f_\alpha(\beta_0,0,\alpha_0,0)}{\partial \dot{\alpha}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{9}$$

Finding characteristic equation corresponding to the eigenproblem $\{\mathbf{A}(\omega) - r\mathbf{I}\}\mathbf{q} = \mathbf{0}$, where $\mathbf{A}$ is the matrix of the set of linearised equations, see Eq. (9), $\mathbf{I}$ - the identity matrix, $\mathbf{q}$ - arbitrary eigenvector, and then solving it one obtains two pairs of complex conjugate roots (eigenvalues). The one of greater real part decides about stability of the system. When the trajectory of that decisive eigenvalue intersects the imaginary axis in the complex plane, then the stability is lost. The ordinate at which the real part becomes zero is the initial frequency of self-excited vibration.

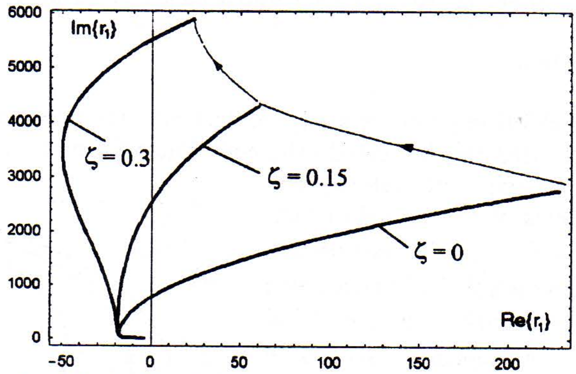

Fig. 6. Trajectories of the decisive eigenvalue for various displacements of the movable shell

Each time the middle bearing shell is elevated the critical rotation speed responsible for the loss of stability grows. This desired effect can be even more clearly presented in a diagram showing the real part of the decisive eigenvalue versus the angular velocity of the journal as well as the relative shift ζ of the movable shell. In Fig. 7a a spatial representation of this eigenvalue is given, Note, that any point at which $\mathrm{Re}\{r_1\} > 0$ means instability (appearance of self-excitation). For better visibility of the safe (stable) area a contour plot is drawn with one contour line only, see Fig. 7b. The borderline between the white and grey regions separates domains of stable and unstable working of the system. This picture is directly derived from Fig. 7a by cutting the $\mathrm{Re}\{r_1\}$ – surface by the $\mathrm{Re}\{r_1\} = 0$ plane. It can be easily seen that the stability domain considerably enlarges for increasing ζ.

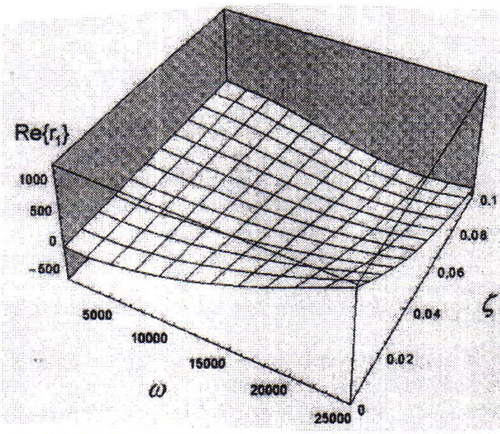

Fig. 7a. Real part of the decisive eigenvalue vs. rotation speed and displacement of the movable shell

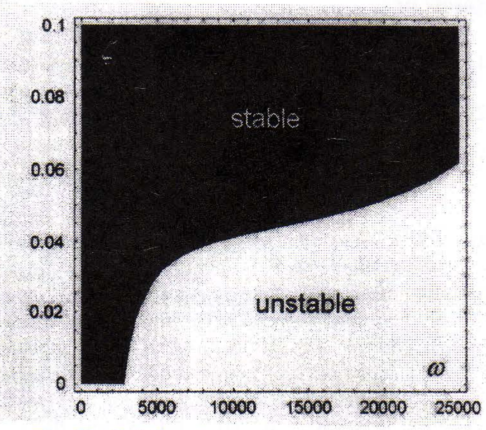

Fig. 7b. Stability domain vs. rotation speed and displacement of the movable shell

4. Concluding remarks

In the paper a method of stabilisation of a single journal bearing system by application of piezoelectric elements is presented. In contrast to other works on stability of such systems with piezoelectric enhancement, where the actuators either change the radial dimension of the oil gap [8], [9] or kinematically affect motion of the entire bearing [7], this study introduces a new concept towards the achievement of the purpose. The idea is based on dividing the bearing shell into three sections – two fixed and the third, the middle one, movable in the direction perpendicular to the bearing axis. The journal center assumes a certain equilibrium position corresponding to current structural and operational conditions (clearance, oil viscosity, mass of the rotor, transverse loading, etc.). In the case of the divided support this position can be changed by displacing the middle part of the bearing and thus forcing the journal to find a new equilibrium position, somewhere within the common space of two intersecting circles of the bearing clearances, which have been mutually shifted. Such a situation is, to some extent, analogous to subjecting the journal to an additional transverse loading, which, as it is well known in the literature, results in stabilisation of the system (heavier rotors are more resistant to self-excitation than their lighter counterparts).

A piezoelectric jack is used for shifting the movable bearing shell in this paper. Piezoelectric materials are very precise instruments for such a task, however any other and purely mechanical way would be equally effective (but probably more expensive). Consequently, further studies should concentrate on the efficiency of a method in which the electric field controlling the piezoelectric actuator is not freely applied but coupled with the rotation speed (via e.g. simple generator). Another, no less interesting problem, will be investigation on the near-critical behaviour of the journal bearing with divided shell controlled by piezoelectric elements.

Acknowledgement

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Notation

c - bearing clearance

d_{ij}^{T} - transposed matrix of the electromechanical coupling coefficients of the piezoelectric material

d_{31}, d_{33} - electromechanical coupling constant corresponding to the transverse and longitudinal piezoelectric effect

E_j - vector of the electric field

f_α, f_β - right-hand sides of the equations of motion

F_α, F_β - hydrodynamic forces in the circumferential and radial direction, respectively, in the fixed part of the bearing

$\tilde{F}_\alpha$, $\tilde{F}_\beta$ - hydrodynamic forces in the circumferential and radial direction, respectively, in the movablepart of the bearing

h_3 - distance between the actuator electrodes

h - dimensional oil gap

H - current dimensionless oil gap

l_1, l_2 - lengths of the piezoélectric actuator in the direction 1, 2, respectively

L - length of the entire bearing

$\tilde{L}$ - length of the movable part of the bearing

m - mass of the rotor

p - oil pressure

Q - transverse load applied to the journal

R - journal radius

$s_{ij}^{(E)}$ - compliance matrix for a fixed electric field E

u_i - i-th dimensionless co-ordinate

α, $\tilde{\alpha}$ - current angular position of the journal center with respect to the fixed and movable part of the bearing, respectively

β, $\tilde{\beta}$ - relative eccentricity in the fixed and movable part of the bearing, respectively

δ - relative oil gap ($\delta = c / R$)

$\Delta 1, \Delta 2, \Delta 3$ - elongations of the piezelectric actuator in the directions 1, 2, 3, respectively

γ - difference between angular positions of the journal center observed in the fixed and movable parts of the bearing

ε_i - i-th component of the strain vector

φ - current angular position at which the oil gap is measured

μ - absolute viscosity of the lubricant

σ_j - j-th component of the stress vector

ω - rotation speed