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THE EFFECT OF THERMAL DISPERSION ON HYDROMAGNETIC FLOW AND HEAT TRANSFER PAST A CONTINUOUSLY MOVING POROUS BOUNDARY WITH TEMPERATURE DEPENDENT VISCOSITY

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The effects of thermal dispersion and magnetic field on the flow and heat transfer past a continuously moving porous plate in a stationary fluid has been analyzed. The fluid viscosity is assumed to vary as an inverse linear function of temperature. By means of the similarity solutions, deviation of the velocity and temperature fields as well as the skin friction and heat transfer results from their constant values are determined numerically by using the shooting method. The effects of thermal dispersion, variable viscosity, magnetic field, and suction (or injection) parameters on the velocity and temperature profiles have been studied..

Keywords: thermal dispersion, heat transfer, hydromagnetic flow.

1. Introduction

Study of heat transfer in porous boundary has been the interest of several researchers, owing to its wide applicability in engineering and geophysical problems, including oil recovery technology, use of fibrous materials for thermal insulations, design of aquifer as an energy storage system, utilization of porous layers for transpiration cooling by water for fire fighting and also Resin Transfer Molding process, in which fiber reinforced polymeric parts are produced in the final shape.

When the effects of the magnetic and fluid forces are considered together, the resulting boundary layer equations became intractable for an analytical treatment idealized models have therefore been investigated in literature with a view to understanding the individual as well as coupled effects of the flow parameter [1–8].

Elbashbeshy [9] studied the free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of the magnetic field. Most of the existing analytical studies of this problem are based on the constant physical properties of the ambient fluid. However, it is known (see Herwig and Gersten[10]) that these properties may change with temperature, especially fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity.

On the other hand, all the above investigations are restricted to MHD flow and heat transfer problems. Seddeek [11] studied the effect of variable viscosity of hydromagnetic flow and heat transfer past continuously moving porous boundary with radiation. Also Seddeek [12] studied the thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature-dependent viscosity and Murthy, Singh [13] studied Thermal dispersion effects on non-Darcy natural convection with lateral mass flux.

The thermal dispersion effect becomes significant as observed in Plumb [14], Hong and Tien [15], Nield and Bejan [16]. Hence, the purpose of the present work is to study the effects of thermal dispersion and variable viscosity on the flow and heat transfer process when the fluid flows past a continuously, moving porous boundary in the presence of magnetic field. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and these have been solved numerically. Results indicate the general trend that the increase in thermal dispersion parameter further increases the heat transfer.

The effects of thermal dispersion, magnetic field and porous boundary on the flow and heat transfer have been shown graphically.

2. Governing equations

Consider a steady, incompressible and electrically conducting fluid past a continuously moving semi-infinite porous plate under the influence of a transversely applied magnetic field.

The fluid properties are assumed isotropic and the fluid viscosity is assumed an inverse linear function of temperature (see Lai and Kulacki [17])

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\alpha[T-T_\infty]] \tag{1}$$

or

$$\frac{1}{\mu}=a(T-T_\Gamma)$$

(2)

where

$$a=\frac{\alpha}{\mu_\infty}, \text{ and, } T_\Gamma=T_\infty-\frac{1}{\alpha}. \tag{3}$$

Both a and T_Γ are constants and their values depend on the reference state and thermal property of the fluid i.e. ∂. In general $a>0$ for liquids and $a<0$ for gases. We consider the magnetic Reynolds number is very small for most fluid used in industrial applications, we assume that the induced magnetic field is negligible. Let the x-axis be taken along the direction of moving plate and y-axis normal to it. If u and v are the velocity components along these directions respectively, then under the usual boundary layer approximation, the hydro-magnetic steady flow and heat transfer for the present problem are governed by the equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\rho B^2(x)}{\rho}u \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right). \tag{6}$$

In the Basic equation α_y is the component of thermal diffusivity α_l in y direction. The suffix and indicate the conditions at the wall and at the outer edge of the boundary layer respectively.

Here is a variable quantity, which is the sum of molecular thermal diffusivity α_l and dispersion thermal diffusivity α_d Following plumb [14], the expression for dispersion thermal diffusivity will be $\alpha d=\gamma du$ where γ is the mechanical dispersion coefficient, its value depends on the experiments and d is the pore diameter. ρ is the fluid density, μ the viscosity coefficient, σ the electrical conductivity, $B(x)$ the magnetic field strength, T the temperature of the fluid, K the thermal conductivity, C_p the specific heat at constant pressure. The boundary conditions are given by:

$$\begin{array}{llll} y=0: & u=U & v=-v_0(x), & T=T_w \\ y\to\infty: & u\to 0: & & T\to T_\infty. \end{array} \tag{7}$$

Where U and T_w are the plate velocity and temperature respectively, $v_0(x)$ the normal velocity at the plate having positive value for suction and negative for blowing, and T_∞ the free-stream temperature.

3. Similarity solution

We define the following similarity variables.

$$\eta=\left(\frac{U}{2x\upsilon}\right)^{\frac{1}{2}}y,\quad v=\left(\frac{\upsilon U}{2x}\right)^{\frac{1}{2}}(\eta f'-f),$$

$$u=Uf'(\eta),\qquad f_w=\left(\frac{2x}{\upsilon U}\right)^{\frac{1}{2}}v_0x,$$

and write the non -dimensional temperature distribution as

$$\theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

This representation is valid since the expressions for velocity components clearly satisfy the continuity equations.

If this set transforms the governing partial differential Eqs. (4)–(6) into ordinary differential equations with x being eliminated and explicitly also from the boundary conditions.

Applying the similarity transformation to the governing Eqs.(4)–(6) we get

$$f''-\frac{1}{\theta-\theta_\Gamma}\theta' f''+\left(\frac{\theta}{\theta_\Gamma}-1\right)\left[Mf'-ff''\right]=0, \tag{9}$$

$$\frac{1}{P_r}\theta'' + f\theta' + \frac{\alpha_d}{\upsilon}\theta' f'' = 0 \tag{10}$$

The transformed boundary conditions for the velocity and temperature field becomes:

$$f(0) = f_w, \quad f'(0) = 1, \quad f'(\infty) = 0, \qquad \theta(0) = 1, \quad \theta(\infty) = 0. \tag{11}$$

where

$$M = \frac{2\sigma x B^2(x)}{(\rho u)^2} \quad P_\Gamma = \frac{\upsilon \rho C_p}{k}. \tag{12}$$

In equations (9-10) prime denotes derivatives with respect to η, M is the magnetic parameter, P_r is the Prandtl number. θ_r is a constant, $f_w>0$ indicates suction and $f_w<0$ denotes injection. It is also important to note that θ_r is negative for liquids and positive for gases. The governing boundary layer equations (9) and (10) subject to the boundary conditions (11) are solved numerically by using shooting method, via the symbolic computation software *Mathematica*.

The physical quantities of interest in the problem are skin friction

$$c_f = \frac{T_w}{\rho U^2}$$

where

$$T_w = -\mu_w \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad c_f (\mathrm{Re})^{\frac{1}{2}} = \frac{2\theta_\Gamma}{\theta_\Gamma - 1} f''(0), \quad \mathrm{Re} = \frac{ux}{2\upsilon}. \tag{13}$$

The effect of the parameters θ_Γ, f_w, M and α_d on the functions $f'(0)$ and $\theta'(0)$ at the surface is tabulated in the Table 1.

Table 1 Values of $f''(0)$ and $\theta'(0)$ for various values of the parameters.

θ_r	f_w	M	α_d	$f''(0)$	$\theta'(0)$
-1.0	0	0.2	0	0.4262	0.2741
-0.1	0	0.2	0	0.3922	0.2923
-0.01	0	0.2	0	0.3831	0.3115
-0.1	-0.5	0.2	0	0.5211	0.3012
-0.1	-0.1	0.2	0	0.5520	0.3232
-0.1	-0.5	0.2	0	0.5934	0.3506
-0.1	0	0.0	0	0.4014	0.2995
-0.1	0	0.5	0	0.5327	0.2721
-0.1	0	1.0	0	0.5751	0.2546
-0.1	0	0.2	0.1	0.4435	0.2869
-0.1	0	0.2	0.2	0.4621	0.2991
-0.1	0	0.2	0.3	0.4775	0.3102

4. Results and Discussion

To study the behavior of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow.

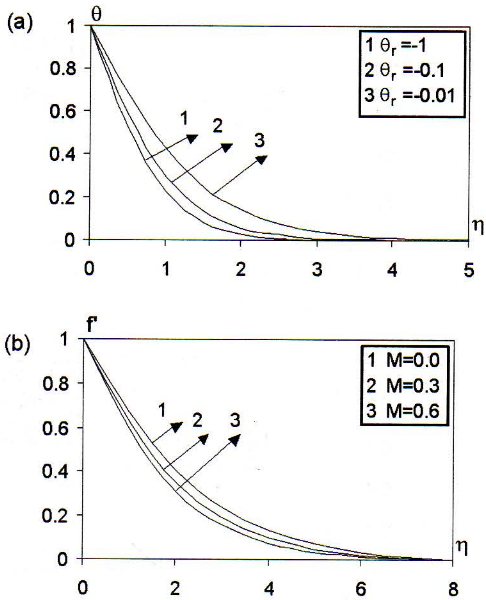

Figure 1 Effect of variable viscosity parameter θ_r on (a) the velocity profiles $f'(\eta)$, and, (b) the temperature profiles $\theta(\eta)$ for p_r=0.72, $f_w = 0$, u=0.2 and α_d=0.1

Fig. 1 display results for the velocity and temperature distribution. It is seen, as expected, that f' decreases with increasing the parameter θ_r, but θ increases with increasing the parameter θ_r. The results presented demonstrate quite clearly that θ_r, which is an indicator of the variation of viscosity with temperature, has a substantial effect on the drag and heat transfer characteristics.

It is seen from Fig. 2 that as expected, the velocity profiles f' decrease monotonically, while the temperature profiles increase with increasing magnetic parameter M. The decrease of f' happens because of an accelerating force, which acts in a direction parallel to the x-axis.

Fig. 3 corresponds to $f`(\eta)$ and $\theta(\eta)$ versus the similarity variable η, by considering and neglecting the thermal dispersion effects for three different values of the non-dimensional mass flux parameter f_w. In both the cases, the velocity and temperature profiles thicken as the mass flux parameter passes from the suction domain to the injection domain.

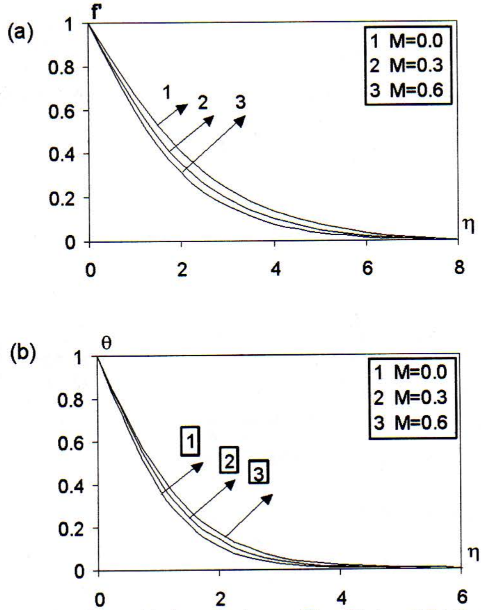

Figure 2 Effect of magnetic parameter M on (a) the velocity profiles $f'(\eta)$ and (b) the temperature profiles $\theta'(\eta)$ for p_r=0.72, f_w =0, h_r=-0.1, and α_d=0.1

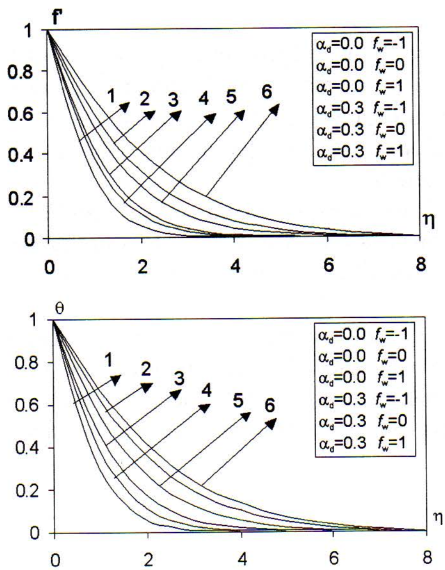

Figure 3 Effect of varying dispersion α_d and mass flux, f_w on (a) the velocity profiles $f'(\eta)$ and (b) the temperature profiles $\theta'(\eta)$ for M=0.2 θ_r=0.1

In the Table 1, we show that $f''(0)$ decreases as θ_Γ increases where as for increasing values of θ_Γ, $\theta'(0)$ increases. To verify the proper treatment of the problem, the solutions have been compared with those of the corresponding constant thermal dispersion case by setting α_d=0. Thus Seddeek [11] have obtained $f''(0)$=0.4296 and $\theta'(0)$=0.2788 at f=0 (without radiation). Our results for $f''(0)$ and $\theta'(0)$ for α_d=0 are 0.4262, 0.2741 respectively and therefore are in very good agreement with reference to [11]. We note that $f''(0)$ increases with increasing the parameters f_w, M and α_d. On the other hand $\theta'(0)$ increases with increasing f_w, and α_d parameters while decreases with increasing the parameter M.

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