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VARIABLE VISCOSITY EFFECTS ON COMBINED HEAT AND MASS TRANSFER BY FREE CONVECTION OVER A TRUNCATED CONE IN POROUS MEDIA: UWT/UWC

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An analysis is presented is to study the heat and mass transfer characteristics of natural convection flow about a truncated cone embedded in a saturated porous medium with uniform surface temperature/concentration under the combined buoyancy effects thermal and mass diffusion. The transformed governing equations are solved by Keller box method. Numerical results for dimensionless temperature and concentration; the local Nuss elt (Sherwood) numbers are presented over a wide range of dimensionless distance ξ, Lewis number Le, buoyancy ratio N and the wall to ambient viscosity ratio v^*. It has been found that the local Nusselt number and Sherwood number decrease with decreasing the wall to ambient viscosity ratio v^*. Furthermore, it is shown that the local Nusselt (Sherwood) numbers of the truncated cone approach those of inclined plate (full cone) for the case of constant viscosity at $\xi = 0$ ($\xi \to \infty$).

Keywords: mass, heat transfer, porous media

1. Introduction

Coupled heat and mass transfer (or double diffusion) driven by buoyancy due to temperature and concentration variations in a saturated porous medium, has several important applications in geothermal and geophysical engineering, e.g., the migration of moisture in fibrous insulation and the underground disposal of nuclear wastes. Comprehensive review of this phenomenon has been reported by Trevisanand and Bejan [1] for various geometries. Bejan and Khair [2] investigated the vertical natural convection boundary layer flow in a saturated porous medium due to the combined heat and mass transfer. Singh and Quenny [3] used the integral method to solve the problem of Bejan and Khair. Lai et al. [4] investigated the coupled heat and mass transfer by natural convection from slender bodies of revolution in porous media. Yih [5] studied coupled heat and mass transfer by free convection over a truncated cone in porous media in the cases: with variable wall temperature/concentration or heat/mass flux. In most of the previous studies on heat transfer in saturated porous media, the thermophysical properties of fluid were assumed to be constant. However, it is known that these properties may change with temperature, especially for fluid viscosity. To accurately predict the heat transfer rate, it is necessary to take into account this variation of viscosity. In spite of its importance in many applications, this effect has received rather little attention. Previous results [6-9] have shown that the variable viscosity has a significant effect on thermal and momentum transport predictions. Blythe and Simpkins [10] applied an integral method to examine the natural convection in a two-dimensional cavity filled with fluid-saturated porous media for the case in which the viscosity is temperature dependent. Ramrez and Saez [11] studied the forced convection boundary layer flow in a saturated porous medium containing a fluid with temperature-dependent viscosity on mixed convection boundary layer flow around a vertical surface on a saturated porous medium. The aim of the present work is to study the variable viscosity effects on a heat and mass transfer characteristics in natural convection flow over a truncated cone subject to uniform wall temperature/concentration embedded in porous media under the coupled heat and mass diffusion. Nonsimilar solutions are obtained for uniform wall temperature/concentration (UWT/UWC).

2. Analysis

Consider the problem of combined heat and mass transfer by free convection flow over a truncated cone (with half angle γ) embedded in a saturated porous medium. We consider the condition at the surface, namely, uniform wall temperature/concentration (UWT/UWC). The physical model and coordinate system are shown in Fig. 1. The origin of the coordinate system is placed at the vertex of the full cone, where x is the coordinate along the surface of cone measured from the origin and y is the coordinate normal to the surface, respectively. The flow is steady and the fluid properties are assumed to be constant except for the density ρ variation in the buoyancy force term and the dynamic viscosity μ variation with temperature. Introducing the boundary layer and Boussinesq approximations, the governing equations based on the Darcy law can be written as follows:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{1}$$

$$\frac{\partial(u)}{\partial y} = \mu u \frac{d}{dy}\left(\frac{1}{\mu}\right) + \frac{\rho_\infty g K \cos\gamma}{\mu}\left(\beta_T \frac{\partial T}{\partial y}\right) + \left(\beta_c \frac{\partial c}{\partial y}\right), \tag{2}$$

$$\frac{u\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$\frac{u\partial c}{\partial x} + \nu \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2}, \tag{4}$$

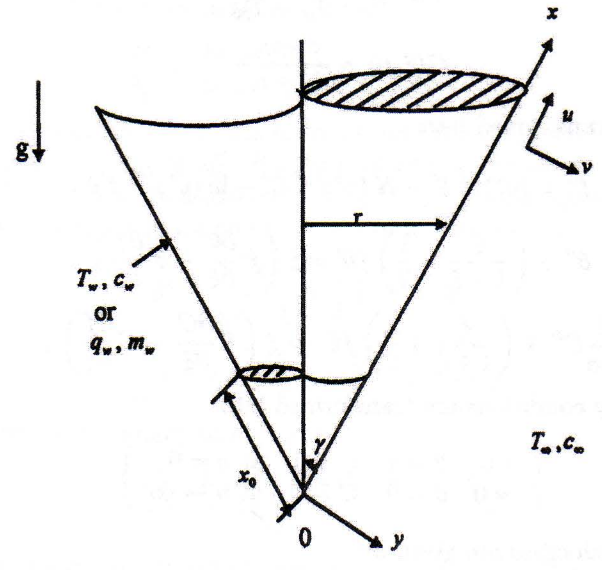

Figure 1 Flow model and physical coordinate system

The boundary conditions are defined as follows:

$$\left.\begin{array}{llll} \nu = 0 & T = T_w & c = c_w & \text{at } y = 0 \\ u = 0 & T = T_\infty & c = c_\infty & \text{at } y \to \infty \end{array}\right\} \quad (5)$$

where u and ν are Darcy velocities in the x and y directions, ρ_∞ is the density of the ambient fluid, g is the gravitational acceleration, K is permeability of the porous medium, β_c is the concentration expansion coefficient, β_T is the thermal expansion coefficient of the fluid, T and c are the temperature and concentration, respectively, and $r = x \sin\gamma$. Equations (1)–(5) are valid in $x_0 \le x < \infty$, where x_0 is the distance of the leading edge of truncated cone measured from the origin. Following [13] the variation of dynamic viscosity with the temperature is written in the form:

$$\left.\begin{array}{l} \mu = \mu_\infty S(T), \\ S(T) = (v^*)^\theta = \exp\left[A(T - T_\infty)\right] \end{array}\right\} \quad (6)$$

where μ_∞ is the absolute viscosity at the ambient temperature and therefore $S(T_\infty = 1$, $v^* = v_w/v_\infty = \exp\left[A(T_w - T_\infty\right]$ is the wall to ambient viscosity ratio parameter. Under the following dimensionless variables:

$$\xi = \frac{x^*}{x_0} = \frac{x - x_0}{x_0}, \quad (7)$$

$$\eta = \frac{y}{x^*}\sqrt{Ra_{x^*}}\,. \quad (8)$$

$$f(\xi, \eta) = \frac{\psi}{\alpha r \sqrt{Ra_{x^*}}} \quad (9)$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

$$C(\xi,\eta) = \frac{c - c_\infty}{c_w - c_\infty}. \tag{11}$$

Eqs. (1)–(4) are transformed into:

$$f'' = (v^*)^{-\theta}\,\theta' + N\,(v^*)^{-\theta}\,C' - \ln(v^*)^{-\theta}\,f'\theta', \tag{12}$$

$$\theta'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) f\theta' = \xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right), \tag{13}$$

$$\frac{1}{Le}C'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) fC' = \xi\left(f'\frac{\partial C}{\partial\xi} - C\frac{\partial f}{\partial\xi}\right), \tag{14}$$

with the boundary conditions are transformed into:

$$\left.\begin{array}{llll} f=0 & \theta=1 & C=1 & \text{at } \eta=0 \\ f'=0 & \theta=0 & C=0 & \text{at } \eta\to\infty \end{array}\right\}. \tag{15}$$

The Darcian velocities are given by:

$$u = \frac{\alpha Ra_{x^*}}{x^*} f' \tag{16}$$

$$\nu = -\frac{\alpha\sqrt{Ra_{x^*}}}{x^*}\left[\left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) f + \xi\frac{\partial f}{\partial\xi} - \frac{1}{2}\eta f'\right]. \tag{17}$$

The local Nusselt number Nu_{x^*}, the local Sherwood number Sh_{x^*} are given by:

$$\frac{Nu_{x^*}}{\sqrt{Ra_{x^*}}} = -\theta'(\xi,0), \tag{18}$$

$$\frac{Sh_{x^*}}{\sqrt{Ra_{x^*}}} = -C'(\xi,0), \tag{19}$$

where the primes denote the differentiation with respect to η and

$$N = \frac{\beta_c(c - c_\infty)}{\beta_T(T_w - T_\infty)}$$

is the buoyancy ratio measuring the relative effect of buoyancy force between mass and thermal diffusion for UWT/UWC. It is clear that N is zero for pure thermal buoyancy induced flow, positive for assisting flow (both thermal buoyancy force and buoyancy force owing to concentration difference act in the same direction) and negative for opposing flow. v^* takes the values 0.1–10, for the case $v^* = 1$ the equations reduce to the two- dimensional base flow equations for constant viscosity evaluated at the ambient temperature; $v^* < 1$ corresponds to the case of liquid heating and $v^* > 1$ corresponds to the case of gas heating [13]. It is interesting to mention that for the case of $N = 0$ at $v^* = 1$, equations (12)–(14) are reduced to those of Cheng et al. [14]. For the case of $\xi = v^* = 0$, equations (12)–(14) are reduced to those of vertical plate [2], where the similarity solutions are obtained.

For the case of $\xi \to \infty$, equations (12)–(14) are reduced to the case of a full cone and the similarity solutions are obtained, since:

$$\frac{\partial f}{\partial \xi} = \frac{\partial \theta}{\partial \xi} = \frac{\partial C}{\partial \xi} = 0, \text{ as } \xi \to \infty.$$

We now obtain approximate solutions of equations (12)–(14) based on local similarity and non-similarity methods [15]. For the first level of truncation the ξ derivatives in equations (13), (14) can be neglected. Thus, the governing equations for the first level of the truncation are equation (12) and the following ones:

$$\theta'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) f\theta' = 0, \tag{20}$$

$$\frac{1}{Le}C'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) fC' = 0, \tag{21}$$

which subject to the boundary conditions:

$$\left.\begin{array}{llll} f=0 & \theta=1 & C=1 & \text{at } \eta=0 \\ f'=0 & \theta=0 & C=0 & \text{at } \eta\to\infty \end{array}\right\} \tag{22}$$

At the second level of truncation, we introduce $\chi = \frac{\partial f}{\partial \xi}$, $\Phi = \frac{\partial \theta}{\partial \xi}$, and $\Omega = \frac{\partial C}{\partial \xi}$ and restore all the neglected terms in the first level of truncation thus, we have equation (12) and the following:

$$\theta'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) f\theta' = \xi\left(f'\Phi - \theta'\chi\right), \tag{23}$$

$$\frac{1}{Le}C'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right) fC' = \xi\left(f'\Omega - C'\xi\right), \tag{24}$$

which subject to the boundary conditions:

$$\left.\begin{array}{llll} f=0, & \theta=1, & C=1 & \text{at } \eta=0; \\ f'=0, & \theta=0, & C=0 & \text{at } \eta\to\infty. \end{array}\right\} \tag{25}$$

The introduction of the three new dependent variables χ, Ω and Φ in the problem requires three equations with appropriate boundary conditions. This can be obtained by differentiating (12), (23), (24) with respect to ξ and neglecting the terms $\frac{\partial^2 f}{\partial \xi^2}$, $\frac{\partial^2 \theta}{\partial \xi^2}$ and $\frac{\partial^2 C}{\partial \xi^2}$, which leads to:

$$\begin{aligned}\chi'' = \; & (v^*)^{-\theta}\,\Omega' - \ln(v^*)\,\Omega\,(v^*)^{-\theta}\,\theta' + N^*(v^*)^{-\theta}\,\Phi' + \\ & - \ln(v^*)\,\Phi\,(v^*)^{-\theta} - \ln(v^*)\left(f'\Omega'\xi'\theta'\right)\end{aligned} \tag{26}$$

$$\Omega'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right)(\chi\theta' + f\Omega') + \frac{1}{(1+\xi)^2} f\theta' = \xi\left(\chi\Omega - \Omega'\chi\right) + \left(f'\Omega - \theta'\chi\right) \tag{27}$$

$$\Phi'' + \left(\frac{\xi}{1+\xi} + \frac{1}{2}\right)(\chi\theta' + f\Phi') + \frac{1}{(1+\xi)^2} f\theta' = \xi\left(\chi\Phi - \Phi'\chi\right) + \left(f'\Phi - \theta'\chi\right) \tag{28}$$

with boundary conditions:

$$\left.\begin{array}{llll} \chi(\xi,0)=0, & \Omega(\xi,0)=0, & \Phi(\xi,0)=0 & \text{at } \eta=0 \\ \chi'(\xi,0)=0, & \Omega(\xi,\infty)=0, & \Phi(\xi,\infty)=0 & \text{at } \eta\to\infty \end{array}\right\}. \tag{29}$$

3. Numerical Results and Discussion

The resulting equations with the boundary conditions have been solved numerically using a very efficient implicit finite difference method known as the modified Keller-box scheme [15]. Numerical results for the temperature, concentration, local Nusselt number and local Sherwood number are presented for the Lewis number Le ranging from 1 to 10, buoyancy ratio N ranging from 0.5 to 4 and v^* ranging from 0.2 to 5.

In order to verify the accuracy of our computer simulation model, we have compared our results with those of Bejan and Khair [2], Sighn and Queeny [3] and Yih [5]. The comparisons in the above cases are found to be in good agreement, as shown in Tables 1–3.

Table 1 Comparison of values of $-\theta'(\xi, 0)$ with $N = 0$ (UWT/UWC).

ξ	← $v^* = 1$ → Yih [5]	present results	$v^* = 0.2$	$v^* = 5$
0	0.4439	0.4439	0.81544	0.2552
0.5	0.5285	0.5413	0.9951	0.3100
1	0.5807	0.5951	1.1011	0.3416
2	0.6373	0.6573	1.2078	0.3756
6	0.7123	0.7220	1.3266	0.4130
10	0.7330	0.7391	1.3501	0.4228
20	0.7500	0.7533	1.3840	0.4309
40	0.7952	0.7608	1.3978	0.4351
∞	0.7686	0.7686	1.4122	0.4396

Table 2 Comparison of values of $-\theta'(\xi, 0)$ for various values of N and Le (UWT/UWC).

N	Le	← $v^* = 1$ → Bejan and Khair [2]	Singh and Queeny [3]	Yih [5]	present results	$v^* = 0.2$	$v^* = 5$
4	1	0.992	0.955	0.9923	0.9924	1.8231	0.5675
	10	0.681	0.634	0.6810	0.6810	1.3071	0.3727
1	1	0.628	0.604	0.6276	0.6276	1.1531	0.3591
	10	0.521	0.495	0.5215	0.5215	0.9745	0.2943
0	1	0.444	–	0.4439	0.4439	0.8154	0.2552
	10	0.444	–	0.4439	0.4439	0.8154	0.2522

Table 3 Comparison of values of $-C'(\xi, 0)$ for various values of N and Le (UWT/UWC).

N	Le	← $v^* = 1$ → Bejan and Khair [2]	Singh and Queeny [3]	Yih [5]	present results	$v^* = 0.2$	$v^* = 5$
4	1	0.992	0.955	0.9923	0.9924	1.8231	0.5675
	10	3.290	3.168	3.2897	3.2897	7.0114	0.5686
1	1	0.628	0.604	0.6276	0.6276	1.1531	0.3591
	10	2.202	2.215	2.2021	2.2021	4.6551	1.0643
0	1	0.444	–	0.4439	0.4439	0.81544	0.2552
	10	1.680	–	1.6802	1.6803	3.5229	0.8225

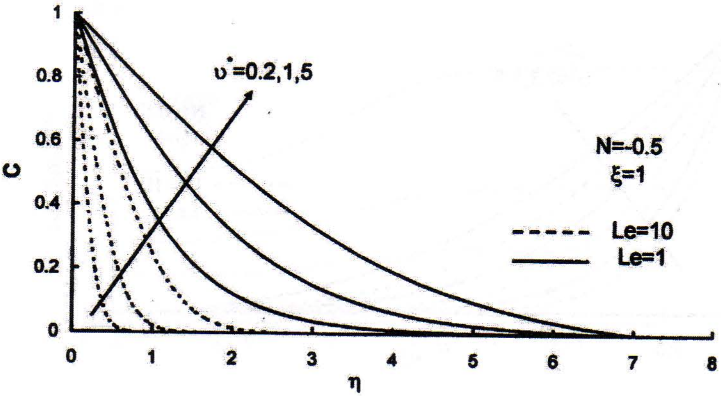

Figure 2 Temperature distribution for $N = -0.5$, $Le = 1, 10$, $v^* = 0.2, 1, 5$ in the case UWT/UWC

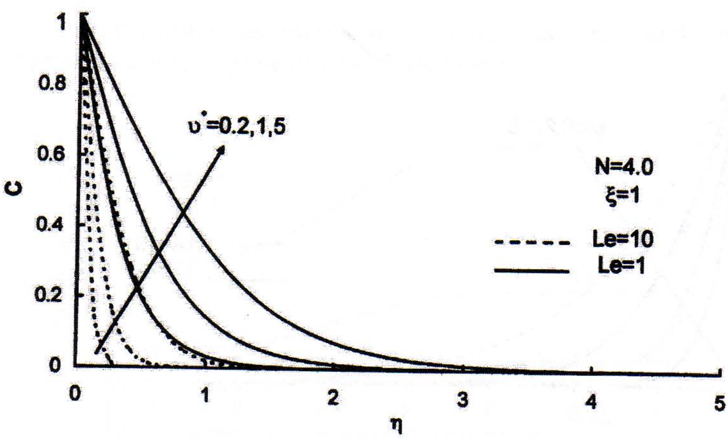

Figure 3 Temperature distribution for $N = 4$, $Le = 1, 10$, $v^* = 0.2, 1, 5$ in the case UWT/UWC

Figures 2 and 3 show that the dimensionless temperature profiles for the various values of the Lewis number, buoyancy ratio N and v^* for UWT/UWC. From these figures we can find that the dimensionless temperature profiles decrease as buoyancy ratio increases for a fixed value of Le and v^*. It is also observed that as the Lewis number Le increases form 1 to 10, the thermal boundary layer thickness increases (decreases) for a positive (negative) buoyancy ratio. For a fixed value of Lewis number and N, the dimensionless temperature increases as v^* increases.

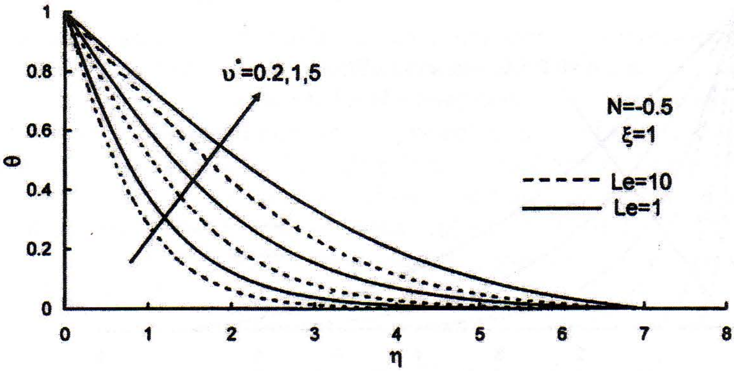

Figure 4 Concentration distribution for $N = -0.5$, $Le = 1, 10$, $v^* = 0.2, 1, 5$ in the case UWT/UWC

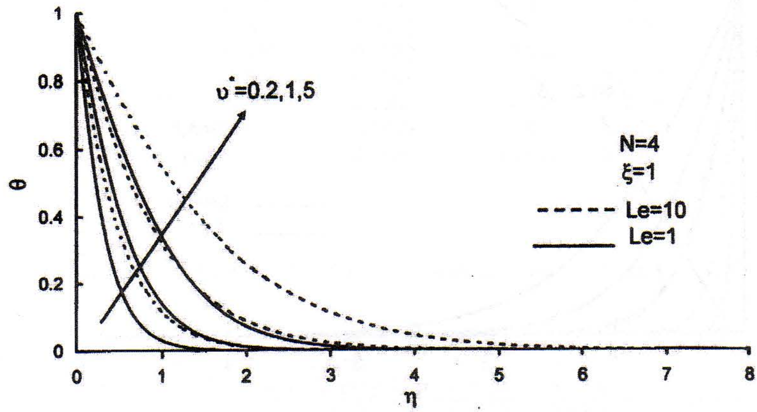

Figure 5 Concentration distribution for $N = 4$, $Le = 1, 10$, $v^* = 0.2, 1, 5$ in the case UWT/UWC

Figures 4 and 5 present dimensionless concentration profiles for the various values of Le, N, and, v^* with UWT/UWC. It is also found that dimensionless concentration profiles, like the dimensionless temperature profiles, decrease monotonically from the surface to the ambient. The concentration boundary layer thickness decreases as buoyancy ratio increases. It is also seen that as the Lewis number Le increases from 1 to 10, the concentration boundary layer decreases, at a fixed value of N and Le, the concentration increases as v^* increases.

Figure 6 displays the local Nusselt number for various values of v^* with UWT/UWC. It is shown that increasing the value of dimensionless distance ξ increases the local Nusselt number. This is because the increase in the value of ξ implies the increase of the buoyancy force which tends to accelerate the flow. It is also observed that the local Nusselt number approaches to constant value when ξ is small and large. We observe that the Nusselt number decreases as v^* increases. Figure 7 illustrates the local Sherwood number for various values of v^* with UWT/UWC.

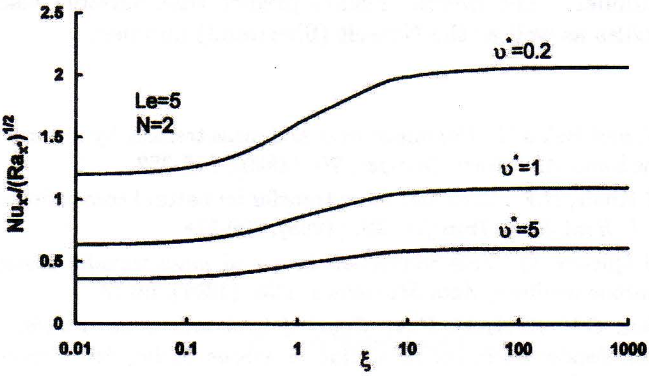

Figure 6 Local Nusselt number distribution for $N = 2$, $Le = 5$, $v^* = 0.2, 1, 5$ in the case UWT/UWC

The local Sherwood number becomes almost constant for small and large ξ. Also, the local Sherwood number decreases as v^* increases.

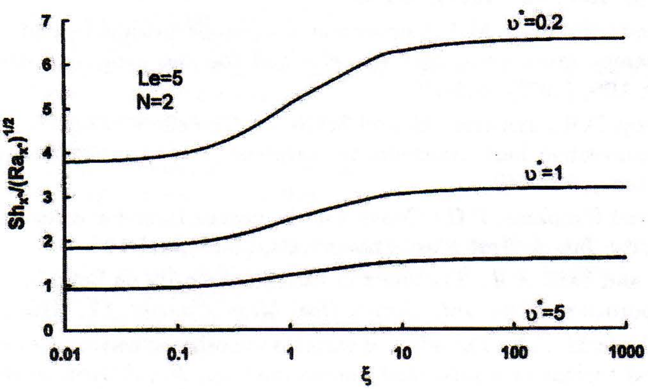

Figure 7 Local Sherwood number distribution for $N = 2$, $Le = 5$, $v^* = 0.2, 1, 5$ in the case UWT/UWC

4. Conclusions

A boundary layer analysis is presented to study the effect of variable viscosity on natural convection flow in a saturated porous medium resulting from combined heat and mass buoyancy effects adjacent to a truncated cone maintained at uniform wall temperature/concentration (UWT/UWC). Numerical solutions are obtained for different values of dimensionless distance ξ buoyancy ratio N, Lewis number Le and wall to ambient ratio v^*. The decay of the dimensionless temperature and concentration profiles has been observed. It is shown that for the increase in the value of N and ξ both the heat and mass transfer increase. It is also found that increasing the wall to ambient ratio parameter decreases the local Nusselt number and Sherwood number. The present results predict that variable viscosity and concentration profiles as well as the Nusselt (Sherwood) numbers.

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Table 4 Nomenclature

c	concentration
C	dimensionless concentration
D	mass diffusivity
f	dimensionless stream function
g	gravitational acceleration
h	local heat transfer coefficient
K	permeability of the porous media
κ	thermal conductivity
Le	Lewis number, α/D
N	buoyancy ratio for UWT/UWC
Nu	local Nusselt number, hx^*/κ
Ra_{x^*}	modified local Rayleigh number, $\rho_\infty g \cos\gamma\beta_T(T_w - T_\infty)Kx^*/(\mu_\infty\alpha)$
r	local radius of truncated cone
Sh_{x^*}	local Sherwood number, $mx^*/D(c_w - c_\infty)$
T	temperature
u	velocity component in x-direction
ν	velocity component in y-direction
x	streamwise coordinate
x^*	distance measured from the leading edge of the truncated cone, $x - x_0$
x_0	distance of the leading edge of the truncated cone measure from the origin
y	transverse coordinate
ρ_∞	density of the ambient fluid
α	thermal diffusivity
Ψ	stream function
μ	dynamic viscosity
θ	dimensionless temperature
η	pseudo-similarity variable
γ	half angle of the truncated cone
ξ	dimensionless distance
w	condition at the wall
∞	condition at infinity