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METHODOLOGY OF THE STEPPER MOTOR ROTATIONAL MOTION INVESTIGATIONS

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Advantages and disadvantages of separate studies on dynamic systems using either theoretical or experimental methods have been emphasized, and a standpoint has been expressed that both methods must definitely be combined. Measurements of vibration in the rotational motion by means of an RWJ instrument served as the methodology of experimental studies, while Lagrange quadratic equations were used for theoretical analysis. The combined experimental-theoretical methodology was adopted for the non-stationary motion of a stepper motor. The analysis and reduction of the system of equations of dynamics for an inverse issue have been presented and selected experimental solutions for the motion of the drive system of a stepper motor has been quoted.

Keywords: rotation motion, investigation methodology.

1. Introduction

As a beginning we would like to put the fundamental and the philosophical question: what and where is truth in the technical sciences? Answers are as follows (Fig. 1):

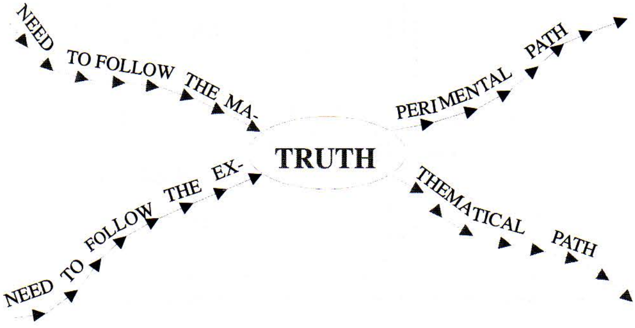

Figure 1 Epistemology interpretation scheme

Studies of the motion of machines and devices in mechanics are generally performed in two independent ways – theoretical (analytical) and experimental. Theoretical methods are based on systems of differential equations of motion dynamics, sensible solutions to which require the knowledge of real initial and boundary conditions, and it is the experimental knowledge that can provide us with such conditions. The lack of such conditions causes solutions to be mathematically correct only formally; however, they are false from the viewpoint of cognition and are useless for the practice of technical sciences. Experimental methods provide us with the true knowledge of a real material system being in motion [10], [11], [12], [14], but they only have partial, fragmentary value, insufficient for the generalized and complete cognition of the motion phenomenon. Thus, maximum cognitive advantages and practical effectiveness can be anticipated when an optimum, in-between method is used. Studies on the motion dynamics of material systems generally are concerned with the determination of motion parameters of this system depending on external loads or assigned initial input functions – these are so called simple tasks.

The experiment still remains a test of the correctness of the results obtained from theoretical analysis. The present work aims to demonstrate a combination of an experiment and an analytical description of a dynamic system considered by way of example, in order to obtain the values of external loads causing motion that cannot be determined theoretically. This is so called inverse task. By determining kinematic parameters of motion of the system analyzed, it is possible to determine the function of an external load from analytic dynamic differential equations of motion. When velocities and accelerations of the particular elements of a system, as well as the values of static, dynamic and geometrical characteristics are known, it is possible to solve a formulated system of equations in relation to e.g. an unknown driving torque. To demonstrate this investigation path, a stepper motor was chosen as a dynamic system of an exceptionally complicated image of non-stationary rotational motion [1], [2], [16].

2. Experimental Investigations

A schematic diagram of a test stand on which measurements of torsional vibration were made is shown in Fig. 2.

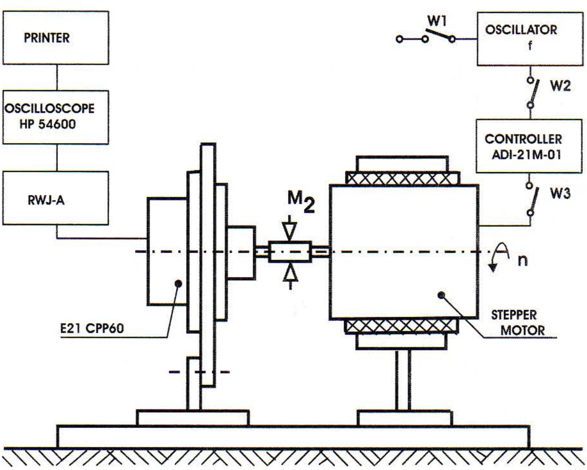

Figure 2 Schematic diagram of the investigation equipment of a stepper motor: RWJ-A – a measuring instrument of torsional vibration of RWJ (A – analogue), E21CPP60 – rotational-pulse converter (mechanical-optical), M2 – external moment applied to the clutch coupling the motor to the converter

A stepper motor has a rotor of a constant moment of inertia J_1 and a variable electrical moment M_1, controlled electrically. The E21CPP60 mechanical-optical has a measuring disk of a moment of inertia J_3, which has slits on its circumference, and is shown in Fig. 3.

The instrument developed enables torsional vibration parameters to be measured: the frequency f, the amplitude of the angle $\varphi(t)$, the velocity $\omega(t)$ and the angular acceleration $\varepsilon(t)$. It can be used for the analysis and diagnostics of a non-stationary rotational motion of shafts and other rotating elements of machines and devices [3], [9]. Special computational programs and procedures allow the rotational moment, resonances, the power spectrum and phase curves to be determined.

Possibilities of a more detailed analysis of the torsional vibration generated are discussed in [8], [10] and [13].

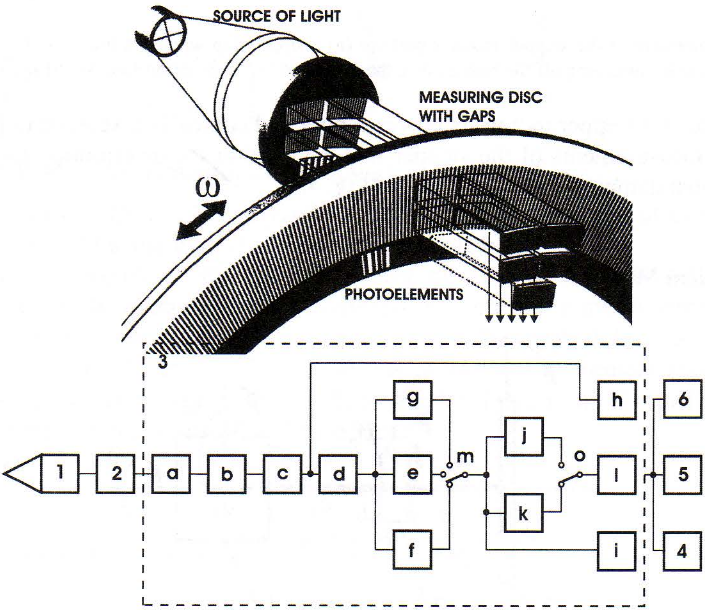

Figure 3 Schematic diagram of mechanical-optical-electrical conversion of the measuring instrument of torsional vibration of RWJ

In Fig. 3 there are shown: 1 – an adaptor (clutch), 2 – an angular displacement-to-electrical signal converter, 3 – an RWJ unit, 4 – a digital oscilloscope, 5 – an analyzer, 6 – a computer (together with its software).

The RWJ unit includes the sub-units: a – a forming system (Schmitt trigger), b – a single pulse generator, c – a filter, d – an amplifier, e – a proportional system, f – an integrating system, g – a differentiating system, h – a rotational speed meter, i – a meter of vibration frequency, j – a detector of a RMS value, k – a detector of a peak value, l – a meter of a vibration level [12].

3. Dynamic Properties of Materials

Its material properties, called static, such as: the modulus of volume elasticity E and of non-dilatational strain G as well as Poisson ration ν, and dynamic material properties, such as: the torsional (angular) rigidity k and the coefficient of torsional (angular) damping c. All these quantities are determined by the experimental method (Fig. 2).

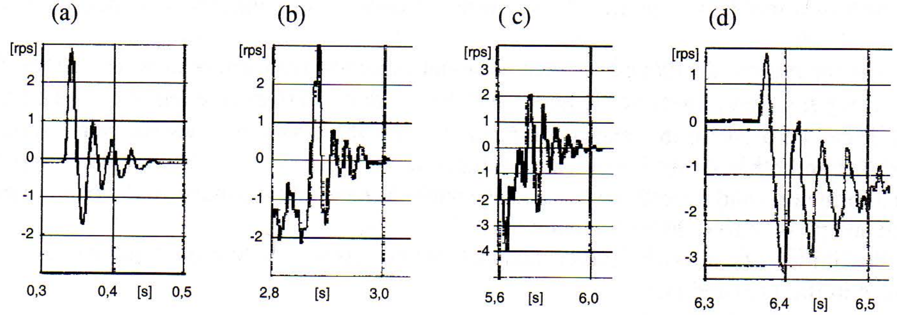

Figure 4 Measurement of the stepper motor vibration: (a) a single step without a load, (b), (c) a "stop", (d) a "start" tests made by switching off the motor (W_1), the generator (W_2), the controller (W_3) (Fig.2)

The structure of a stepper motor, its power supply and control is discussed in [15].

Sample measurements of the stepper motor vibration for determining the coefficient of the torsional damping c are presented in Fig. 4.

4. Analytical Model

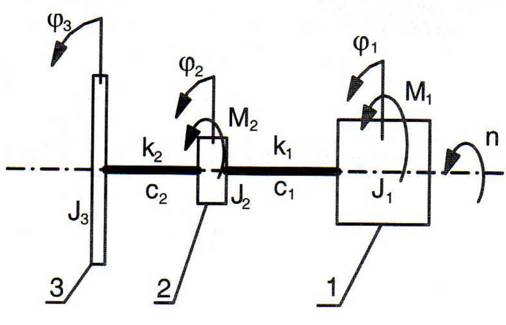

Figure 5 Computational schematic diagram of the object studied: 1 – a stepper motor, 2 – a clutch and a device for imparting the external moment M_2, 3 – the disk of an RWJ measuring instrument of torsional vibration

Dynamic equations of motion for a discrete system shown in Fig.5 were written on the basis of Lagrange second-order equations for holonomic systems of ideal constraints [4], [6], [7]. This equation has the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}+\frac{\partial D}{\partial \dot{q}_j}=-\frac{\partial V}{\partial q_j} \tag{1}$$

For the description of the motion of the discrete system shown in Fig. 1 a vector of the generalized coordinates in the form

$$\mathbf{q_j}=[\varphi_1 \quad \varphi_2 \quad \varphi_3]^{\mathrm{T}} \tag{2}$$

where: ϕ_1 – the angle of torsion of the rotating element of a stepper motor, ϕ_2 – the angle of torsion of the clutch 2 (Fig. 5), ϕ_3 – the angle of torsion of the measuring disk.

Following the determination of the kinetic and potential energy of the system, as well as of the dissipation function and the virtual operation of external moments acting on the system, appropriate mathematical procedures were performed so that a system was obtained of three differential dynamic equations of motion of the form

$$
\begin{aligned}
&J_1\ddot{\varphi}_1 - c_1(\dot{\varphi}_2 - \dot{\varphi}_1) = k_1(\varphi_2 - \varphi_1) + M_1 \\
&J_2\ddot{\varphi}_2 + c_1(\dot{\varphi}_2 - \dot{\varphi}_1) + c_2(\dot{\varphi}_2 - \dot{\varphi}_3) = -k_1(\varphi_2 - \varphi_1) - k_2(\varphi_2 - \varphi_3) + M_2 \\
&J_3\ddot{\varphi}_3 - c_2(\dot{\varphi}_2 - \dot{\varphi}_3) = k_2(\varphi_2 - \varphi_3)
\end{aligned}
\tag{3}
$$

In the vector notation the system of differential equations formulated has the form

$$\mathbf{J}\cdot\ddot{\mathbf{q}} + \mathbf{c}\cdot\dot{\mathbf{q}} + \mathbf{k}\cdot\mathbf{q} = \mathbf{M}, \tag{4}$$

where the vector of generalized coordinates, the damping matrix, the rigidity matrix and the vector of external loads are, respectively

$$c = \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1 + c_2 & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix};\ k = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k & k_1 + k_2 & k_2 \\ 0 & -k_2 & k_2 \end{bmatrix};\ Q = \begin{bmatrix} M_1 \\ M_2 \\ 0 \end{bmatrix}.$$

5. Theoretical - Experimental Methodology

Systems of equation (3) can be analyzed and solved in real conditions of motion owing to the combination of the theoretical and experimental method.

Material parameters, i.e. the matrices of: the masses and moments of inertia J, the flexibility l/k and the damping c are determined by performing measurements. Only the remaining quantities are unknown functions of time. The methodology of measurement of a non-stationary rotational motion by means of an RWJ instrument allows the functions of time ϕ_1, ϕ_2, ϕ_3 to be determined. From the third equation (3) a differential linear equation for the motion of the clutch 2 (see Fig. 5).

$$\dot{\varphi}_2 + a_2\varphi_2 + b_2 = 0, \tag{5}$$

where

$$a_2 = \frac{k_2}{c_2} = \text{const};\quad b_2 = -\frac{J_3\ddot{\varphi}_3 + c_2\dot{\varphi}_3 + k_2\varphi_3}{c_2} = f(t).$$

Equation (5) is not difficult to solve when the boundary conditions are determined experimentally. The second equation (3) must be solved for two variants

1) M_2=0;
2) $M_2(t)\neq 0$.

The moment M_2 can be represented by a braking moment or another moment, e.g. caused by a failure in the power transmission system. For solution 1) from the second equation (3), a differential linear equation for the stepper motor rotor motion will be obtained

$$\dot{\varphi}_1 + a_1\varphi_1 + b_1 = 0, \tag{6}$$

where

$$a_1 = \frac{k_1}{c_1} = \text{const};\quad b_1 = -\frac{J_2\ddot{\varphi}_2 + c_1\dot{\varphi}_2 + c_2(\dot{\varphi}_2 - \dot{\varphi}_3) + k_1\varphi_1 + k_2(\varphi_2 - \varphi_3)}{c_1} = f(t).$$

Putting the solution to equation (6) to the first equation (3), a sought-for function of the moment of an electric stepper motor M_1 will be obtained.

For the case of the solution for variant 2) in equation (6), b_1 will then be an unknown function due to the unknown function $M_2(t)$, i.e.

$$\dot{\varphi}_1 + a_1\varphi_1 + b_1' = 0$$

where

$$b_1' = -\frac{J_2\ddot{\varphi}_2 + c_1\dot{\varphi}_2 + c_2(\dot{\varphi}_2 - \dot{\varphi}_3) + k_1\varphi_1 + k_2(\varphi_2 - \varphi_3) - c_1M_2(t)}{c_1}.$$

The first equation (3) is transformed to the form

$$\ddot{\varphi}_1 + a_0\dot{\varphi}_1 + a_1'\varphi_1 + b_1' = 0,$$

where

$$a_0 = \frac{c_1}{J_1} = \text{const};\ a_1 = \frac{k_1}{J_1} = \text{const};\ b_1' = -\frac{c_1\dot{\varphi}_2 + k_1\varphi_2 + M_1(t)}{J_1} = f(t).$$

It is necessary to study the solutions to systems of equations (7) and (8). One can expect to obtain help from those studying electric machines, in which case, by measuring the voltage $U(t)$ and intensity $i(t)$ of electric current supplied to a stepper motor, when magnetic properties of the motor are known, it is possible to determine the driving torque $M_1(t)$ with sufficient approximation.

6. Selected Experimental Investigations

The RWJ measuring instrument allows a wide range of measurements [13], [15] to be made on the test stand shown in Fig. 2. In Fig. 6 the curves of the rotational speed n [rps] of the motor (proportional to the voltage [V]) for so called test of "rundown", when there is a slow increase in the frequency f up to the maximum boundary value f_g until the motor rotor stops. In the case of absence of a load (M_2=0), (Fig. 6a) the boundary frequency was f_g=500÷520 Hz; when the braking moment occurred (M_2>0), (Fig. 6b), the frequency f_g=350÷370 Hz.

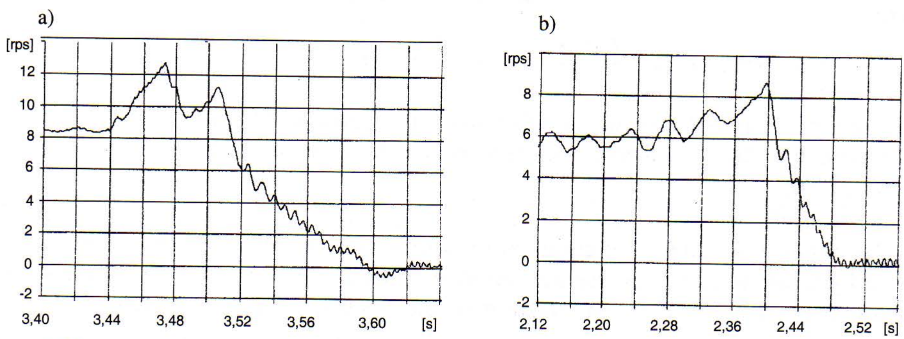

Figure 6 Measurements of the stepper motor vibration during the test of "rundown" with an increase in the frequency f: a) – without a load; b) – with a of a permanent braking moment M_2 (Figs2 and 5)

The determination of the moment $M_2(t)$ is very difficult to measure; it can only be determined making use of the analytical method discussed in 4. In Fig. 7 simulations of the surge moment M_2 of a step character are presented by way of example. In Fig. 7a) a non-stationary transient state is shown, when the moment M_2 acted in the reverse direction, while in Fig. 7b) in agreement with the rotational speed n.

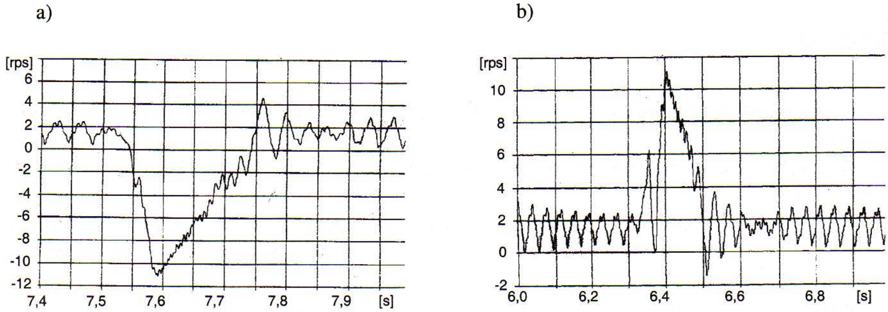

Figure 7 Measurements of the stepper motor vibration for f=100 Hz during the tests of acting with the surge moment M_2 (Figs 2 and 5): a) – in the reverse direction, b) – in agreement with the direction of rotations n

The curves in Figs 6 and 7 present example experimental functions $\dot{\varphi}_3 = 2\pi n$, which when differentiated and integrated will yield $\ddot{\varphi}_3$ and φ, respectively as fundamental experimental functions for solving system of equations (3) discussed in [5].

7. Summary

a) The authors emphasize advantages and disadvantages of using experimental or theoretical (analytical) methods separately in the practice of scientific investigations on rotating material systems of machines and mechanical devices.

b) It is postulated that the theoretical-experimental methodology be used both for fundamental research and applied research, the major advantages being pointed out to be:
   - the experiment provides us with real boundary conditions for integrating differential equations, while calculations broaden, deepen and generalize partial results of the measurements;
   - the experiment and theory check and falsify each other, which leads to the reliable results of experiments; results with the smallest error and of the greatest correctness i.e. good conformance with real phenomena of motion.

c) The choice of experimental-theoretical methodology of the research requires a well-thought-out development of:
   - an experimental physical model and a measuring test stand,
   - a mathematical model in the form of a system of equations of motion dynamics satisfying the principle: the minimum of the experiment and the minimum of mathematical complication to obtain the maximum of reliable and true knowledge or the maximum of the technological usability.

d) In the paper, an analytical method based on Lagrange second-order equations and the method of measurements of the rotational motion by means of an RWJ instrument of the authors' own construction have been proposed. The combination of both methods leads from a system of equations (3) to the practical analysis of equations (5), (6), (7) and (8) and then more readily to a mathematical solution. A system with a stepper motor of an exceptionally complicated form of rotational motion was selected as a technical object. Some selected examples of experimental investigations of practical importance have been presented.

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Nomenclature

c – torsional damping coefficient [Nmrds^{-1}]
D – dissipation energy function [Nmrds^{-1}]
f – controller excitation frequency [Hz]
J – mass inertia moment [kgm^2]
k – torsional stiffness coefficient [Nmrd^{-1}]
M – external force moment [Nm]
n – rotation speed [rps]
Q_j – generalized forces [Nm]
T – kinetic energy function [Nm]
V – potential energy function [Nm]