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Axisymmetric MHD Oscillation of a Compressible Hollow Jet

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The capillary oscillation of compressible hollow Jet under the action of the electromagnetic force for axisymmetric perturbation is discussed. Some published results are recovered as limiting cases on utilizing the dispersion relation of the present model. The capillary force has a destabilizing effect in a small range ($0 \leq \chi < 1$, where χ is the dimensionless longitudinal wavenumber) of perturbation. While it is stabilizing as $1 \leq \chi < \infty$. The compressibility has a strong destabilizing tendency of the model for all wavenumber values $\chi \neq 0$. Each of the magnetic fields pervaded in the gas and liquid regions has a stabilizing effect for all short and. long wavelengths. As that effect is so strong, the capillary instability is suppressed and stability sets in. But in fact here the model is unstable due to the strong destabilizing effect of the compressibility.

Keywords: convection, solar, magnetic field, stress-free, exact solution, nonlinear.

1. Introduction

The oscillation of a liquid cylinder is comprehensively studied by Rayleigh (1945) and Chandrasekhar (1981). The stability investigation of a gas cylinder penetrated into a liquid is suggested by Chandrasekhar (1981) for axisymmetric perturbation and simultaneously by Drazin and Reid (1980), see also Cheng (1985). Kendall (1986) made nice experiments for discussing the oscillation of a such model, and he did attract the attention for investigating its stability under different factors. Radwan (1991) examined the effect of the magnetic field on the capillary oscillation of a hollow jet, see also Radwan (2000). In all previous works the liquid is considered to be incompressible with solenoidal velocity.

Here we intend to study the axisymmetric MHD oscillation of a compressible hollow jet, where the liquid velocity is not solenoidal any more and the liquid density is not constant.

2. Basic equations

Here we consider a gas cylinder of radius R_0 pervaded into a compressible liquid. The gas medium is pervaded by $\underline{H}_0^g = (0, 0, \alpha H_0)$ while the liquid one penetrated by $\underline{H}_0 = (0, 0, H_0$ where $\underline{H}_0$ is the intensity of the magnetic field in the liquid and α is some parameter. The model is acting upon the surface pressure, kinetic pressure of liquid and electromagnetic force.

The basic equations are given as follows. In the liquid

$$\rho\left(\frac{d\underline{u}}{dt}\right) = -\nabla P + \mu\left(\nabla \wedge \underline{H}\right) \wedge \underline{H} \tag{1}$$

$$\left(\frac{d\rho}{dt}\right) + \rho\left(\nabla \cdot \underline{u}\right) = 0, \left(\frac{d}{dt}\right) = \left(\frac{\partial}{\partial t}\right) + \left(\underline{u} \cdot \nabla\right) \tag{2}$$

$$\rho C_v\left(\frac{dT}{dt}\right) = -P\left(\nabla \cdot \underline{u}\right) \tag{3}$$

$$P = \mathcal{K}\rho^{\gamma}\,, \quad P = \rho R^c T \tag{4}$$

$$\nabla \cdot \underline{H} = 0 \tag{5}$$

$$\left(\frac{d\underline{H}}{dt}\right) = \left(\underline{H} \cdot \nabla\right)\underline{u} - \underline{H}\left(\nabla \cdot \underline{u}\right) \tag{6}$$

In the gas

$$\nabla \cdot \underline{H}^g = 0\,, \tag{7}$$

$$\nabla \wedge \underline{H}^g = 0\,. \tag{8}$$

Along the gas–liquid interface

$$P_s = S\left(\nabla \cdot \underline{N}\right)\,. \tag{9}$$

Here ρ, $\underline{u}$ and P are the liquid density , velocity vector and pressure, $\underline{H}$ and μ are the magnetic field intensity and the permeability coefficient, T the temperature, γ the ratio of specific heats C_v and C_p, $\mathcal{K}$ and R^c are constants. $\underline{H}^g$ the magnetic field intensity in the gas region, P_s the pressure surface due to capillary force, S the surface tension coefficient and $\underline{N}$ is the unit outward vector normal to the gas cylinder surface.

3. Fourier Analysis

For small departures from the initial state, the perturbed variables may be expressed as

$$Q = Q_0(r) + \epsilon_0 Q_1(r, \phi, z, t) + \dots \tag{10}$$

with ϵ_0 is the initial amplitude of the perturbation. The amplitude at time t is given by

$$\epsilon = \epsilon_0 \exp(\sigma t) \tag{11}$$

where σ is the growth rate. Here Q stands for ρ, $\underline{u}$, P, $\underline{H}$, $\underline{H}^g$, N, T and the radial distance of the gas cylinder. The latter, for a single Fourier term and based on the linear perturbation technique is being

$$r = R_0 + \epsilon_0 R_1 + \dots$$

with

$$R_1 = \exp(\sigma t + ikz) \tag{12}$$

the elevation of the surface wave in the axisymmetric perturbation where k is the longitudinal wavenumber.

By substituting from (10) into the basic equations (1)–(9), the unperturbed and perturbed systems of equations are obtained. The unperturbed system of equations is solved and the liquid pressure is given by

$$P_0 = \frac{1}{2}\mu H_0^2\left(\alpha^2 - 1\right) - \frac{S}{R_0} + P_0^g\,. \tag{13}$$

The perturbed system of equations is solved and the required boundary conditions are applied. Applying the compatibility condition that the normal component of the total stress tensor must be continuous, across the gas-liquid interface at $r = R_0$. The eigenvalue relation is derived as

$$\sigma^2 = \frac{\mu H_0^2}{\rho R_0^2}\left[-\chi^2 - \alpha^2\chi y\frac{I_0(\chi)K_1(y)}{I_1(\chi)K_0(y)}\right] + \frac{S}{\rho R_0^3}\left(1-\chi^2\right)\frac{yK_1(\chi)}{K_0(\chi)}\,, \tag{14}$$

where $\chi = kR_0$ and $y = \eta R_0$ are the dimensionless longitudinal ordinary and compressible wave numbers with

$$\eta^2 = k^2 + \left[\frac{\sigma^2}{a^2\zeta}\right], \zeta = 1 + \frac{\mu H_0^2}{\rho_0\sigma^2}\left[\frac{\sigma^2}{a^2} + k^2\right]\,,$$

while I_0 and K_0 are the modified Bessel functions of the first and second kind of zero order and idem I_1 and K_1, of order one.

4. Discussion and results

In order to determine the stable and unstable domains for different wavelengths, we have to discuss the dispersion relation (14). In view of the behaviour and characteristics of the modified Bessel functions (see Abramowitz and Stegun (1970), for $\chi \neq 0$)

$$I_m(\chi) > 0 \text{ and } K_m(\chi) > 0\,, \tag{15}$$

with m is the azimuthal wavenumber where $I_m(\chi)$ is monotonically increasing while $K_m(\chi)$ is monotonically decreasing but never negative. We see, for $\chi \neq 0$, $y \neq 0$, that

$$\chi\left(\frac{I_0(\chi)}{I_1(\chi)}\right) > 0 \text{ and } y\left(\frac{K_1(y)}{K_0(y)}\right) > 0\,. \tag{16}$$

Consequently, we find that

$$\left.\begin{array}{lll} \sigma^2\left(\frac{S}{\rho R_0^3}\right)^{-1} > 0 & \text{as} & 0 < \chi < 1 \\ \sigma^2\left(\frac{S}{\rho R_0^3}\right)^{-1} \leq 0 & \text{as} & 1 \leq \chi < \infty \end{array}\right\} \text{ for } H_0 = 0\,, \tag{17}$$

from which we conclude that the capillary force is destabilizing.

Also

$$\frac{\sigma^2}{\frac{\mu H_0^2}{\rho R_0^2}} < 0 \text{ for } \chi > 0 \text{ for } S = 0\,. \tag{18}$$

This means that the electromagnetic force is strongly stabilizing. Moreover, the discussions of the relation (14), reveal that the compressibility has a destabilizing tendency.

As the effect of the electromagnetic force is so strong, the capillary destabilizing effect is suppressed and the stability sets in. However, it is found that the compressibility has a strong destabilizing tendency which lead to collapsing the model varicosity.

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