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Natural Convection Flows with Variable Viscosity, Heat and Mass Diffusion Along a Vertical Plate

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The aim of this paper is a numerical study of laminar double difiusive free convection viscous flows adjacent to a vertical plate, taking into account the variation of the viscosity and double-diffusive heat and mass transfer with temperature. The governing conservation equations of mass, momentum, energy and chemical species arc non-dimensionalized by using appropriate transformations. The resulting equations are solved numerically by using the fourth order Runge-Kutta integration scheme along with the Nachtsheim-Swiger shooting technique. It is noticed that both the velocity and concentration of air are increasing as the parameter β_2, (the species diffusion parameter) increases, but an opposite effect for the velocity is observed at a certain distance far from the plate. It is also observed that the temperature decreases as the parameter β_2 increases. The shearing stress at the plate, the local Nusselt number and the local Sherwood number are obtained. The friction coefficient at the plate, of heat and mass transfer at the plate, the momentum, thermal and concentration boundary layers thickness $(\delta, \delta_T, \delta_C)$ have been estimated for different values of α, Sc and N.

Keywords: convection flow, mass diffusion, viscoelastic fluid.

1. Introduction

Natural convection flows driven by temperature differences have been extensively studied by *Gebhart te al.* (1971), Gebhart and Pera (1971); Jaluria and Gebhart (1974); Jaluria (1980); Ostrach (1980); Elbashbeshy and Ibrahim (1993); Mongruel *et al.* (1996); Kuan-Tzong Lee (1999); Saddeek (2000) and other authors. Gebhart and Pera (1971) studied laminar natural convection flows driven by thermal and concentration buoyancy forces adjacent to a vertical plate. They presented similarity solutions and also investigated the laminar stability of such flows. Pera and Gebhart (1972) extended their previous work to flows for horizontal plate. Williams, *et al.* (1987) assumed a plate temperature that varies with time and position and found possible semi similar solutions for a variety of classes of wall temperature distribution. Eltayeb and Loper (1991) studied the stability of verical orienited double diffusive interfaces having an imposed vertical stable temperature gradient. The influence of variable viscosity on laminar boundary layer flow and heat transfer due to a continuously moving flat plate is examined by Pop, *et al.* (1992). Ibrahim and Ibrahim (1984) obtained a solution for the variable viscosity flow of a dilute suspension between two parallel plates taking into account two different forms for the viscosity-temperature relation.

A numerical study of natural convection flows due to the combined buoyancy of heat and mass diffusion in a thermally stratified medium was obtained by Angirasa and Srinivasan (1989). They assumed that the viscosity, the thermal diffusivity and the mass diffusion coefficients are constants. The problem of free convection flow of a Newtonian fluid having variable viscosity and thermal diffusivity along an isothermal vertical plate was studied by Elbashbeshy and Ibrahim (1993). Mongruel, *et al.* (1996) investigated natural convection driven by two buoyancy sources, such as heat and mass, in vertical boundary layer starting from the integral equations and using scale analysis to derive the different asymptotic flow regimes encountered with different buoyancy forces and diffusion coefficients. The natural convection heat and mass transfer in vertical parallel plates with discrete heating has been studied by Kuan-Tzong Lee (1999). The effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation is studied by Saddeek, (2000). He assumed that the fluid viscosity varies as an inverse linear function of temperature.

In all the above studies the effects of variable viscosity, thermal and mass diffusivities (together) on the flow field have not been considered yet.

In the present study, we extend the works of Angirasa and Srinivasan (1989); Elbashbeshy and Ibrahim (1993) and Mongruel, et al. (1996), taking into account the dependence of mass diffusivity on temperature because it is well known that in many natural and technological processes the temperature and concentration differences occur simultaneously. Such processes occur in cleaning operations, drying, crystal growth, solar ponds and photosynthesis. For this reason we will see the effect of the variable concentration on the motion of the fluid. The Boussinesq approximation is used in the equation of motion.

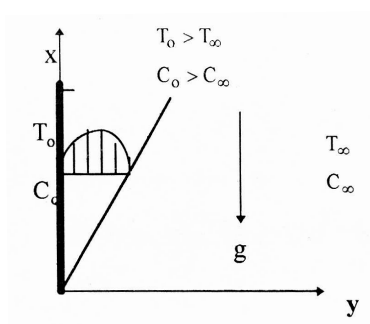

Figure 1 Flow near the plate

2. Formulation of the problem

We consider convection flow near a vertical plate with variable viscosity, thermal and mass diffusivities. Let the Cartesian coordinates x and y move in the direction of the plate vertically upward and normal to the plate respectively as in Figure 1. Consider the temperature of the plate is T_0 and the concentration of the diffusing species at the plate is C_0. Let the temperature and concentration at infinity are T_∞ and C_∞ respectively. We assume that the density of the fluid (Mongruel *et al.*, (1996)) is

$$\rho = \rho_\infty \left[1 - \beta_T(T - T_\infty - \beta_C(C - C_\infty)\right] ,$$

where ρ is the density of the fluid, ρ_∞ is the density at infinity, β_T and β_C are the coefficients of the thermal and mass expansion, respectively; T is the temperature of the fluid; C is the concentration. The flat plate is heated in such way, that the pressure in each horizontal plane is equal to the hydrostatic pressure and thus it is constant. By using the Boussinesq approximation one can write the basic governing equations which are the conservation of mass, momentum, heat and mass as (Gebhart (1971) and Elbashbeshy and Ibrahim (1993)):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$\rho_\infty \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho_\infty g\beta_t(T - T_\infty) + \rho_\infty g\beta_C(C - C_\infty)\,, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) , \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right) , \tag{4}$$

where u, v are the components of the velocity of the fluid in x and y directions respectively; μ is the viscosity coefficient of the fluid; g is the constant acceleration due to gravity; K is the thermal diffusivity; D is the mass diffusivity. The boundary conditions which are associated with equations (1)–(4) are

$$u = v = 0\,, \quad T = T_0\,, \quad C = C_0\,, \quad \text{at } y = 0 \tag{5}$$

and

$$u \to 0\,, \quad v \to 0\,, \quad T \to T_\infty\,, \quad C \to C_\infty \text{ at } y \to \infty\,. \tag{6}$$

The continuity equation (1) is satisfied by the stream function $\Psi(x, y)$, which is defined by

$$u = \frac{\partial \Psi}{\partial y}\,, \quad v = -\frac{\partial \Psi}{\partial x}\,. \tag{7}$$

To transform the partial differential equations (2)–(4) into a set of ordinary differential equations, the following dimensionless variables are introduced (Gebhart (1971)):

$$\begin{aligned} \eta &= \frac{y[Gr(x)]^{\frac{1}{4}}}{x}\,, \quad \Psi(\eta) = 4\nu_\infty\,[Gr(x)]^{\frac{1}{4}}\,\phi(\eta)\,, \\ \theta &= \frac{T - T_\infty}{T_0 - T_\infty}\,, \quad \gamma = \frac{C - C_\infty}{C_0 - C_\infty}\,, \end{aligned} \tag{8}$$

where η is the dimensional distance from the plate, ν_∞ is the kinematic viscosity at infinity, $Gr(x) = \frac{g\beta_T(T-T_\infty)x^3}{4\nu_\infty^2}$ is the Grashof number, $\phi(\eta)$ is the dimensionless stream function, θ is the dimensionless temperature, γ is the dimensionless concentration. The variations of viscosity, thermal diffusivity and mass diffusion coefficients with dimensionless temperature are written in the form: (Schlichting (1968); Pop, *et al.* (1992) and Elbashbeshy and Ibrahim (1993))

$$\frac{\mu}{\mu_\infty} = \mathrm{e}^{-\alpha\theta}\,, \tag{9}$$

$$\frac{K}{K_\infty} = 1 + \beta_1\theta\,, \tag{10}$$

$$\frac{D}{D_\infty} = 1 + \beta_2\theta\,, \tag{11}$$

where α, β_1 and β_2 are parameters depending on the nature of the fluid and μ_∞, K_∞ and D_∞ are the values of μ, K and D as $\eta \to \eta_\infty$ (where η_∞ is the maximum value of η). By using the transformations (8) and equations (9), (10) and (11) equations (2)–(4) transform into

$$\phi''' + \phi''[3\phi\mathrm{e}^{\alpha\theta} - \alpha\theta'] + \mathrm{e}^{\alpha\theta}[\theta + N\gamma - 2\phi'^2] = 0\,, \tag{12}$$

$$\theta'' + \left(\frac{\beta_1}{1+\beta_1\theta}\right)\theta'^2 + \left(\frac{3Pr}{1+\beta_1\theta}\right)\phi\theta' = 0\,, \tag{13}$$

$$\gamma'' + \left(\frac{\beta_2}{1+\beta_2\theta}\right)\theta'\gamma' + \left(\frac{3Sc}{1+\beta_2\theta}\right)\phi\gamma' = 0\,, \tag{14}$$

where N, Pr and Sc are buoyancy ratio, Prandtl and Schmidt numbers, respectively. They are given by

$$N = \frac{\beta_2(C_0 - C_\infty)}{\beta_1(T_0 - T_\infty)}\,, \quad Pr = \frac{\nu_\infty}{K_\infty} \quad \text{and } Sc = \frac{\nu_\infty}{D_\infty}\,. \tag{15}$$

The boundary conditions (5) and (6) transform to

$$\phi = 0\,, \quad \phi' = 0\,, \quad \gamma = 1\,, \quad \text{at} \quad \eta = 0\,, \tag{16}$$

$$\phi' \to 0\,, \quad \theta \to 0\,, \quad \gamma \to 0\,, \quad \text{as} \quad \eta \to \infty\,. \tag{17}$$

Neglecting the effect of the variation of concentration on the motion of the fluid, with $N = 0$, we will obtain the same equations as those obtained by Elbashbeshy and Ibrahim (1993). Also in the case of $\alpha = \beta_1 = \beta_2 = 0$, equations (12)–(14) will reduce to those equations of Mongruel, *et al.* (1996) with the same boundary conditions (16) and (17).

3. The primary physical quantities of interest

(i) Boundary layer thickness (δ) has been regarded as that distance from the plate where the velocity at its end (ϕ') has approximate value equal to 0.01 [Schlichting (1968)]

$$\delta = \eta|_{\phi'=0.01}\,. \tag{18}$$

(ii) Thermal boundary layer thickness (δ_T) is defined in the same manner as

$$\delta_T = \eta|_{\theta=0.01} \,. \tag{19}$$

(iii) Concentration boundary layer thickness (δ_C) is also defined as

$$\delta_C = \eta|_{\gamma=0.01} \,. \tag{20}$$

(iv) The shearing stress on the plate is given by

$$\tau_w = \left[\mu \frac{\partial u}{\partial y}\right]_{y=0} = \frac{4A^{\frac{3}{4}}\mu_0^2}{\rho} x^{\frac{1}{4}} \mathrm{e}^{-\alpha}\phi''(0) \,, \tag{21}$$

where $\phi''(0)$ is the friction coefficient at the plate. The dimensionless shearing stress at the plate is defined as

$$\tau_w^* = \frac{\rho\tau_w}{4A^{\frac{3}{4}}\mu_0^2 x^{\frac{1}{4}}} = \mathrm{e}^{-\alpha}\phi''(0) \,. \tag{22}$$

(v) The local Nusselt number $Nu(x)$ for heat transfer is defined as

$$Nu(x) = \frac{-x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -A^{\frac{1}{4}} x^{\frac{3}{4}} \theta'(0) = -\left[Gr(x)\right]^{\frac{1}{4}} \theta'(0) \,, \tag{23}$$

where $\theta'(0)$ is the rate of heat transfer at the plate.

(vi) The local Sherwood number $Sh(x)$ is finally defined as

$$Sh(x) = \frac{-x\left(\frac{\partial C}{\partial y}\right)_{y=0}}{C_w - C_\infty} = -A^{\frac{1}{4}} x^{\frac{3}{4}} \gamma'(0) = -\left[Gr(x)\right]^{\frac{1}{4}} \gamma'(0) \,, \tag{24}$$

where $\gamma'(0)$ is the rate of mass transfer at the plate.

4. The numerical solution

The set of nonlinear ordinary differential equations (12)–(14) along with the boundary conditions (16) and (17) have been solved using a fourth order Runge-Kutta integration scheme with the Nachtsheim-Swiger shooting technique (1965). This problem is a mixed condition, in which both conditions at the plate ($\eta = 0$) and at infinity (η_∞) are given. The initial conditions $\phi''(0)$, $\phi'(0)$, $\phi(0)$, $\theta(0)$, $\theta'(0)$, $\gamma'(0)$ and $\gamma(0)$ must be specified to start the integration. But we have only the conditions at the plate ($\eta = 0$): $\phi'(0) = 0$, $\phi(0) = 0$, $\theta(0) = 1$ and $\gamma(0) = 1$. We notice that the values $\phi''(0)$, $\theta'(0)$ and $\gamma'(0)$ are unknowns at the plate ($\eta = 0$), and we have the conditions at infinity (η_∞): $\phi'(\eta_\infty) = 0$, $\theta(\eta_\infty) = 0$ and $\gamma(\eta_\infty) = 0$. Nachtsheim-Swigert method has been used to solve this problem. The procedure is to estimate the unknown values of $\phi''(0)$, $\theta'(0)$ and $\gamma'(0)$ by iterations satisfying the conditions:

$$\begin{aligned} \phi''(\eta_\infty) = \delta_1 \,, \quad \phi'(\eta_\infty) = \delta_2 \,, \quad \theta'(\eta_\infty) = \delta_3 \,, \\ \theta(\eta_\infty) = \delta_4 \,, \quad \gamma'(\eta_\infty) = \delta_5 \,, \quad \gamma(\eta_\infty) = \delta_6 \,, \end{aligned} \tag{25}$$

where δ_i $(i = 1, 2, \ldots, 6)$ are very small quantities (errors) of order 10^{-5} (say). Equations (25) depend on the unknown surface conditions $\phi''(0)$, $\theta'(0)$ and $\gamma'(0)$. In order to obtain correction equations for the values $\phi''(0)$, $\theta'(0)$ and $\gamma'(0)$ it is required to perform the first order Taylor's series expansion. In the Nachtsheim-Swigert (1965) iteration scheme the success estimate of $\phi''(0)$, $\theta'(0)$ and $\gamma'(0)$ are obtained in such a way that the sum of the squares of the errors $\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 + \delta_5^2 + \delta_6^2$ is minimal with respect to variations in $\Delta\phi''(0)$, $\Delta\theta'(0)$ and $\Delta\gamma'(0)$. Let

$$E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 + \delta_5^2 + \delta_6^2 \,, \tag{26}$$

then by using the relations

$$\frac{\partial E}{\partial\left[\Delta\phi''(0)\right]} = 0 \,, \quad \frac{\partial E}{\partial\left[\Delta\theta'(0)\right]} = 0 \,, \quad \frac{\partial E}{\partial\left[\Delta\gamma'(0)\right]} = 0 \,, \tag{27}$$

we can obtain three algebraic equations for the three unknowns $\Delta\phi''(0)$, $\Delta\theta'(0)$ and $\Delta\gamma'(0)$. Hence, an improved iteration can be obtained for the initial guessed values of $\phi''(0)$, $\theta'(0)$ and $\gamma'(0)$, by adding the corresponding values $\Delta\phi''(0)$, $\Delta\theta'(0)$ and $\Delta\gamma'(0)$. This process can be repeated several times until the accuracy is obtained. In order to verify the accuracy of our present method, we have compared our results with those of Elbashbishy and Ibrahim (1993). For special case that there is no variation of concentration and the viscosity is constant, $\alpha = \beta_2 = 0$, $\beta_1 = 0.12$ and $Pr = 4$, our results are $\phi''(0) = 0.51163$ and $-\theta'(0) = 0.83848$ but their results were $\phi''(0) = 0.5115$ and $-\theta'(0) = 0.8384$. So our results are in good agreement with them.

5. The governing parameters

We have six important parameters depending on the nature of the fluid, which are α, β_1, β_2, N, Pr and Sc. For positive values of α the viscosity of the fluid decreases with an increase in the temperature. This is the case for fluids such as water and lubrication oils, while for negative values of α, the viscosity, of the fluid increases with an increase of the temperature and this is the case for air. β_1 is a parameter appearing in equation (10). The positive values of β_1, mean that the thermal diffusivity increases with an increase in temperature and this is the case for fluid such as water or air. Similarly, β_2 is the constant rate of change of the chemical diffusivity with temperature which is induced in $\frac{D}{D_\infty} = 1 + \beta_2\theta$. The behavior of β_2 is similar to the behavior of β_1. The constant N (the buoyancy ratio) which measures the amplitude and the direction of concentration and thermal forces (buoyancy forces). When $N = 0$ and $N = \infty$ we recover the case where a single scalar is diffusing. When $N < 0$ buoyancy forces derive the flow in opposite direction. When $N > 0$, buoyancy forces are cooperating and derive the flow in the same direction.

The Prandtl number $Pr = \frac{\nu_\infty}{K_\infty}$, is the ratio between the kinematic viscosity and the thermal diffusivity. We consider Pr as a constant in the air and take it in our calculation as ($Pr = 0.733$). The last constant is Schmidt number $Sc = \frac{\nu}{D}$, it is the ratio between the kinematic viscosity and the mass diffusivity. From the discussions the range of variations of the parameters of the flow can be taken as follows: (Schlichting (1968); Gebhart (1971); Pop, *et al.* (1992) and Elbashbeshy and Ibrahim (1993))

(i) for air:

$$-0.7 \le \alpha \le 0\,, \quad 0 \le \beta_1 \le 6\,, \quad 0 \le \beta_2 \le 4\,,$$
$$-1 \le N \le 3\,, \quad Pr = 0.73\,, \quad Sc = 1.2\,.$$

(ii) For water:

$$0 \le \alpha \le 0.6\,, \quad 0 \le \beta_1 \le 0.12\,, \quad 0 \le \beta_2 \le 0.1\,,$$
$$-1 \le N \le 3\,, \quad 2 \le Pr \le 6\,, \quad 3 \le Sc \le 10\,.$$

6. Results and discussions

It is clear from Figures 2 and 3 that the velocity of the fluid at any vertical plane increases as the parameter α increases (decreasing of the viscosity of the fluid) in interval $0 < \eta < \eta_0 \simeq 1.2$ but we get an opposite behavior after this interval for both air and water respectively. These results are expected for water and air because as α increases, the fluid particles will be under two opposite forces; the first force increases the velocity due to the decreasing in the viscosity, and the second force decreases the velocity due to the decreasing of the temperature and concentration. Near the plate where $0 \le \eta \le \eta_0$, θ and C are high, so the first force will be dominant and the velocity ϕ' creases as α increases. On the other hand a far from the plate $\eta \ge \eta_0$, where θ and C are low, the second force will be dominant and the velocity ϕ' decreases as α increases.

It is observed from Figure 4 the effect of α on the temperature. The temperature decreases as the parameter α increases. It is noticed from Figure 5 that as the constant β_1 increases (the thermal diffusivity increases), the velocity of the particles increases. Also from Figure 6, we observe that as β_1 increases, the temperature increases and the same effect of β_1 on the concentration but the effect is wake Figure 7. These effects of β_1 occur only for air because for water β_1 has very small effect.

From Figure 8, it is clear that the dimensionless velocity $\phi'(\eta)$ increases as the constant term of concentration β_2 increases in $0 \le \eta \le \eta_0 \cong 2.1$, but an opposite effect is noticed at a certain distance far from the plate $\eta \approx 2.1$. Figure 9 represents that as β_1 increases, the temperature θ decreases. We can notice from Figure 10 that as the constant β_2 increases, the concentration increases.

Figures 11–13 represent the effects of the buoyancy ratio N on the velocity of the particles, the temperature and the concentration for air respectively. It is clear from Figure 11 that as N increases ($N > 0$), the velocity of the particles $\phi'(\eta)$ also increases. This is because thermal and solute forces drive the flow in the same direction. When $N < 0$, these forces drive the flow in opposite direction and the flow field can reverse. So an opposite effect for N is observed at a certain distance from the plate ($\eta \ge$).

From Figures 12 and 13, it is clear that as N increases, the temperature and the concentration increase respectively. Figure 14 represents the effects of N on the velocity $\phi'(\eta)$ but for liquids (water). The effects of N on water is similar as its effects on air. From Figures 15 and 16, it is noticed the effects of Prandtl number Pr on the velocity and the temperature, respectively. The velocity and the temperature of the fluid decrease as Pr increases. Figures 17 and 18 describe the effect of Schmdit number Sc on the velocity and the concentration, for water and air, respectively. It is clear from Figure 17 that the velocity at any vertical plane near the plate decreases as Schmidt number Sc increases. But an opposite effect is noticed at a certain distance from the plate, $\eta_0 = 1.9$. In Figure 18 a fluid with a higher Sc value has a lower species diffusion coefficient. This reduces the mass diffusion rate and hence reduces the concentration driven buoyancy.

In the present work if K and D are taken constants (they do not depend on the temperature), our results agree very well with those of Mongruel, Cloiture and Alladin (1996). Also if the motion of the fluid is steady, we will obtain the same results and the same figures as Angirasa and Srinivasan (1989), Angirasa (1989). And if there is no concentration ($C = 0$), i.e., ($N = 0$), our results agree those of Elbashbeshy and Ibrahim (1993).

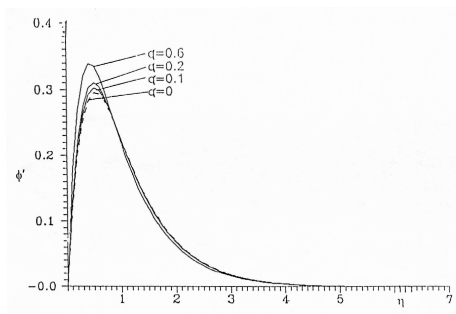

Figure 2 The variation of the dimensionless velocity ϕ' with α for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_1 = 4.0$, $\beta_2 = 3.0$, $\alpha = -0.1, -0.2, -0.3, 0.4$

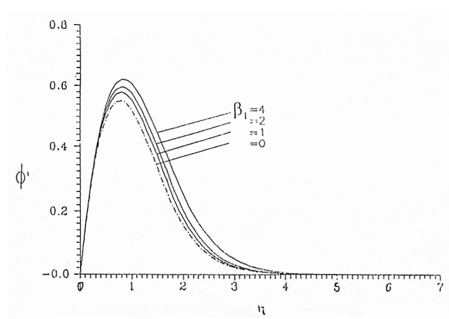

Figure 3 The variation of the dimensionless velocity ϕ' with α for $Pr = 4$, $Sc = 10$, $N = 3$, $\beta_1 = 0.12$, $\beta_2 = 0.1$, $\alpha = 0, 0.1, 0.2, 0.6$

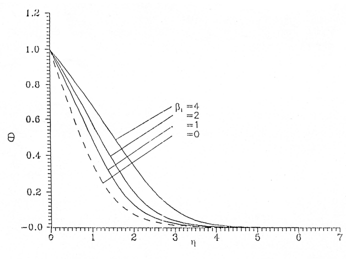

Figure 4 The variation of the dimensionless temperature θ with α for $Pr = 4$, $Sc = 10$, $N = 3$, $\beta_2 = 0.12$, $\beta_2 = 0.1$, $\alpha = 0, 0.6$

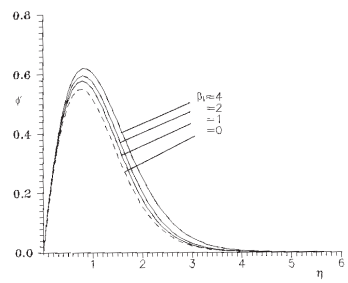

Figure 5 The variation of the dimensionless velocity ϕ' with β_1 for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_2 = 3.0$, $\alpha = -0.4$, $\beta_1 = 0, 1, 2, 4$

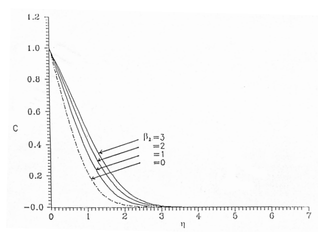

Figure 6 The variation of the dimensionless temperature θ with β_1 for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_2 = 3$, $\alpha = 0.6$, $\beta_1 = 0, 1, 2, 4$

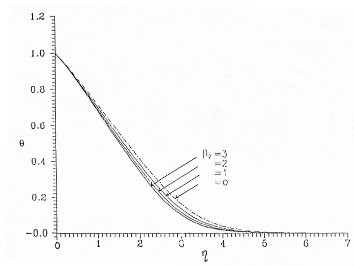

Figure 7 The variation of the dimensionless concentration γ with β_1 for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_2 = 3.0$, $\alpha = -0.4$, $\beta_1 = -0, 1, 2, 4$

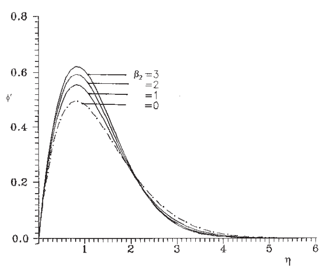

Figure 8 The variation of the dimensionless velocity ϕ' with β_2 for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_1 = 4.0$, $\alpha = -0.4$, $\beta_2 = 0, 1, 2, 3$

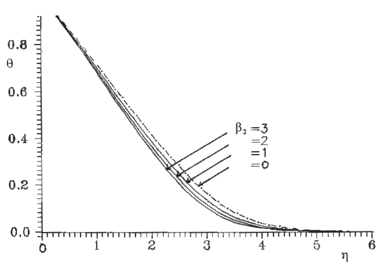

Figure 9 The variation of the dimensionless temperature θ with β_2 for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_1 = 4.0$, $\alpha = -0.4$, $\beta_2 = 0, 1, 2, 3$

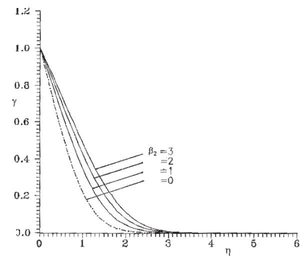

Figure 10 The variation of the dimensionless concentration γ with β_2 for $Pr = 0.73$, $Sc = 1.4$, $N = 3$, $\beta_1 = 4.0$, $\alpha = -0.4$, $\beta_2 = 0, 1, 2, 3$

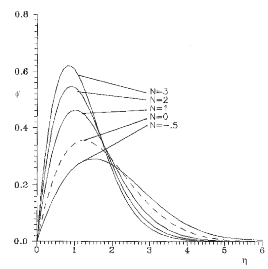

Figure 11 The variation of the dimensionless velocity ϕ' with N for $Pr = 0.73$, $Sc = 1.4$, $\beta_1 = 4.0$, $\alpha = -0.4$, $\beta_2 = 3.0$, $N = -0.5, 0, 1, 2, 3$

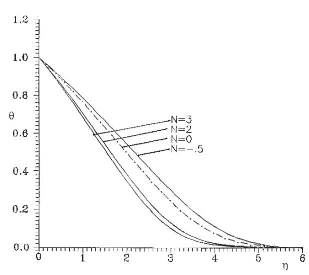

Figure 12 The variation of the dimensionless temperature θ with N for $Pr = 0.73$, $Sc = 1.4$, $\alpha = -0.4$, $\beta_2 = 3.0$, $N = -0.5, 0, 1, 2, 3$

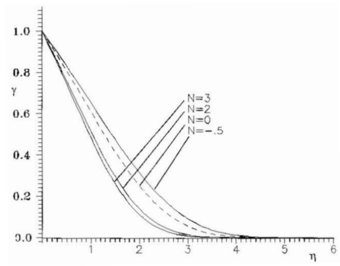

Figure 13 The variation of the dimensionless concentration γ with N for $Pr = 0.73$, $Sc = 1.4$, $\beta_2 = 3.0$, $N = -0.5, 0, 1, 2, 3$

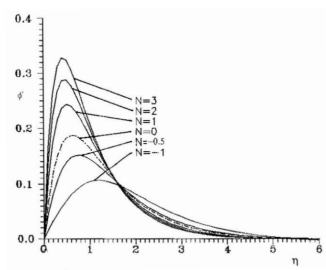

Figure 14 The variation of the dimensionless velocity ϕ' with N for $Pr = 4$, $Sc = 10$, $\beta_2 = 0.1$, $\alpha = 0.6$, $\beta_2 = 0.1$, $N = -1, -0.5, 0, 1, 2, 3$

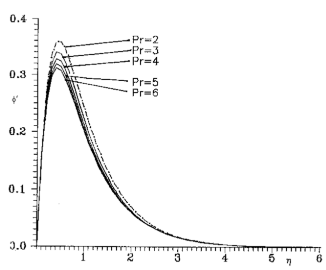

Figure 15 The variation of the dimensionless velocity ϕ' with Pr for $N = 3$, $Sc = 10$, $\beta_1 = 0.12$, $\alpha = 0.6$, $\beta_2 = 0.1$, $Pr = 2, 3, 4, 5, 6$

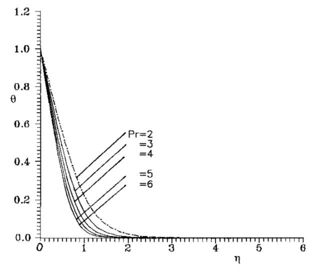

Figure 16 The variation of the dimensionless temperature θ with Pr for $N = 3$, $Sc = 10$, $\alpha = -0.4$, $\beta_1 = 0.12$, $\beta_2 = 0.1$, $Pr = 2, 3, 4, 5, 6$

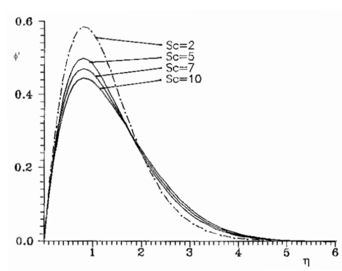

Figure 17 The variation of the dimensionless velocity ϕ' with Sc for $N = 3$, $Pr = 4$, $\beta_1 = 0.12$, $\alpha = 0.6$, $\beta_2 = 0.1$, $Sc = 2, 5, 7, 10$

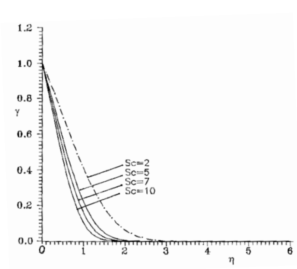

Figure 18 The variation of the dimensionless concentration γ with Sc for $N = 3$, $Pr = 4$, $\beta_1 = 0.12$, $\beta_2 = 0.1$, $\alpha = 0.6$, $Sc = 2, 5, 7, 10$

Table 1 The values of $\phi''(0)$, $-\theta'(0)$, $-\gamma'(0)$, δ, δ_T and δ_C for different values of parameters of the flow α, Sc and N with $Pr = 7$, $\beta_1 = 0.12$ and $\beta_2 = 0.1$ in water

Part a

α	$\phi''(0)$	$-\theta'(0)$	$-\gamma'(0)$	δ	δ_T	δ_C
0.0	1.3660	0.9768	1.4711	4.8	2.50	1.50
0.2	1.5409	1.1568	1.5022	4.7	2.15	1.49
0.3	1.6547	1.1744	0.5278	4.5	2.10	1.45
0.6	2.0463	1.2291	1.6050	4.4	2.00	1.40

Part b

Sc	$\phi''(0)$	$-\theta'(0)$	$-\gamma'(0)$	δ	δ_T	δ_C
8	2.0080	1.2135	1.6831	4.51	2.00	1.20
10	1.9760	1.1882	1.8202	4.81	2.09	1.19
15	1.8420	1.1452	2.0941	4.85	2.19	0.99
20	1.7727	1.1174	2.3109	4.90	2.23	0.89

Part c

N	$\phi''(0)$	$-\theta'(0)$	$-\gamma'(0)$	δ	δ_T	δ_C
-1	0.1538	0.6136	0.7737	3.3	5.5	2.0
0	1.7820	0.9181	1.1911	2.5	5.1	1.9
1	1.2500	1.0540	1.3731	2.3	4.8	1.6
2	1.6644	1.1518	1.5032	2.1	4.7	1.5

From Table 1, we can observe the effect of α, Sc and N on the primary physical quantities. It is observed that (Table 1a) and (Table 1c), when α or N increase, $\theta''(0)$ increases, $-\theta'(0)$ and $-\gamma(0)$ also increase, Table 1b shows that as Sc increases, $\phi''(0)$ and $-\theta'(0)$ also increases but $-\gamma(0)$ decreases. From Table 2 explains the effect of the thermal diffusivity β_1, mass diffusivityβ_2 and buoyancy force N on δ, δ_T and δ_C. The value of N has the same effect on air as on water. With increase of β_1 value the values of $\phi''(0)$, $\theta'(0)$ and δ_C increase but the value of $-\theta'(0)$ (the rate of heat transfer) decreases. Also as β_2 increases the values of $\phi''(0)$, $\theta'(0)$ and δ_C increase, but the values of $-\gamma'(0)$, δ and δ_t decrease.

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Table 2 The values of $\phi''(0)$, $-\theta'(0)$, $-\gamma'(0)$, δ, δ_T and δ_C for different values of parameters of the flow β_1, β_2 and N with $Pr = 0.7$, $\alpha = -0.4$ and $Sc = 1.4$ in air

Part a

β_1	$\phi''(0)$	$-\theta'(0)$	$-\gamma'(0)$	δ	δ_T	δ_C
0	1.5380	0.70377	0.40613	4.41	4.3	3.32
1	1.5922	0.46714	0.43618	4.46	4.5	3.28
2	1.6231	0.37279	0.45572	4.51	4.7	3.41
3	1.6441	0.31841	0.46987	4.62	4.9	3.45

Part b

β_2	$\phi''(0)$	$-\theta'(0)$	$-\gamma'(0)$	δ	δ_T	δ_C
0.0	1.4402	0.25316	0.91324	5.1	5.4	2.6
0.5	1.5038	0.26024	0.76382	4.9	5.3	2.7
1.0	1.5492	0.26592	0.67051	4.8	5.2	3.0
2.0	1.6144	0.27489	0.55391	4.7	5.0	3.3

Part c

N	$\phi''(0)$	$-\theta'(0)$	$-\gamma'(0)$	δ	δ_T	δ_C
-0.5	0.4122	0.19768	0.32216	5.8	5.9	4.5
0.0	0.6340	0.21682	0.36141	5.6	5.7	4.3
1.0	1.0139	0.24495	0.41342	5.3	5.5	3.9
2.0	1.6599	0.28183	0.48073	4.7	5.1	3.4

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