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Relaxation Effects on Thermal Shock Problems in an Elastic Half-Space of Generalized Magneto-Thermoelastic Waves

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The propagation of electromagneto-thermo-elastic disturbances produced by thermal shock based on Lord-Shulman (L-S), Green-Lindsay (G-L) and classical dynamical coupled (CD) theories in a perfectly conducting half-space are studied. There acts an initial magnetic field parallel to the plane boundary of the half-space. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically for each case. A comparison is made with the results predicted by the coupled theory. It is found that the magnetic field has decreasing effect.

Keywords: relaxation effects, thermal shock, normal mode analysis.

1. Introduction

In the last few decades a new domain has been developed, which investigates the interactions between the strain and electromagnetic fields. This discipline is called magneto-elasticity. A stimulus for its development was the possibility of its applications to geophysical problems, certain topics in optics, acoustics, investigations on damping of acoustic waves in a magnetic field, etc.

Shadwick and Sneddon [1] have shown how the form of plane waves traveling in infinite elastic solid is affected by its thermal properties. Biot [2] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equation for both theories is of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. Lord and Shulman [3] introduced the theory of generalized thermo-elasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier's law. This law contains the heat flux vector as well as its time's derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, ensuring finite speeds of propagation for heat and elastic wave. The remaining governing equations for this theory, namely, the equations of motion and the constitutive relation remain the same as those for the coupled and uncoupled theories. Dhaliwal and Sherief [4] extended this theory to general anisotropic media in the presence of heat sources. Because of the complicated nature of these equations, few attempts have been made to solve them. Sherief [5] solved a spherically symmetric problem with a point source of heat, and Sherief and Anwar [6] solved a cylindrically symmetric problem with a line source of heat.

Müller [7] first introduced the theory of generalized thermoelasticity with two relaxation times. A more explicit version was then introduced by Green and Laws [8], Green and Lindsay [9] and independently by Suhubi [10]. In this theory, the temperature rates are considered among the constitutive variables. This theory also predicts finite speeds of propagation as in Lord and Shulman's theory. It differs from the latter in that Fourier's law of heat conduction is not violated if the body under consideration has a center of symmetry. Erbay and uhubi [11] studied wave propagation in cylinder. Ignaczak [12] studied a strong discontinuity wave and obtained a decomposition theorem [13].

Many authors have considered the propagation of electromagneto-thermoelastic waves in an electrically and thermally conducting solid. Ezzat and Othman established the model of the two-dimensional equations of generalized magneto-thermoelasticity with two relaxation times in a perfectly conducting medium. Paria [15] discussed the propagation of plane magneto-thermoelastic waves in anisotropic unbounded medium under the influence of a uniform thermal field and with a magnetic field acting transversely to the direction of the propagation. Paria used the classical Fourier law of heat conduction and neglected the electric displacement. A comprehensive review of the earlier contributions to the subject can be found in [16]. Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Nasser [17], who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. They obtained the governing equations in the general case and the solution for some particular cases. Choudhuri [18] extended these results to rotating media. Lately, Othman [19] constructs the model of generalized thermo-elasticity in an isotropic elastic medium under the dependence of the modulus of elasticity on the reference temperature with one relaxation time. Tomita and Shindow [20] have studied Rayleigh waves in magneto-thermoelastic solids with thermal relaxation.

In the present paper, a comparison is carried out between temperature distribution, displacement components and thermal stresses as calculated from the generalized thermoelasticity L-S and G-L theories for the problem under consideration. It appears; in particular, that the results obtained from G-L theory tends to those of L-S theory as the values of the two relaxation times become closer to each other. The classical dynamic coupled (CD) theory is recovered as a special case. The second relaxation time is well pronounced when it becomes larger than the first one. The results obtained in this study put in evidence the effects of the thermal relaxation times involved in the theories and the effect of magnetic field on the displacement and stress.

2. Formulation of the problem and basic equations

We consider the problem of a thermoelastic half-space ($x \geq 0$). A magnetic field with constant intensity **H** acts parallel to the boundary plane (taken as the direction of the z-axis). The surface of the half-space is subjected at time ($t = 0$) to a thermal shock that is a function of y and t. Thus all the quantities considered will be functions of the time variable t and of the coordinates x and y. We begin our consideration with linearized equations of electrodynamics of slowly moving medium

$$J = \operatorname{curl} h - \varepsilon_0 \dot{E}\,, \tag{1}$$

$$\operatorname{curl} E = -\mu_0 \dot{h}\,, \tag{2}$$

$$E = -\mu_0 (\dot{u} \wedge H)\,, \tag{3}$$

$$\nabla \cdot \mathbf{h} = 0\,. \tag{4}$$

The above equations are supplemented by the displacement equations of the theory of elasticity, taking into account the Lorentz force

$$\rho \ddot{u}_i = \sigma_{ij,j} + \mu_0 \left(J \wedge H\right)_i\,, \tag{5}$$

The equation of heat conduction

$$kT_{,ii} = \rho C_E(\dot{T} + \tau \ddot{T}) + \gamma T_0(\dot{u}_{i,i} + \tau \delta \ddot{u}_{i,i})\,, \tag{6}$$

Stress-strain-temperature constitutive relations

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma\left(T - T_0 + \nu \dot{T}\right)\delta_{ij}\,, \tag{7}$$

Strain-displacement constitutive relations

$$e_{xx} = \frac{\partial u}{\partial x}\,, \quad e_{yy} = \frac{\partial v}{\partial y}\,, \quad e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\,, \quad e_{xz} = e_{yz} = e_{zz} = 0\,, \tag{8}$$

together with the previous equations, constitute a complete system of generalized magneto-thermoelasticity with thermal relaxation times equations for a medium with a perfect electric conductivity.

In the above equations a dot denotes differentiation with respect to time, and a comma followed by a subscript denotes partial differentiation with respect to the corresponding coordinates. The summation notation is used. We shall consider only the simplest case of the two-dimensional problem. We assume that all causes producing the wave propagation are independent of the variable z, and that waves are propagated only in the xy-plane. Thus all quantities appearing in equations (1)–(8) are independent of the variable z. Then the displacement vector has components ($u(x, y, t)$, $v(x, y, t)$, 0).

Assume now that the initial conditions are homogeneous and the initial magnetic field has components$(0, 0, H_0)$. Then equations (1)–(4) yield

$$E = \mu_0 H_0(-\dot{v}, \dot{u}, 0)\,, \tag{9}$$

$$\mathbf{h} = -H_0(0, 0, e)\,. \tag{10}$$

Moreover the use of the relaxation times τ, ν and a parameter δ marks the aforementioned fundamental equations possible for the tree different theories:

1. Classical Dynamical theory (1956): $\tau = \nu = 0$, $\delta = 0$.

2. Lord and Shulman's theory (1967): $\nu = 0$, $\tau > 0$, $\delta = 1$.

3. Green and Lindsay's theory (1972): $\nu \leq \tau > 0$, $\delta = 0$.

Eliminating σ_{ij} from Eq. (5) and (7) and using Eq. (8), one may get

$$\rho \ddot{u}_i = (\lambda + \mu) u_{k,ki} + \mu \nabla^2 u_i - \gamma \left(T + \nu \dot{T} \right)_{,i} + \mu_0 (J \wedge H)_i \,. \tag{11}$$

Expressing the components of the vector **J** in terms of displacement, by eliminating from equation (1) the quantities **h** and **E** and introducing them into the displacement equations (11), we arrived at:

$$\beta_0^2 u_{,xx} + (\beta_0^2 - 1) v_{,xy} + u_{,yy} - \beta^2 (\theta_{,x} + \nu \theta_{,xt}) = \alpha u_{,tt} \,, \tag{12}$$

$$\beta_0^2 v_{,yy} + (\beta_0^2 - 1) u_{,xy} + v_{,xx} - \beta^2 (\theta_{,y} + \nu \theta_{,yt}) = \alpha v_{,tt} \,, \tag{13}$$

where e is the cubical dilatation, it can be expressed as the following form

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \,, \tag{14}$$

the heat conduction equation

$$\nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2} \right) \theta + \varepsilon \left(\frac{\partial}{\partial t} + \tau \delta \frac{\partial^2}{\partial t^2} \right) e \,, \tag{15}$$

and the components of the stress are

$$\sigma_{xx} = (\beta^2 - 2) e + 2 u_{,x} - \beta^2 \left(1 + \nu \frac{\partial}{\partial t} \right) \theta \,, \tag{16}$$

$$\sigma_{yy} = (\beta^2 - 2) e + 2 v_{,y} - \beta^2 \left(1 + \nu \frac{\partial}{\partial t} \right) \theta \,, \tag{17}$$

$$\sigma_{xy} = u_{,y} + v_{,x} \,. \tag{18}$$

Differentiating Eq. (12) with respect to x, and Eq. (13) with respect to y, then adding we obtain

$$\left(\nabla^2 - \alpha_1 \frac{\partial^2}{\partial t^2} \right) e - \beta_1^2 \left(1 + \nu \frac{\partial}{\partial t} \right) \nabla^2 \theta = 0 \,, \tag{19}$$

In the equations above the following non-dimensional variables are used: $x = c_1 \eta x'$, $y = c_1 \eta y'$, $u = c_1 \eta u'$, $v = c_1 \eta v'$, $t = c_1^2 \eta t'$, $\tau = c_1^2 \eta \tau'$, $\nu = c_1^2 \eta \nu'$,

$$\theta = \frac{\gamma \left(T' - T_0 \right)}{\lambda + 2\mu} \,, \quad \sigma_{ij} = \frac{\sigma'_{ij}}{\mu} \,, \quad \beta_1^2 = \frac{c_1^2}{c_0^2} \,, \quad \alpha_1 = \frac{\alpha}{\beta_0^2} \,.$$

3. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as following form

$$[u, v, e, \theta, \sigma_{ij}](x, y, t) = \left[u^*(x), v^*(x), e^*(x), \theta^*(x), \sigma^*_{ij}(x)\right] \exp(\omega t + iay) . \tag{20}$$

where ω is the (complex) time constant and a is the wave number in the y-direction.

By using Eq. (20), we can obtain the following equations from Eqs. (15) and (19) respectively

$$\left[D^2 - a^2 - \omega(1 + \tau\omega)\right] \theta^*(x) = \varepsilon\omega(1 + \tau\omega\delta)e^*(x) , \tag{21}$$

$$\left[D^2 - a^2 - \alpha_1\omega^2\right] e^*(x) = \beta_1^2(1 + \nu\omega)(D^2 - a^2)\theta^*(x) . \tag{22}$$

where, $D = \frac{d}{dx}$.

Eliminating $\theta^*(x)$ in Eqs. (21) and (22), we get

$$(D^4 - a_1 D^2 + a_2)e^*(x) = 0 . \tag{23}$$

where,

$$a_1 = 2a^2 + b_1 , \tag{24}$$

$$a_2 = a^4 + a^2 b_1 + b_2 , \tag{25}$$

$$b_1 = \omega_2 + \alpha_1\omega^2 + \varepsilon\omega_1\beta_1^2(1 + \tau\omega\delta) , \tag{26}$$

$$b_2 = \alpha_1\omega^2\omega_2 , \tag{27}$$

$$\omega_1 = \omega(1 + \nu\omega) , \quad \omega_2 = \omega(1 + \tau\omega) . \tag{28}$$

Equation (23) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)e^*(x) = 0 . \tag{29}$$

where,

$$k_{1,2}^2 = \frac{1}{2}\left[a_1 \pm \sqrt{a_1^2 - 4a_2}\right] \tag{30}$$

is the root of the following characteristic equation

$$k^4 - a_1 k^2 + a_2 = 0 , \tag{31}$$

The solution of equation (29) has the form

$$e^*(y) = \sum_{i=1}^{2} e_i^*(x) . \tag{32}$$

where $e_i^*(x)$ is the solution of the equation

$$(D^2 - k_i^2)e_i^*(x) = 0 , \quad i = 1, 2 . \tag{33}$$

The solution of Eq. (33) which is bounded as $x \to \infty$ is given by

$$e_i^*(x) = A_i(a, \omega)e^{-k_i x} . \tag{34}$$

Thus $e^*(x)$ has the form

$$e^*(x) = \sum_{i=1}^{2} A_i(a,\omega) e^{-k_i x} \,. \tag{35}$$

In a similar manner, we get

$$\theta^*(x) = \sum_{i=1}^{2} B_i(a,\omega) e^{-k_i x} \,, \tag{36}$$

where $A_i(a,\omega)$ and $B_i(a,\omega)$ are some parameters depending on a and ω.

Substituting from Eqs. (34) and (35) into Eq. (20), we obtain the following relation

$$B_i = \frac{\varepsilon\omega(1+\tau\omega\delta)}{[k_i^2 - a^2 - \omega_2]} A_i \,, \quad i = 1, 2 \,. \tag{37}$$

Substituting from Eq. (37) into Eq. (36), we get the following relation

$$\theta^*(x) = \sum_{i=1}^{2} \frac{\varepsilon\omega(1+\tau\omega\delta)}{[k_i^2 - a^2 - \omega_2]} A_i e^{-k_i x} \,. \tag{38}$$

In order to obtain the displacement u, in terms of Eq. (20), from Eqs. (12) and (14)

$$(D^2 - a^2 - \alpha\omega^2)u^* + (\beta_0^2 - 1)De^* - \beta^2(1+\nu\omega)D\theta^* = 0 \,. \tag{39}$$

Using Eqs. (35) and (38) we get the following partial differential equation satisfied by

$$(D^2 - m^2)u^* = \sum_{i=1}^{2} \left[(\beta_0^2 - 1) - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{[k_i^2 - a^2 - \omega_2]} \right] k_i A_i(a,\omega) e^{-k_i x} \,, \tag{40}$$

where,

$$m = \sqrt{a^2 + \alpha\omega^2} \,. \tag{41}$$

The solution of Eq. (40) bounded as $x \to \infty$, is given by

$$u^*(x) = Ce^{-mx} + \sum_{i=1}^{2} \left[(\beta_0^2 - 1) - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{[k_i^2 - a^2 - \omega_2]} \right] \frac{k_i A_i(a,\omega)}{(k_i^2 - m^2)} e^{-k_i x} \,. \tag{42}$$

where $C = C(a,\omega)$ is some parameter depending on a and ω.

In terms of Eq. (20), from Eqs. (14) we get

$$v^* = -\frac{i}{a} \left(e^* - \frac{\partial u^*}{\partial x} \right) \,. \tag{43}$$

Substituting from Eqs. (35) and (42) into the right-hand side of Eq. (43), we get

$$v^*(x) = \frac{i}{a} \left\{ \sum_{i=1}^{2} \frac{1}{k_i^2 - m^2} \left(m^2 - k_i^2 \left[\beta_0^2 - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2 - a^2 - \omega_2} \right] \right) A_i e^{-k_i x} - mCe^{-mx} \right\} \tag{44}$$

In terms of Eq. (20), substituting from equations (35), (38), (42) and (44) into Eqs. (16)–(18), we get

$$\sigma^*_{xx}(x) = -2mCe^{-mx} - \sum_{i=1}^{2}\left\{\frac{k_i^2}{k_i^2-m^2}\left[(\beta_0^2-1) - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right] - (\beta^2-2) + \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right\}A_i e^{-k_i x}\,, \tag{45}$$

$$\sigma^*_{yy}(x) = 2mCe^{-mx} + \sum_{i=1}^{2}\left\{\beta^2 + \frac{2k_i^2}{k_i^2-m^2}\left[(\beta_0^2-1) - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right] - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right\}A_i e^{-k_i x}\,, \tag{46}$$

$$\sigma^*_{xy}(x) = \frac{i}{a}\left\{(m^2+a^2)Ce^{-mx} + \sum_{i=1}^{2}\frac{k_i}{k_i^2-m^2}\left[(k_i^2+a^2)\left(\beta_0^2 - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right) - (m^2+a^2)\right]A_i e^{-k_i x}\right\}. \tag{47}$$

The normal mode analysis is, in fact, to look for the solution in Fourier transformed domain. Assuming that all the relations are sufficiently smooth on the real line such that the normal mode analysis of these functions exists.

In order to determine the parameters A_i, $i = 1, 2$ and C, we need to consider the boundary conditions at $x = 0$. We consider two kinds of boundary conditions respectively, and the details are described as the following

3.1. *Case 1*

Thermal boundary condition that the surface of the half-space subjected to a thermal shock

$$\theta(0, y, t) = n(y, t)\,. \tag{48}$$

Mechanical boundary condition that the surface of the half-space is traction free

$$\sigma_{yy}(0, y, t) = 0\,, \tag{49}$$

$$\sigma_{xy}(0, y, t) = 0\,. \tag{50}$$

Using Eq. (20) and substituting from the expressions of considered variables into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{i=1}^{2}\frac{\varepsilon\omega(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}A_i = n^*(a,\omega)\,, \tag{51}$$

$$2mC + \sum_{i=1}^{2}\left\{\beta^2 + \frac{1}{k_i^2-m^2}\left[2k_i^2(\beta_0^2-1) - (3k_i^2-m^2)\frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right]\right\}A_i = 0\,, \tag{52}$$

$$(m^2+a^2)C+\sum_{i=1}^{2}\frac{k_i}{k_i^2-m^2}\left\{(k_i^2+a^2)\left(\beta_0^2-\frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right)-\left(m^2+a^2\right)\right\}A_i=0 \tag{53}$$

Solving Eqs. (48), (49) and (50) we get the parameters A_i, $i=1,2$ and C with the following form respectively

$$A_1=\frac{n^*[(a^2+m^2)M_2-2mN_2]}{(a^2+m^2)(M_2S_1-M_1S_2)-2m(N_2S_1-N_1S_2)}\,, \tag{54}$$

$$A_2=\frac{-n^*[(a^2+m^2)M_1-2mN_1]}{(a^2+m^2)(M_2S_1-M_1S_2)-2m(N_2S_1-N_1S_2)}\,, \tag{55}$$

$$C=\frac{n^*[M_1N_2-M_2N_1]}{(a^2+m^2)(M_2S_1-M_1S_2)-2m(N_2S_1-N_1S_2)}\,. \tag{56}$$

where

$$S_i=\frac{\varepsilon\omega(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\,, \tag{57}$$

$$M_i=\beta^2+\frac{1}{k_i^2-m^2}\left[2k_i^2(\beta_0^2-1)-(3k_i^2-m^2)\frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right]\,, \tag{58}$$

$$N_i=\frac{k_i}{k_i^2-m^2}\left\{(k_i^2+a^2)\left(\beta_0^2-\frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right)-(m^2+a^2)\right\}\,. \tag{59}$$

3.2. *Case 2*

Thermal boundary condition that the surface of the half-space subjected to a thermal shock

$$\theta(0,y,t)=n(y,t)\,, \tag{60}$$

Displacement boundary condition that the surface of the half-space is rigidly fixed

$$u(0,y,t)=0\,, \tag{61}$$

$$v(0,y,t)=0\,. \tag{62}$$

Using Eq. (20) and substituting from the expressions of considered variables into the boundary conditions (60)–(62), we can obtain the following equations satisfied by the parameters

$$\sum_{i=1}^{2}\frac{\varepsilon\omega(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}A_i=n^*(a,\omega)\,, \tag{63}$$

$$C+\sum_{i=1}^{2}\frac{k_i}{k_i^2-m^2}\left[(\beta_0^2-1)-\frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right]A_i=0\,, \tag{64}$$

$$\sum_{i=1}^{2}\frac{1}{k_i^2-m^2}\left(m^2-k_i^2\left[\beta_0^2-\frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2-a^2-\omega_2}\right]\right)A_i-mC=0\,. \tag{65}$$

From Eqs. (62)–(64), we get

$$A_1 = \frac{-n^*[mL_2 + G_2]}{m(L_1S_2 - L_2S_1) + G_1S_2 - G_2S_1}\,, \tag{66}$$

$$A_2 = \frac{n^*[mL_1 + G_1]}{m(L_1S_2 - L_2S_1) + G_1S_2 - G_2S_1}\,, \tag{67}$$

$$C = \frac{n^*[G_2L_1 - G_1L_2]}{m(L_1S_2 - L_2S_1) + G_1S_2 - G_2S_1}\,. \tag{68}$$

where

$$L_i = \frac{k_i}{k_i^2 - m^2)}\left[(\beta_0^2 - 1) - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2 - a^2 - \omega_2}\right]\,, \tag{69}$$

$$G_i = \frac{1}{k_i^2 - m^2}\left(m^2 - k_i^2\left[\beta_0^2 - \frac{\varepsilon\omega_1\beta^2(1+\tau\omega\delta)}{k_i^2 - a^2 - \omega_2}\right]\right)\,. \tag{70}$$

4. Numerical results

The thermal shock $n(y,t)$ applied on the surface, is taken of the form

$$n(y,t) = \theta_0 H(L - |y|)\exp(-bt)\,, \tag{71}$$

where H is the Heaviside unit step function, b and θ_0 are constants. This means that heat is applied on the surface of the half-space on narrow band of width $2L$ surrounding the y- axis to keep it at temperature θ_0, while the rest of the surface is kept at zero temperature.

The copper material was chosen for numerical evaluations. In the calculation process, the material constants necessary to be known can be found in [21].

Since we have $\omega = \omega_0 + i\zeta$, $e^{\omega t} = e^{\omega_0 t}(\cos\zeta t + i\sin\zeta t)$, where i is imaginary unit, and for small values of time, we can take $\omega = \omega_0$ (real). The other constants of the problem are taken as $L = 2$, $\nu = 0.02$, $\tau = 0.05$, $\theta_0 = 1$, $\omega_0 = 2$, $a = 1.2$.

Considering the distributions of displacement component u and stress components σ_{xx} and σ_{yy} for $y = 0$ at $t = 0.1$. The computations were carried out in the absence ($\alpha = 1$) and in the presence ($\alpha = 1.8$) of external magnetic field, when the medium is a perfect electric conductor. Calculated results of the real part of the non-dimensional displacement and stresses are shown in Figs. 1–6 respectively. The graph shows the sixth curves predicted by the different theories of thermo-elasticity. In these figures the solid line represents the solution corresponding to using the coupled (CD) of heat conduction ($\nu = \tau = 0$ and $\delta = 0$), the dotted lines represent the solution for (L-S) theory ($\nu = 0$, $\tau = 0.05$ and $\delta = 1$) and the dashed lines represent the solution for (G-L) theory ($\nu = 0.02$, $\tau = 0.05$ and $\delta = 0$). Due to symmetries of geometrical shape and thermal boundary condition, the displacement component v and the stress component σ_{xy} are zero when $y = 0$.

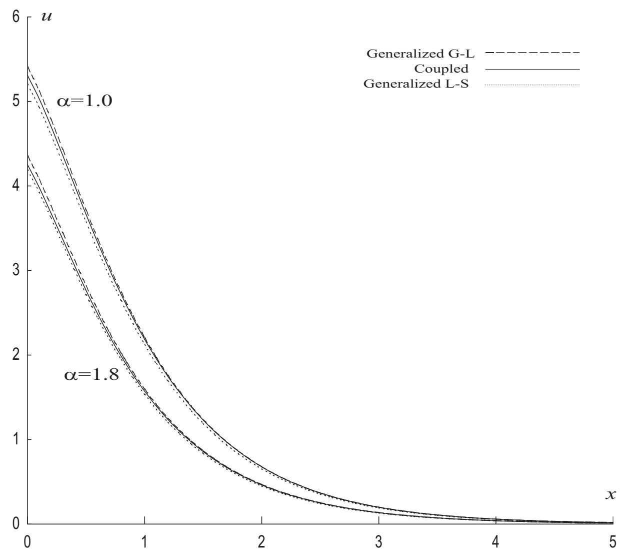

Figure 1 Horizontal displacement distribution for case 1

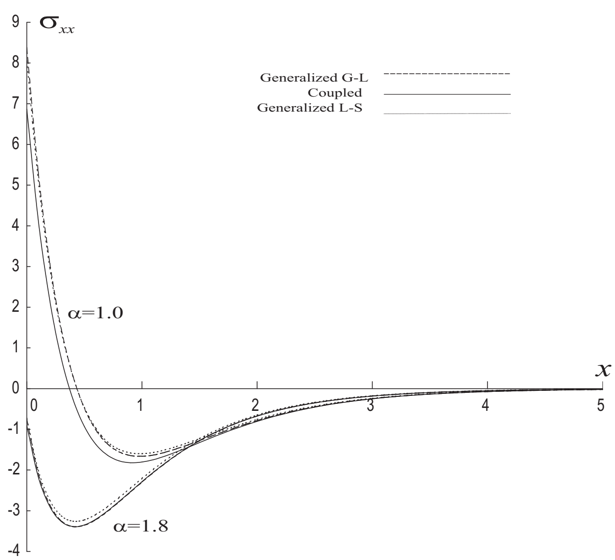

Figure 2 The distribution of stress components σ_{xx} for case 1

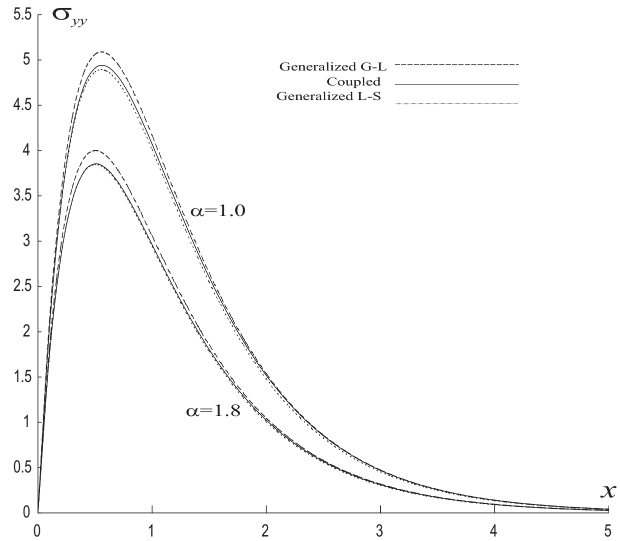

Figure 3 The distribution of stress components σ_{yy} for case 1

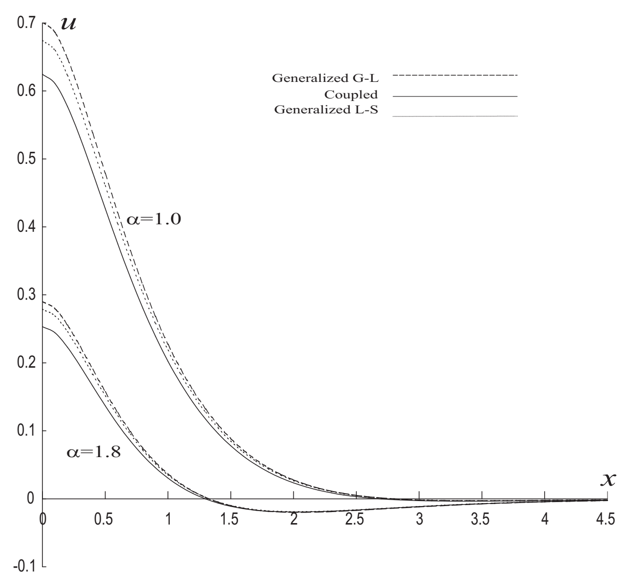

Figure 4 Horizontal displacement distribution for case 2

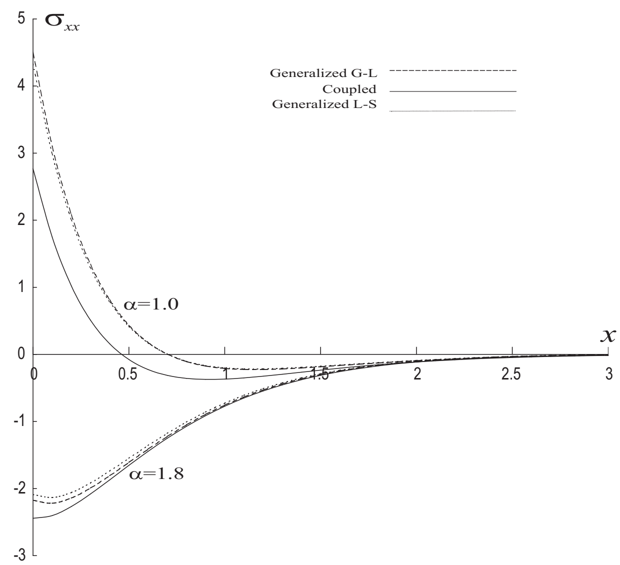

Figure 5 The distribution of stress components σ_{xx} for case 2

In all figures, it is clear that all the distributions considered have a non-zero value only in a bounded region of space. Outside this region the values vanish identically and this means that the region has not felt thermal disturbance yet. From the distribution of displacement component and the stress components, it is found that the magnetic field has a decreasing effect.

5. Concluding remarks

Owing to the complicated nature of the governing equations for the generalized electromagneto-thermoelasticity with thermal relaxation few attempts have been made to solve problems in this field [13-18], these attempts utilized approximate methods valid for only a specific range of some parameters.

In this work, the method of normal mode analysis is introduced in the field of magneto-thermo-elasticity and applied to two specific cases in which the temperature, stress, displacement and magnetic field are coupled. This method gives exact solutions without any assumed restrictions on either the applied magnetic field or the temperature and stress distributions.

References

[1] **Chadwick, P** and **Sneddon, IN**: Plane waves in an elastic solid conducting heat, *J. Mech. Physics Solids*, (1958), **6**, 223-230.

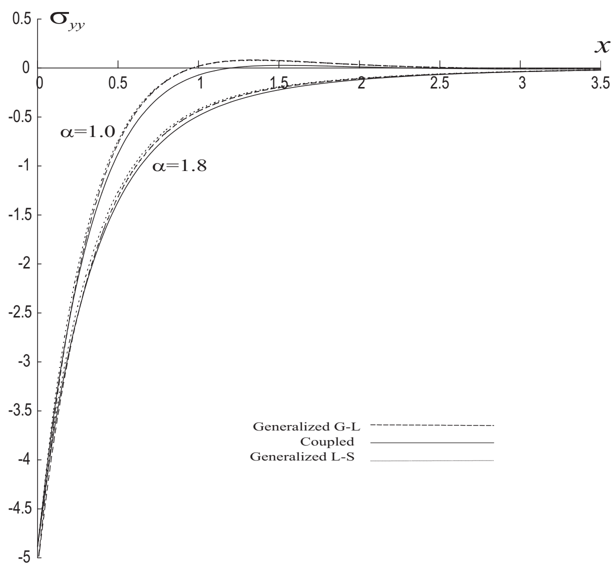

Figure 6 The distribution of stress components σ_{yy} for case 2

[2] **Biot, MA**: Thermoelasticity and Irreversible Thermodynamics, *J. Appl. Phys.*, (1956), **27**, 240-253.

[3] **Lord, H** and **Shulman, A**: Generalized Dynamical Theory of Thermoelasticity, *J. Mech. Phys. Solid*, (1967), **15**, 299-309.

[4] **Dhaliwal, R** and **Sherief, HH**: Generalized Thermoelasticity for an Isotropic Media, *Quart. Appl. Math.*, (1980), **33**, 1-8.

[5] **Sherief, HH**: Fundamental Solutions of the Generalized Thermoelastic Problem for short Times, *J. Thermal Stresses*, (1986), **9**, 151-164.

[6] **Sherief, HH** and **Anwar, MA**: Two-Dimensional Generalized Thermoelasticity Problem for an Infinitely Long Cylinder, *J. Thermal Stresses*, (1994), **17**, 213-227.

[7] **Müller, IM**: The Coldness, A Universal Function in Thermo-elastic Solids, *Arch. Rational Mech. Anal.*, (1971), **41**, 319.

[8] **Green, AE** and **Laws, N**: On the Entropy Production Inequality, *Arch. Rational Mech. Anal.*, (1972), **45**, 47.

[9] **Green, AE** and **Lindsay, KA**: Thermoelasticity, J. Elast., (1972), **2**, 1.

[10] **Suhubi, ES**: *Thermoelastic Solids*, in: *Continuum Physics II*, Chapter 2, **Eringen, AC**, ed., (1975), Academic, Press, New York,.

[11] **Erby, S** and **Suhubi, S**: Longitudinal Wave Propagation in a Generalized Elastic Cylinder, *J. Thermal Stresses*, (1986), **9**, 279.

[12] **Ignaczak, J**: A Strong Discontinuity Wave in Thermoelastic with Relaxation Times, *J. Thermal Stresses*, (1985), **8**, 25.

[13] **Ignaczak, J**: Decomposition Theorem for Thermoelasticity with Finite Wave Speeds, *J. Thermal Stresses*, (1978), **1**, 41.

[14] **Ezzat, MA** and **Othman, MI**: Electromagneto-thermoelastic Plane Waves with Two Relaxation Times in a Medium of Perfect Conductivity, *Int. J. Engng. Sci.*, (2000), **38**, 107-120.

[15] **Paria, G**: On Magneto-thermo-elastic plan waves, Proceedings, *Cambridge Philosophical Society*, (1962), **58**, 52-531.

[16] **Paria, G**: Magneto-elasticity and Magneto-thermoelasticity, *Adv. Appl. Mech.*, (1967), **10**, 73.

[17] **Nayfeh, AH** and **Nasser, SN**: Electromagneto-thermoelastic Plane Waves in Solids with Thermal Relaxation, *J. Appl. Mech.*, (1972), **113**, 108-113.

[18] **Choudhuri, S**: Electromagneto-thermoelastic Plane Waves in Rotating Media with Thermal Relaxation, *Int. J. Engng. Sci.*, (1984), **22**, 519-530.

[19] **Othman, MI**: Lord-Shulman Theory under the Dependence of the Modulus of Elasticity on the Reference Temperature in Two-Dimensional Generalized Thermoelasticity, *Int. J. Thermal Stresses*, (2002), **25**, 1027-1045.

[20] **Tomita, S** and **Shindow, Y**: Rayleigh Waves in Magneto-thermoelastic Solids with Thermal Relaxation, *Int. J. Engng. Sci.*, (1979), **17**, 227.

[21] **Sherief, HH** and **Helmy, KA**: A Two-dimensional Problem for a Half-space in Magneto-thermoelasticity with Thermal Relaxation, *Int. J. Engng. Sci.*, (2002), **40**, 587.

Nomenclature

λ, μ	Lamé's constants,
ρ	density,
C_E	specific heat at constant strain,
t	time,
T	absolute temperature,
T_0	reference temperature chosen so that $\|T - T_0\| \ll 1$,
σ_{ij}	components of stress tensor,
e_{ij}	components of strain tensor,
u_i	components of displacement vector,
k	thermal conductivity,
μ_0	magnetic permeability,
ε_0	electric permeability,
a_0^2	$\frac{\mu_0 H_0^2}{\rho}$, Alfen velocity,
c_1^2	$\frac{\lambda+2\mu}{\rho}$,
c_0^2	$c_1^2 + a_0^2$,
c_2	$\sqrt{\frac{\mu}{\rho}}$ velocity of transverse waves,
c	$\sqrt{\frac{1}{\mu_0 \varepsilon_0}}$ the velocity of light,
α	$1 + \frac{a_0^2}{c^2}$,
β^2	$\frac{c_1^2}{c_2^2}$,
α_0	$\alpha\beta^2$,
β_0^2	$\frac{c_0^2}{c_2^2}$,
τ, ν	relaxation times,
α_t	coefficient of linear thermal expansion,
γ	$(3\lambda + 2\mu)\alpha_t$,
ε	$\frac{\gamma^2 T_0}{\rho C_E(\lambda+3\mu)}$,
η	$\frac{\rho C_E}{k}$.