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Elastodynamic response of Mechanical and Thermal sources in Generalized Thermoelastic Half-space with Voids

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The dynamic response of a homogeneous, isotropic, generalized thermoelastic half- space with voids subjected to normal, tangential force and thermal source is investigated. The displacements, stresses, temperature distribution and change in volume fraction field so obtained in the physical domain are computed numerically and illustrated graphically. The numerical results of these quantities for magnesium crystal-like material are illustrated to depict voids effect in the Lord-Shulman (L-S) theory and Green-Lindsay (G-L) theory for an insulated boundary and temperature gradient boundary.

Keywords: Generalized thermoelasticity, voids, mechanical and thermal sources, Laplace and Fourier transforms.

1. Introduction

Theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical utility for investigating various types of geological and biological materials for which elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the void volume is included among the kinematics variables and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity.

A nonlinear theory of elastic materials with voids was developed by Nunziato and Cowin [1]. Later, Cowin and Nunziato [2] developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory b y investigating the response of the materials to homogeneous deformations, pure bending of beams and small amplitudes of acoustic waves. Puri and Cowin [3] studied the behavior of plane waves in a linear elastic materials with voids. Domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and

Wang [4]. Scarpetta [5] studied well posedness theorems for linear elastic materials with voids. Birsan [6] established existence and uniqueness of weak solution in the linear theory of elastic shells with voids. Ciarletta, Iovane and Sumbatyan [7] studied stress analysis for cracks in elastic materials with voids.

Rusu [8] studied the existence and uniqueness in thermoelastic materials with voids. Saccomandi [9] presented some remarks about the thermoelastic theory of materials with voids. Ciarletta and Scalia [10] discussed the nonlinear theory of non simple thermoelastic materials with voids. Ciarletta and Scarpetta [11] discussed some results on thermoelasticity for dielectric materials with voids. Dhaliwal and Wang [12] developed a heat flux dependent theory of thermoelasticity with voids. Marin [13,14] studied uniqueness and domain of influence results in thermoelastic bodies with voids. Marin [15] presented the contributions on uniqueness in thermoelastodynamics on bodies with voids. Marin and Salca [16] obtained the relation of Knopoff-de Hoop type in thermoelasticity of dipolar bodies with voids. Chirita and Scalia [17] studied the spatial and temporal behavior in linear thermoelasticity of materials with voids. Pompei and Scalia [18] studied the asymptotic spatial behavior in linear thermoelasticity of materials with voids. The present investigation is to determine the components of displacement, stress, temperature distribution and change in volume fraction field in a homogeneous, isotropic, generalized thermoelastic half-space with voids due to normal, tangential force and thermal source.

2. Basic equations

Following Lord-Shulman [19], Green-Lindsay [20] and Iesan [21] the field equations and constitutive relations in generalized thermoelastic solid with voids without body forces, heat sources and extrinsic equilibrated body force can be written as

$$\mu\nabla^2\overrightarrow{u} + (\lambda+\mu)\nabla(\nabla.\overrightarrow{u}) + b\nabla\phi - \beta\nabla(T+\delta_{2k}\tau_1\dot{T}) = \rho\ddot{\overrightarrow{u}}\,, \tag{1}$$

$$\alpha\nabla^2 2\phi - b(\nabla.\overrightarrow{u}) - \zeta_1\phi + mT = \rho\xi\ddot{\phi}\,, \tag{2}$$

$$K\nabla^2 T - \beta T_0(\nabla.\ddot{\overrightarrow{u}} + \tau_0 delta_{1k}\nabla.\ddot{\overrightarrow{u}} - mT_0\dot{\phi} = \rho c_e(\dot{T}+\tau_0\ddot{T})\,. \tag{3}$$

and

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j}u_{j,i}) + b\phi\delta_{ij} - \beta(T+\delta_{2k}\tau_1\dot{T})\delta_{ij}\,,(i,j=x,y,x)\,. \tag{4}$$

In equations (1)–(4), we have used the notations: λ, μ – Lame's constants, α, b, ζ_1, m, ξ - material constants due to presence of voids, T – temperature change, $\beta = (3\lambda+2\mu)\alpha_t$, α_t – linear thermal expansion, $\overrightarrow{u}$ – displacement vector, t_{ij} – stress tensor, ρ, c_e – density and specific heat at constant strain, respectively, K – thermal conductivity, ϕ – change in volume fraction field, δ_{ij} – Kronecker delta, T_0 – uniform temperature; a superposed dot represents differentiation with respect to time variable t, τ_0, τ_1 are thermal relaxation times. For L-S theory, $\tau_1 0$, $\delta_{1k}=1$ and for G-L theory $\tau_1 > 0$, $\delta_{1k}=0$ (i.e., $k=1$ for L-S theory and $k=2$ for G-L theory). The thermal relaxations satisfy the inequality $\tau_1 \geq \tau_0 > 0$ for the G-L theory only,

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

are the gradient and Laplacian operators respectively.

3. Formulation of the problem

We consider a homogeneous, isotropic, thermally conducting elastic half-space with voids in the undeformed state at uniform temp erature T_0. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the plane surface $z = 0$ with z-axis pointing vertically into the medium is introduced. A concentrated, uniformly and linearly distributed normal, tangential force or thermal source, is assumed to be acting at the origin of the rectangular Cartesian co-ordinates.

For two-dimensional problem, we assume $\overrightarrow{u} = (u, 0, w)$ in equations (1)–(4). The initial and regularity conditions are given by:

$$\begin{aligned} u(x,z,0) &= 0 = \dot{u}(x,z,0)\,, \\ w(x,z,0) &= 0 = \dot{w}(x,z,0)\,, \\ \phi(x,z,0) &= \dot{\phi}(x,z,0)\,, \\ T(x,z,0) &= \dot{T}(x,z,0) \text{ for } z \geq 0\,, \quad -\infty < x < \infty\,, \end{aligned} \tag{5}$$

$$u(x,z,t) = w(x,z,t) = \phi(x,z,t) = T(x,z,t) = 0 \text{ for } t > 0\,, \quad \text{when } z \to \infty\,. \tag{6}$$

We consider two types of boundary conditions

- **Mechanical sources on the surface of half-space.**
  The boundary conditions in this case are

$$\begin{aligned} t_{zz}(x,z,t) &= -\psi(x)H(t)\,, \quad t_{zx}(x,z,t) = -\zeta(x)H(t)\,, \\ \frac{\partial \phi}{\partial z} &= 0\,, \quad \frac{\partial T}{\partial z} + hT = 0 \text{ at } z = 0\,, \end{aligned} \tag{7}$$

  where $H(t)$ is the Heaviside unit step function and $\psi(x)$ specify the vertical and horizontal source distribution functions respectively along the x-axis. h is the heat transfer coefficient.

- **Thermoelastic interactions due to thermal point sources.**
  The boundary conditions in this case are

$$t_{zz} = 0\,, \quad t_{zx} = 0\,, \quad \frac{\partial \phi}{\partial z} = 0 \text{ at } z = 0\,,$$

$$\frac{\partial T}{\partial z}(x, z = 0) = r(x,t) \text{ at } z = 0$$

  for the temperature gradient boundary, or,

$$T(x, z = 0) = r(x,t) \text{ at } z = 0\,, \tag{8}$$

  for the temperature input boundary, where $r(x,t) = \eta(x)H(t)$.

4. Solution of the problem

To transform the equations (1)–(4) to nondimensional form, we define the following variables

$$\begin{aligned}
x' &= \frac{\omega_1^*}{c_2}x\,, \quad z' = \frac{\omega_1^*}{c_2}z \quad t' = \omega_1^* t\,,\\
u' &= \frac{\omega_1^*}{c_2}u\,, \quad w' = \frac{\omega_1^*}{c_2}w \quad T' = \frac{T}{T_0}\,,\\
\phi' &= \frac{\omega_1^*\xi}{c_2^2}\,, \quad \epsilon_1 = \frac{\beta c_2^2}{K\omega_1^*}\,, \quad \tau_0' = \omega_1^*\tau_0\,,\\
\tau_1' &= \omega_1^*\tau_1\,, \quad a' = \frac{\omega_1^*}{c_2}a\,,
\end{aligned} \tag{9}$$

$$t'_{zz} = \frac{t_{zz}}{\beta T_0}\,, \quad t'_{zx} = \frac{t_{zx}}{\beta T_0}\,, \quad h' = \frac{hc^2}{\omega_1^*}\,. \tag{10}$$

where

$$c_2 = \left(\frac{\mu}{\rho}\right)^{\frac{1}{2}} \text{ and } \omega_1^* = \frac{\rho c_e c_2^2}{K}\,.$$

The displacement components can be written as

$$u = \frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_2}{\partial z}\,, \quad w = \frac{\partial\psi_1}{\partial z} + \frac{\partial\psi_2}{\partial x}\,, \tag{11}$$

where $\psi_1(x,z,t)$, and $\psi_2(x,z,t)$ are scalar potential functions.

Applying the Laplace and Fourier transforms

$$\begin{aligned}
\hat{f}(x,z,p) &= \int_0^\infty e^{-pt}dt\\
\tilde{f}(\xi,z,p) &= \int_{-\infty}^\infty \tilde{f}(x,z,p)e^{i\xi x}dx\,.
\end{aligned} \tag{12}$$

on equations (1)–(3), after using (9), (11) (suppressing the primes for convenience) and eliminating $\tilde{\psi}$, $\tilde{\phi}$, $\tilde{T}$ and $\tilde{\psi_2}$ from the resulting expressions, we obtain

$$\left(\frac{d^6}{dz^6} + Q\frac{d^4}{dz^4} + N\frac{d^2}{dz^2} + I\right)\left(\tilde{\psi_1}, \tilde{\phi}, \tilde{T}\right) = 0\,, \tag{13}$$

and

$$\left(\frac{d^2}{dz^2} - \lambda_4^2\right)\tilde{\psi_2} = 0\,, \tag{14}$$

where

$$
\begin{aligned}
Q &= \frac{1}{b_1}\left[b_1(b_3-b_5-3\zeta^2)-p^2+a_2a_4-b_2b_4\epsilon_1\right],\\
N &= \frac{1}{b_1}\left[(a_6a_8p-b_3b_5-2b_3\zeta^2+2b_5\zeta+3\zeta^4)b_1-p^2(b_3-b_5-2\zeta^2)+\right.\\
&\quad \left. a_2(a_4b_5-2a_4\zeta^2-a_6\epsilon_1b_4)+b_2(\epsilon_1b_3b_4-2\epsilon_1b_4\zeta^2-a_4a_8p)\right]\\
I &= -\frac{1}{b_1}\left\{b_1\zeta^6-\zeta^4(p^2+b_1b_3-b_1b_5+a_2a_4+b_2b_4)+\zeta^2\left[b_3p^2+b_5p^2+\right.\right.\\
&\quad \left.\left. b_1a_6a_8p-b_1b_3b_5-a_2a_4b_5-\epsilon_1b_4(a_2a_6-b_2b_3)\right\}+p^2(a_6a_8p+b_3b_5)\right],\\
b_1 &= (1+a_1), \quad b_2=a_3(1+\tau_1\delta_{2k}\omega_1^*p), \quad b_3=a_7p^2+a_5,\\
b_4 &= (p+p^2\tau_0\delta_{ik}\omega_1^*), \quad b_5=(p+p^2\tau_0\omega_1^*), \quad \lambda_4^2=\zeta^2+p^2,\\
a_1 &= \frac{\lambda+\mu}{\mu}, \quad a_2=\frac{bc_2^2}{\chi\mu\omega_1^*}, \quad a_3=\frac{\beta T_0}{\mu}, \quad a_4=\frac{b\chi}{\alpha}, \quad a_5=\frac{\zeta_1c_2^2}{\omega_1^{*2}\alpha},\\
a_6 &= \frac{m\chi T_0}{\alpha}, \quad a_7=\frac{\rho\chi c_2^2}{\alpha}, \quad a_8=\frac{mc_2^4}{\omega_1^{*3}\chi K}.
\end{aligned}
$$

The roots of equations (13) and (14) are $\pm\lambda_l$, $l=1,2,3,4$. Assuming the regularity condition (6), the solutions of equations (13) and (14) may be written as

$$\tilde{\psi}_1=A_1\overline{e}^{\lambda_1z}+A_2\overline{e}^{\lambda_2z}+A_3\overline{e}^{\lambda_3z}, \tag{15}$$

$$\tilde{\phi}=d_1A_1\overline{e}^{\lambda_1z}+d_2A_2\overline{e}^{\lambda_2z}+d_3A_3\overline{e}^{\lambda_3z} \tag{16}$$

$$\tilde{T}=e_1A_1\overline{e}^{\lambda_1z}+e_2A_2\overline{e}^{\lambda_2z}+e_3A_3\overline{e}^{\lambda_3z} \tag{17}$$

$$\tilde{\psi}_2=A_4\overline{e}^{\lambda_4z} \tag{18}$$

where

$$
\begin{aligned}
e_l &= \frac{U^*\lambda_l^2+V^*}{a_2\lambda_l^2+T^*}, \quad d_l=\frac{P^*\lambda_l^2+Q^*}{R^*\lambda_l^2+S^*}, \quad (l=1,2,3),\\
U^* &= a_2b_4\epsilon_1+pb_1a_8, \quad V^*=-p^*-\zeta^2(b_1+\epsilon_1a_2b_4),\\
T^* &= b_2a_8p-a_2\left(\zeta^2+b_5\right), \quad p^*=\frac{a_4}{a_6}-\frac{b_1}{b_2},\\
R^* &= \frac{1}{a_6}, \quad Q^*=\frac{1}{b_2}(\zeta^2b_1+p^2)-\frac{a_4\zeta^2}{a_6},\\
S^* &= \frac{a_2}{b_2}+\left(\frac{b_3-\zeta^2}{a_6}\right),
\end{aligned}
$$

with A_l, $l=1,2,3,4$ being arbitrary constants.

5. Applications

5.1. *Mechanical sources on the surface of half-space*

Applying the Laplace and Fourier transforms defined by (12) to the boundary conditions (7), we get

$$\tilde{t}_{zz}(\xi,z,p)=-\frac{\tilde{\psi}(\xi)}{p}, \quad \tilde{t}_{zx}(\xi,z,p)=-\frac{\tilde{\zeta}(\xi)}{p}, \quad \frac{d\tilde{\phi}}{dz}=0, \quad \frac{d\tilde{T}}{dz}=h\tilde{T}=0 \text{ at } z=0. \tag{19}$$

Making use of equations (4)–(5), (9)–(11), applying the transforms defined by (12) and substituting the values of $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{T}$, $\tilde{\phi}$ from equations (15)–(18) in the boundary conditions (19), we obtain the expressions for the components of displacement, stress, temperature distribution and change in volume fraction field as

$$
\begin{aligned}
\tilde{u} &= -\frac{i\xi}{p\Delta}\Big[\tilde{\psi}(\xi)(\Delta_1 e^{-\lambda_1 z} - \Delta_3 e^{-\lambda_2 z} + \Delta_5 e^{-\lambda_3 z} - \Delta_7 e^{-\lambda_4 z}) \\
&\quad + \tilde{\zeta}(\xi)(\Delta_2 e^{-\lambda_1 z} - \Delta_4 e^{-\lambda_2 z} + \Delta_6 e^{-\lambda_3 z} - \Delta_8 e^{-\lambda_4 z})\Big]\,, \\
\tilde{w} &= -\frac{1}{p\Delta}\Big[\tilde{\psi}(\xi)(\lambda_1\Delta_1 e^{-\lambda_1 z} - \lambda_2\Delta_3 e^{-\lambda_2 z} + \lambda_3\Delta_5 e^{-\lambda_3 z} - i\xi\Delta_7 e^{-\lambda_4 z}) \\
&\quad + \tilde{\zeta}(\xi)(\lambda_1\Delta_2 e^{-\lambda_1 z} - \lambda_2\Delta_4 e^{-\lambda_2 z} + \lambda_3\Delta_6 e^{-\lambda_3 z} - i\xi\Delta_8 e^{-\lambda_4 z})\,, \\
\tilde{t}_{zz} &= \frac{1}{p\Delta}\Big[\tilde{\psi}(\xi)(s_1\Delta_1 e^{-\lambda_1 z} - s_2\Delta_3 e^{-\lambda_2 z} + s_3\Delta_5 e^{-\lambda_3 z} - s_4\Delta_7 e^{-\lambda_4 z}) \\
&\quad + \tilde{\zeta}(\xi)(s_1\Delta_1 e^{-\lambda_1 z} - s_2\Delta_4 e^{-\lambda_2 z} + s_3\Delta_6 e^{-\lambda_3 z} - s_4\Delta_8 e^{-\lambda_4 z})\,, \\
\tilde{t}_{zx} &= \frac{1}{p\Delta}\Big[\tilde{\psi}(\xi)(\lambda_1\Delta_1 e^{-\lambda_1 z} - \lambda_2\Delta_3 e^{-\lambda_2 z} + \lambda_3\Delta_5 e^{-\lambda_3 z} + n_1\Delta_7 e^{-\lambda_4 z}) \\
&\quad + \tilde{\zeta}(\xi)(\lambda_1\Delta_2 e^{-\lambda_1 z} - \lambda_2\Delta_4 e^{-\lambda_2 z} + \lambda_3\Delta_6 e^{-\lambda_3 z} + n_1\Delta_8 e^{-\lambda_4 z})\,, \\
\tilde{T} &= \frac{1}{p\Delta}\Big[\tilde{\psi}(\xi)(e_1\Delta_1 e^{-\lambda_1 z} - \Delta_3 e^{-\lambda_2 z} + \Delta_5 e^{-\lambda_3 z} - \Delta_7 e^{-\lambda_4 z}) \\
&\quad + \tilde{\zeta}(\xi)\Big[\tilde{\psi}(\zeta)(e_1\Delta_2 e^{-\lambda_1 z} - e_2\Delta_4 e^{-\lambda_2 z} + e_3\Delta_6 e^{-\lambda_3 z}) \\
\tilde{\phi} &= \frac{1}{p\Delta}\Big[\tilde{\psi}(\xi)(d_1\Delta_1 e^{-\lambda_1 z} - d_2\Delta_3 e^{-\lambda_2 z} + d_3\Delta_5 e^{-\lambda_3 z}) \\
&\quad + \tilde{\zeta}(\xi)(d_1\Delta_2 e^{-\lambda_1 z} - d_2\Delta_4 e^{-\lambda_2 z} + d_3\Delta_6 e^{-\lambda_3 z})\Big]\,,
\end{aligned}
\tag{20}
$$

where

$$
\begin{aligned}
\Delta &= \Delta_1^* + d\Delta_2^*\,, \\
\Delta_1^* &= \lambda_2\lambda_3(s_4\lambda_1 - n_1 s_1)(d_3 e_2 - d_2 e_3)\lambda_1\lambda_3(s_4\lambda_2 + n_1 s_2)(d_1 e_3 + e_1 d_3) + \\
&\quad \lambda_1\lambda_2(s_4\lambda_3 + n_1 s_3)(d_1 e_2 - e_1 d_2)\,, \\
\Delta_2^* &= (s_4\lambda_1 - n_1 s_1)(e_3\lambda_2 d_2 - e_2 d_3\lambda_3) + (e_3\lambda_1 d_1 - e_1\lambda_3 d_3)(s_4\lambda_2 + n_1 s_2) - \\
&\quad (s_4\lambda_3 + n_1 s_3)(e_1\lambda_2 d_2 - e_2\lambda_1 d_1)\,, \\
\Delta_{1,2} &= (n_1, s_4)\Delta_{10}\,, \quad \Delta_{3,4} = (-n_1, s_4)\Delta_{20}\,, \quad \Delta_{5,6} = (-n_1, s_4)\Delta_{30}\,, \\
\Delta_7 &= \{\lambda_1\lambda_2\lambda_3[e_3(d_1 - d_2) + e_2(d_3 - d_1) - e_1(d_3 - d_2)] + h\,[\lambda_1(e_3 d_2\lambda_2 - e_2 d_3\lambda_3) - \\
&\quad [\lambda_2(e_3\lambda_1 d_1 - e_2\lambda_3 d_3) + \lambda_3(\lambda_1 d_1 e_2 - e_1\lambda_2 d_2)]\}\,(d_1 e_2 - e_1 d_2)\,, \\
\Delta_8 &= \{[s_2\lambda_1\lambda_3(d_1 e_3 - e_1 d_2) + s_1\lambda_2\lambda_3(e_2 d_3 - e_3 d_2) - s_3\lambda_2\lambda_1(e_1 d_2 - e_2 d_1)] + \\
&\quad h[s_1(e_3 d_2\lambda_2 - e_2 d_3\lambda_3) - s_2(e_3\lambda_1 d_1 - e_1\lambda_3 d_3) + s_3(\lambda_1 d_1 e_2 - e_1\lambda_2 d_2)]\}\,, \\
\Delta_{10} &= (e_2 d_3 - d_2 e_3)\lambda_2\lambda_3 + h(e_3\lambda_2 d_2 - \lambda_3 d_3 e_2)\,, \\
\Delta_{20} &= (e_1 d_3 - d_1 e_3)\lambda_1\lambda_3 + h(e_3\lambda_1 d_1 - e_1\lambda_3 d_3)\,,
\end{aligned}
$$

$$
\begin{aligned}
s_l &= \frac{-i\xi\lambda}{\beta T_0}\,, \quad s'_{20} = \frac{-(\lambda+2\mu)}{\beta T_0}\,, \quad s'_{30} = \frac{bc_2^2}{\beta T_0 \omega_1^* \chi}\,,\\
s'_{40} &= (1+p\tau_1\delta_{2k})\,, \quad n_1 = \frac{\lambda_4^2+\xi^2}{2i\xi}\,.
\end{aligned}
$$

5.1.1. *Case: Concentrated force*

In this case

$$\psi(x) = P\delta(x)\,, \quad \zeta(x)P\delta(x)\,,$$

with

$$\tilde{\psi}(\xi) = P\,, \quad \tilde{\zeta}(\xi)P\,, \tag{21}$$

where P is the magnitude of the force and $\delta(x)$ is Dirac delta function.

5.1.2. *Case: Uniformly distributed force*

The solution due to uniformly distributed force applied on the half-space surface is obtained by setting

$$\{\psi(x), \zeta(x)\} = \begin{cases} 1 & \text{if} \quad |x| \le a\,,\\ 0 & \text{if} \quad |x| > a\,. \end{cases}$$

in equation (7). The Laplace and Fourier transforms with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width $2a$ applied at origin of the coordinate system ($x = z = 0$) in dimensionless form after suppressing the primes becomes

$$\{\psi(x), \zeta(x)\} = \left[2\sin\left(\frac{\frac{\xi c_2 a}{\omega_1^*}}{\xi}\right)\right]\,, \quad \xi \neq 0\,. \tag{22}$$

5.1.3. *Case: Linearly distributed force*

The solution due to linearly distributed force applied on the half-space surface is obtained by setting

$$\{\psi(x), \zeta(x)\} = \begin{cases} 1-\frac{|x|}{a} & \text{if} \quad |x| \le a\,,\\ 0 & \text{if} \quad |x| > a\,. \end{cases}$$

in equation (7), where $2a$ is the width of strip load. Using equations (9)–(10) (after suppressing the primes) and applying the transforms defined by equation (12), we get

$$\left\{\tilde{\psi}(\xi), \tilde{\zeta}(\xi)\right\} = \frac{2\left[1-\cos\left(\frac{\xi c_2 a}{\omega_1^*}\right)\right]}{\frac{\xi^2 c_2 a}{\omega_1^*}} \tag{23}$$

The expressions for displacements, stresses, temperature distribution and change in volume fraction field can be obtained for concentrated, uniformly and linearly distributed force by replacing $\tilde{\psi}(\xi)$, $\tilde{\zeta}(\xi)$ from equations (21)–(23), respectively, in equation (20).

5.1.4. *Particular case [5.1.1]*

If we neglect the voids effect i.e., ($\alpha = b = \xi_1 = m = \chi = 0$) in equation (20), we obtain the components of displacement, stress and temperature distribution in generalized thermoelastic half-space as

$$\begin{aligned}
\tilde{u} &= -\frac{1}{p\Delta^{**}}\Big[\tilde{\psi}(\xi)(-i\xi[\Delta_3^{**}e^{-\lambda_1^* z} - \Delta_4^{**}e^{-\lambda_2^* z}) + \Delta_5^{**}\lambda_4 e^{-\lambda_4 z}]+ \\
&\quad +\tilde{\zeta}(\xi)(i\xi(\Delta_6^{**}e^{-\lambda_1^* z} + \Delta_7^{**}e^{-\lambda_2^* z}) - \Delta_8^{**}e^{-\lambda_4 z})\Big]\,, \\
\tilde{w} &= -\frac{1}{p\Delta^{**}}\Big[\tilde{\psi}(\xi)(\lambda_1^*\Delta_3^{**}e^{-\lambda_1^* z} + \lambda_2\Delta_4^{**}e^{-\lambda_2^* z} + i\xi\Delta_5^{**}e^{-\lambda_4 z})- \\
&\quad +\tilde{\zeta}(\xi)(\lambda_1^*\Delta_6^{**}e^{-\lambda_1^* z} + \lambda_2^*\Delta_7^{**}e^{-\lambda_2^* z} - i\xi\Delta_8^{**}e^{-\lambda_4 z})\,, \\
\tilde{t}_{zz} &= \frac{1}{p\Delta^{**}}\Big[\tilde{\psi}(\xi)(s_1^*\Delta_3^{**}e^{-\lambda_1^* z} + s_2^*\Delta_4^{**}e^{-\lambda_2^* z} + s_3\Delta_5^{**}e^{-\lambda_4 z})- \\
&\quad +\tilde{\zeta}(\xi)(s_1^*\Delta_6^{**}e^{-\lambda_1^* z} + s_2^*\Delta_7^{**}e^{-\lambda_2^* z} + s_3^*\Delta_8^{**}e^{-\lambda_4 z})\,, \\
\tilde{t}_{zx} &= \frac{1}{p\Delta^{**}}\Big[\tilde{\psi}(\xi)(\lambda_1^*\Delta_3^{**}e^{-\lambda_1^* z} + \lambda_2^*\Delta_4^{**}e^{-\lambda_2^* z} - n_1\Delta_5^{**}e^{-\lambda_4 z}) \\
&\quad +\tilde{\zeta}(\xi)(\lambda_1^*\Delta_6^{**}e^{-\lambda_1^* z} + \lambda_2^*\Delta_7^{**}e^{-\lambda_2^* z} - n_1\Delta_8^{**}e^{-\lambda_4 z})\,, \\
\tilde{T} &= \frac{1}{p\Delta^{**}}\Big[\tilde{\psi}(\xi)(e_1^*\Delta_3^{**}e^{-\lambda_1^* z} + e_2^*\Delta_4^{**}e^{-\lambda_2^* z}) \\
&\quad +\tilde{\zeta}(\xi)\Big[\tilde{\psi}(\zeta)(e_1^*\Delta_6^{**}e^{-\lambda_1^* z} + e_2^*\Delta_7^{**}e^{-\lambda_2^* z})\ .
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
\Delta^{**} &= \Delta_1^{**} + d\Delta_2^{**}\,, \\
\Delta_1^{**} &= -(s_1^*e_2^*\lambda_2 - s_2^*e_1^*\lambda_1)n_1(s_3^*e_2^* - e_1^*)\lambda_1^*\lambda_2^*\,, \\
\Delta_2^{**} &= (s_1^*e_2^* - s_2^*e_1^*)n_1 + s_3^*e_2^*\lambda_1^* - e_1^*\lambda_2^*\,, \\
\Delta_3^{**} &= -(\lambda_2^* - h)e_2^*n_1\,, \quad \Delta_4^{**} = -(\lambda_1^* + h)e_1^*n_1\,, \\
\Delta_5^{**} &= (e_1^* - e_2^*)\lambda_1^*\lambda_2^* + h(e_2^*\lambda_1^* - e_1^*\lambda_2^*)\,, \quad \Delta_6^{**} = (\lambda_2^* - h)e_2^*s_3^*\,, \\
\Delta_7^{**} &= (\lambda_1^* + h)e_1 s_3\,, \quad \Delta_8 = -(e_1^*s_2^*\lambda_1^* - e_2^*s_1^*\lambda_2^*) + h(e_2 s_3 - s_2 e_1)\,, \\
\lambda_l^{*2} &= \frac{-A + (-1)^{l+1}\sqrt{A^2 - 4B}}{2}\,; \quad l = 1,2 \\
A &= \frac{p^2 + (2\xi^2 + b_5)b_1 + \epsilon_1 b_2 b_4}{b_1}\,, \quad B = \frac{(p^2 + \xi^2 b_1)(\xi^2 + b_5) + \epsilon_1 b_2 b_4 \xi^2}{b_1}\,, \\
s_l^* &= -i\xi s'_{10} - \lambda_l^{*2}s'_{20} - e_l^*\,, \quad l = 1,2\,, \quad s_3^* = s'_{10} - i\xi s'_{20})\lambda_4\,, \\
e_l^* &= \frac{b_1\lambda_l^{*2} - p^2 - b_1\xi^2}{b_2}\,, \quad l = 1,2\,.
\end{aligned}$$

The expressions for displacements, stresses and temperature distribution can be obtained for concentrated, uniformly and linearly distributed force by replacing $\tilde{\psi}(\xi,\ \tilde{\zeta}(\xi)$ from equations (21)–(23), respectively, in equation (24).

5.2. *Thermoelastic interactions due to thermal source*

Applying the Laplace and Fourier transforms defined by (12), to the boundary conditions (8) we get

$$\tilde{t}_{zz} = 0\,, \quad \tilde{t}_{zx} = 0\,, \quad \text{at } z = 0\,,$$

or

$$\frac{d\tilde{T}}{dz}(\xi.z = 0) = r(\xi, p) \quad \text{at } z = 0\,,$$

for the temperature gradient boundary,

$$\tilde{T}(\xi, z = 0) = r(\xi, p) \quad \text{at } z = 0\,, \tag{25}$$

for the temperature input boundary, where

$$r(\xi, p) = \frac{\tilde{\eta}(\xi)}{p}\,.$$

Making use of equations (4)–(5), (9)–(11) applying the transforms defined by (12) and substituting the values of $\tilde{\psi}_1$, $\tilde{\psi}_2$, $\tilde{T}$, $\tilde{\phi}$ from equations (15)–(18) in the boundary conditions (25), we obtain the expressions for the components of displacement, stress, temperature distribution and change in volume fraction field as

$$\begin{aligned}
\tilde{u} &= -\eta_1\left[i\xi(\Delta_1' e^{-\lambda_1 z} - \Delta_2' e^{-\lambda_2 z} + \Delta_3' e^{\lambda_3 z}) - \Delta_4'\lambda_4 e^{-\lambda_4 z}\right] \\
\tilde{w} &= -\eta_1\left(\lambda_1\Delta_1' e^{-\lambda_1 z} - \lambda_2\Delta_2' e^{-\lambda_2 z} + \lambda_3\Delta_3' e^{-\lambda_3 z} - i\xi\Delta_4' e^{-\lambda_4 z}\right)\,, \\
\tilde{t}_{zz} &= -\eta_1\left(s_1\Delta_1' e^{-\lambda_1^* z} - s_2\Delta_2' e^{-\lambda_2 z} + s_3\Delta_3' e^{-\lambda_3 z} - s_4\Delta_4' e^{-\lambda_4 z}\right)\,, \\
\tilde{t}_{zx} &= -\eta_1\left(\lambda_1\Delta_1' e^{-\lambda_1 z} - \lambda_2\Delta_2' e^{-\lambda_2 z} + \lambda_3\Delta_3' e^{-\lambda_3 z} + n_1\Delta_4' e^{-\lambda_4 z}\right)\,, \\
\tilde{T} &= -\eta_1\left(e_1\Delta_1' e^{-\lambda_1 z} - e_2\Delta_2' e^{-\lambda_2 z} + e_3\Delta_3' e^{-\lambda_3 z}\right)\,, \\
\tilde{\phi} &= -\eta_1\left(d_1\Delta_1' e^{-\lambda_1 z} - d_2\Delta_2' e^{-\lambda_2 z} + d_3\Delta_3' e^{-\lambda_3 z}\right)\,,
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
\Delta_1' &= s_4\lambda_2\lambda_3(d_3 - d_2) + n_1(s_2\lambda_3 d_3 - s_3\lambda_2 d_2)\,, \\
\Delta_2' &= s_4\lambda_1\lambda_3(d_3 - d_1) + n_1(s_1\lambda_3 d_3 - s_3\lambda_1 d_1)\,, \\
\Delta_3' &= s_4\lambda_1\lambda_2(d_2 - d_1) + n_1(s_1\lambda_2 d_2 - s_2\lambda_1 d_1)\,, \\
\Delta_4' &= s_1\lambda_2\lambda_3(d_3 - d_2) + s_2\lambda_1\lambda_3(d_1 - d_3) + s_3\lambda_1\lambda_2(d_2 - d_1))\,, \\
\eta_1 &= \frac{\tilde{\eta}(\xi)}{p\Delta}\,.
\end{aligned}$$

On replacing Δ, by Δ_1^* and Δ_2^* respectively, we obtain the expressions for temperature gradient boundary and temperature input boundary.

5.2.1. *Case: Thermal point source*

In this case

$$\eta(x) = p\eta(x)\,,$$

with

$$\tilde{\eta}(\xi) = P\,, \tag{27}$$

where P is the magnitude of constant temperature applied on the boundary.

5.2.2. *Case: Uniformly distributed thermal source*

In this case

$$\eta(x) = \begin{cases} 1 & \text{if} \quad |x| \le a\,, \\ 0 & \text{if} \quad |x| > a\,. \end{cases}$$

with

$$\{\tilde{\eta}(\xi) = \left[2\sin\left(\frac{\frac{\xi c_2 a}{\omega_1^*}}{\xi} \right) \right], \quad \xi \neq 0\,. \tag{28}$$

5.2.3. *Case: Linearly distributed thermal source*

In this case

$$\eta(x) = \begin{cases} 1 - \frac{|x|}{a} & \text{if} \quad |x| \le a\,, \\ 0 & \text{if} \quad |x| > a\,. \end{cases}$$

with

$$\tilde{\eta}(\xi) = \frac{2\left[1 - \cos\left(\frac{\xi c_2 a}{\omega_1^*}\right)\right]}{\frac{\xi^2 c_2 a}{\omega_1^*}}\,. \tag{29}$$

Replacing $\tilde{\eta}(\xi)$ from (27)–(29) in (26), we obtain the corresponding expressions for thermal point source, uniformly and linearly distributed, respectively.

5.2.4. *Particular case [*5.2.1*]*

Neglecting the voids effect, the expressions for displacement components, stresses and temperature distribution in a generalized thermoelastic half-space are obtained by replacing Δ_i^{**} with Δ_i^{***}, $(i = 6, 7, 8)$, $\tilde{\zeta}(\xi)$ with $\tilde{\eta}(\xi)$ and $\Delta_3^{**} = \Delta_4^{**} = \Delta_5^{**} = 0$ equation (24) respectively, where

$$\Delta_6^{***} = s_2^* n_1 + s_3^* \lambda_2^*\,, \quad \Delta_7^{***} = s_1^* n_1 + s_3^* \lambda_1^*\,, \quad \Delta_8^{***} = s_1^* \lambda_2^* + s_2^* \lambda_1^*\,.$$

On replacing Δ^{**} by Δ_1^{**} and Δ_2^{**} in equation (24), we obtain the expressions for temperature gradient boundary and temperature input boundary respectively.

<u>Sub-case 1:</u> If $h \to 0$ in eq. (20), we obtain the corresponding expressions of displacements, stresses, temperature distribution and change in volume fraction field for the insulated boundary.

<u>Sub-case 2:</u> If $h \to \infty$ in eq. (20), we recover the corresponding expressions of displacements, stresses, temperature distribution and change in volume fraction field for the isothermal boundary.

<u>Special case 1:</u> By putting $k = 1$ and $\tau_1 = 0$ in eqs (20),(24) and (26), we obtain the corresponding expressions of thermoelastic half-space with and without voids, respectively, for L-S theory.

<u>Special case 2:</u> For G-L theory, we recover the corresponding expressions of thermoelastic half-space with and without voids, respectively, by substituting $k = 2$ in

eqs (20),(24) and (26).

Special case 3: In case of coupled theory of thermoelasticity, the thermal relaxation times vanish i.e. $\tau_0 = \tau_1 = 0$ and consequently, we obtain the corresponding expressions of thermoelastic half-space with and without voids, respectively, with following changed values in eqs (20), (24) and (26)

$$
\begin{aligned}
Q &= \frac{1}{b_1}\left[b_1(b_3 - p - 3\xi^2) - p^2 + a_2a_4 - a_3p\epsilon_1\right] , \\
N &= \frac{1}{b_1}\left[(a_6a_8p - b_3p - 2b_3\xi^2 + 2p\xi^2 + 3\xi^4)b_1 - p^2(b_3 - p - 2\xi^2) + \right. \\
&\quad \left. a_2(a_4p - 2a_4\xi^2 - a_6\epsilon_1p) + a_3p(\epsilon_1b_3 - 2\epsilon_1\xi^2 - a_4a_8)\right] , \\
I &= -\frac{1}{b_1}\left\{b_1\xi^6 - \xi^4(p^2 + b_1b_3 - b_1p + a_2a_4 + a_3p) + \xi^2\left[b_3p^2 + p^3 + \right.\right. \\
&\quad \left.\left. b_1a_6a_8p - b_1b_3p - a_2a_4p - \epsilon_1p(a_2a_6 - a_3b_3)\right\} + p^3(a_6a_8 + b_3)\right] , \\
b_2 &= a_3 , \quad b_4 = b_5 = p , \quad U^* = p(a_2\epsilon_1 + b_1a_8) , \\
V^* &= -p^2 - \xi^2(b_1 + \epsilon_1a_2p) , \quad T^* = a_3a_8p - a_2(\xi^2 + p) , \quad P^* = \frac{a_4}{a_6} - \frac{b_1}{a_3} , \\
Q^* &= \frac{1}{a_3}(\xi^2b_1 + p^2) - \frac{a_4\xi^2}{a_6} , \quad S^* = \frac{a_2}{a_3} + \frac{b_3 - \xi^2}{a_6} , \qquad (30) \\
s_l &= -i\xi s'_{10} - \lambda_l^2s'_{30} + e_l , \quad l = 1, 2, 3 , \\
\lambda_l^{*2} &= \frac{-A + (-1)^{l+1}\sqrt{A^2 - 4B}}{2} , \quad l = 1, 2 , \qquad (31) \\
A &= -\frac{p^2 + (2\xi^2 + p)b_1 + \epsilon_1a_3p}{b_1} , \quad B = \frac{(p^2 + \xi^2b_1)(\xi^2 + p) + \epsilon_1a_3p\xi^2}{b_1} . \qquad (32)
\end{aligned}
$$

Special case 4: Taking $\epsilon_1 = 0$ in (30), we recover the corresponding expressions of thermoelastic half-space with and without voids, respectively, for uncoupled theory of thermoelasticity.

6. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in equations (20), (24) and (26), for the two theories, i.e., L-S and G-L. These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ, respectively, and hence are of the form $\tilde{f}(\xi, z, p)$ in the physical domain, first we invert the Fourier transform using

$$\hat{f}(x, z, p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\tilde{f}(\xi, z, p)d\xi = \frac{1}{\pi}\int_0^{\infty} (\cos(\xi x)f_e - i\sin(\xi x)f_0)\, d\xi , \quad (33)$$

where f_e and f_0 are, respectively, even and odd parts of the function $\tilde{f}(\xi, z, p)$

Thus, expression (31) gives us the Laplace transform $\hat{f}(x, z, p)$ of the function $f(x, z, t)$. Following Honig and Hirdes [22], the Laplace transform function $\hat{f}(x, z, p)$ can be inverted to $f(x, z, t)$. The last step is to calculate the integral in equation

(31). The method for evaluating this integral is described by Press *et al.* [23], which involves the use of Romberg's integration with adaptive step size. This, also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

Following Dhaliwal and Singh [24] we take the case of magnesium crystal-like material for numerical calculations. The physical constants used are:

$$\begin{array}{lll} \lambda = 2.17\times 10^{10}\ \mathrm{Nm}^{-2} & \mu = 3.278\times 10^{10}\ \mathrm{Nm}^{-2} & \rho = 1.74\times 10^{3}\ \mathrm{kgm}^{-3}, \\ c_e = 1.04\times 10^{3}\ \mathrm{Jkg}^{-1}\mathrm{deg}^{-1} & \omega_1^* = 3.58\times 10^{1}1\ \mathrm{s}^{-1} & P = 1 \\ K = 1.7\times 10^{2}\ \mathrm{Wm}^{-1}\mathrm{deg}^{-1} & \beta = 2.68\times 10^{6}\ \mathrm{Nm}^{-2}\ \mathrm{deg}^{-1} & T_0 = 298\ \ \mathrm{K} \end{array}$$

and void parameters are:

$$\begin{array}{lll} \xi = 1.753\times 10^{-15}\ \mathrm{m}^2 & \alpha = 3.688\times 10^{-5}\ \mathrm{N} & \xi_1 = 1.475\times 10^{10}\ \mathrm{Nm}^{-2} \\ b = 1.13849\times 10^{10} & m = 2\times 10^{6}\ \mathrm{Nm}^{-2}\mathrm{deg}^{-1}\,. & \end{array}$$

The comparison of dimensionless normal stress t_{zz}, boundary temperature field T and change in volume fraction field ϕ with distance x for Lord-Shulman theory with void (LSV), Green-Lindsay theory with void (GLV) and Lord-Shulman theory without void (LSWV), Green-Lindsay theory without void (GLWV) for concentrated source (CS) uniformly distributed source (UDS) linearly distributed source (LDS) are shown graphically in Figures 1–6, for non-dimensional relaxation times $\tau_0 = 0.02$ $\tau_1 = 0.05$, The computations were carried out for time $t = 0.5$ at $z = 1.0$ in the range $0 \le x \le 10$. The solid and dashed lines with center symbols are denoted by LSWV and GLWV, without center symbol are denoted by LSV and GLV. In Figures 7 and 14 the solid and dashed lines without center symbols corresponds to L-S theory and with center symbol corresponds to G-L theory for all the three sources. The results for a distributed source (mechanical) and distributed thermal source are presented for dimensionless width $a = 1$. The Figures are depicted for normal force and thermal source respectively.

7.1. *Normal force on the boundary of half-space(Insulated boundary)*

7.1.1. *Concentrated Normal force*

Figure 1 depicts the variation of normal stress t_{ZZ} with distance x. Near the source application due to the presence of voids the values of t_{ZZ} for LSV and GLV are greater than LSWV and GLWV. The values of t_{ZZ} for LSV and GLV decrease sharply and then become oscillatory in the whole range.

Figure 2 shows the variation of temperature distribution T with distance x. Due to voids effect the values of T for LSV and GLV decrease in the ranges: $0 \le x \le 2$, $4 \le x \le 5$, $7 \le x \le 8$ and increase in rest of the ranges. The values of T for GLV are greater than LSV in the whole range. For LSWV and GLWV the values of T decrease in the range $0.5 \le x \le 6$ and increase in rest of the range.

The variations of normal stress and temperature distribution for uniformly and linearly distributed force are same as those of concentrated force with difference in their magnitudes for both the theories.

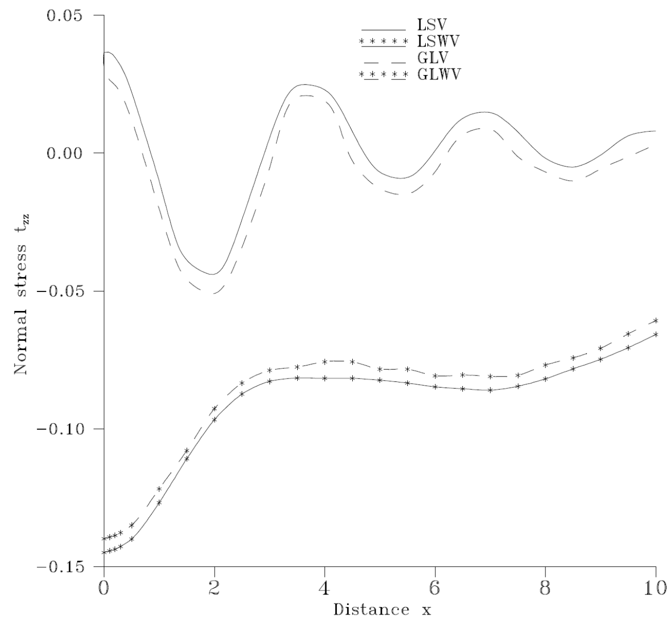

Figure 1 Variation of normal stress t_{zz} with distance x at time $t = 1.0$ due to concentrated force along normal direction (insulated boundary)

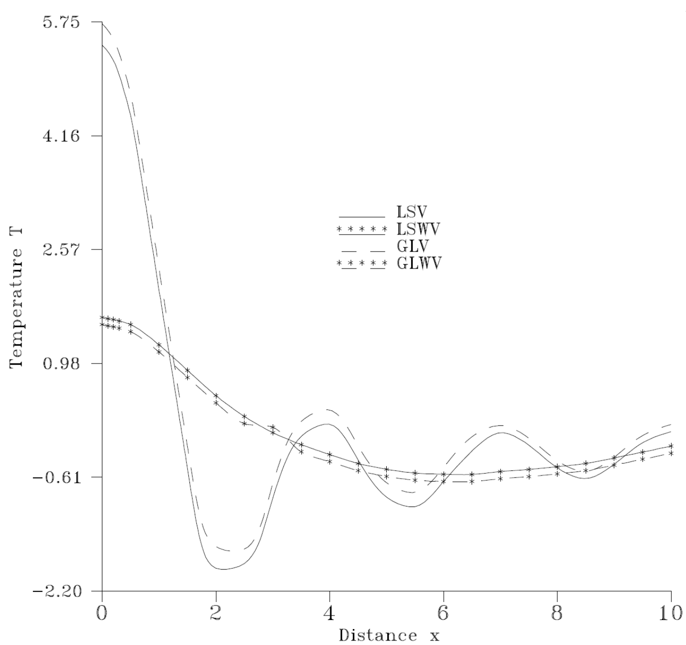

Figure 2 Variation of temperature T with distance x at time $t = 0.5$ due to concentrated force along normal direction (insulated boundary)

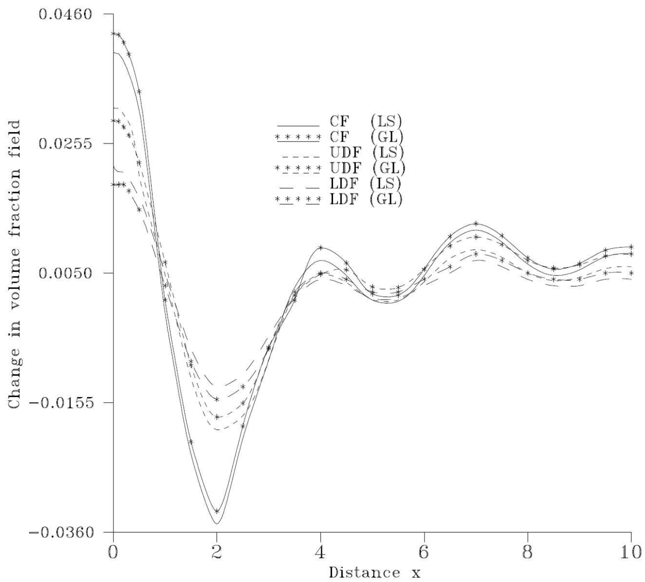

Figure 3 Variation of change in volume fraction field with distance x

Figure 3 depicts the variation of change in volume fraction field ϕ with distance x. Initially, the values of ϕ for linearly distributed force is smaller than those for concentrated force and uniformly distributed force for both the theories. For fixed time $t = 0.5$, the values of ϕ for uniformly distributed force are less than concentrated force and greater than linearly distributed force in the ranges $0 \leq x \leq 1$ and $3 \leq x \leq 10$ and reveals reverse pattern in rest of the range for both the theories.

7.2. *Thermoelastic interactions due to thermal source (Temperature gradient boundary)*

7.2.1. *Thermal point source*

Figure 4 shows the variations of normal stress t_{zz} with distance x. Due to voids effect, the values of t_{zz} for LSV and GLV increase in range $0 \leq x \leq 6$ and then decrease steadily. For LSWV and GLWV the values of t_{zz} decrease slowly in the whole range.

Figure 5 depicts the variation of temperature distribution T with distance x. Near the source application, the values of T for LSWV and GLWV increase sharply and then become oscillatory in the whole range. If we fix the point of observation i.e., the value of distance x, the values of T increase or decrease with reference to x for LSWV and GLWV. Due to voids effect for LSV and GLV the values of T decrease slowly in the whole range.

The variations of normal stress and temperature distribution for uniformly and linearly distributed thermal source are same as those of thermal point source with a difference in their magnitudes.

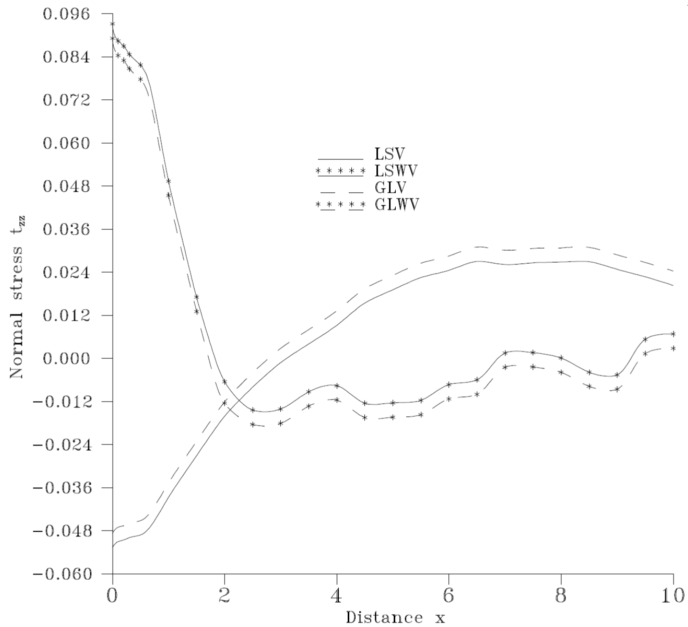

Figure 4 Variation of normal stress t_{zz} with distance x at time $t = 0.5$ due to thermal point source (temperature gradient boundary)

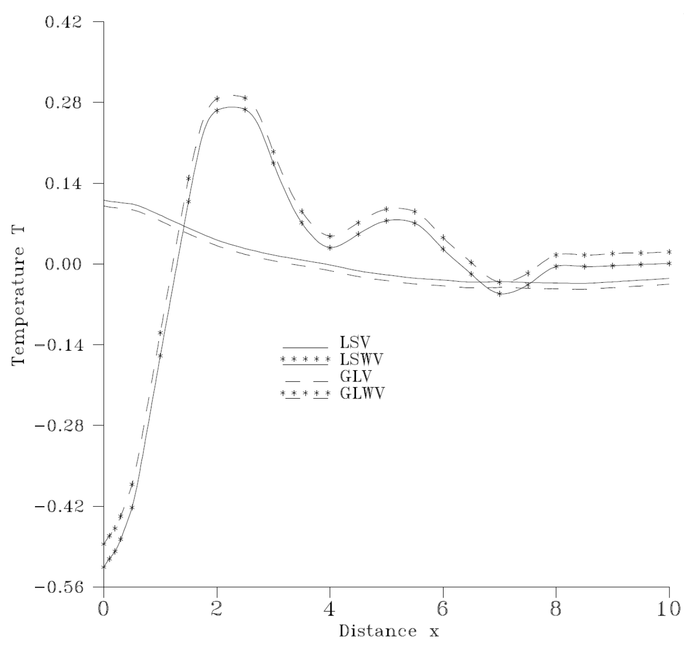

Figure 5 Variation of temperature T with distance x at time $t = 0.5$ due to thermal point source (temperature gradient boundary)

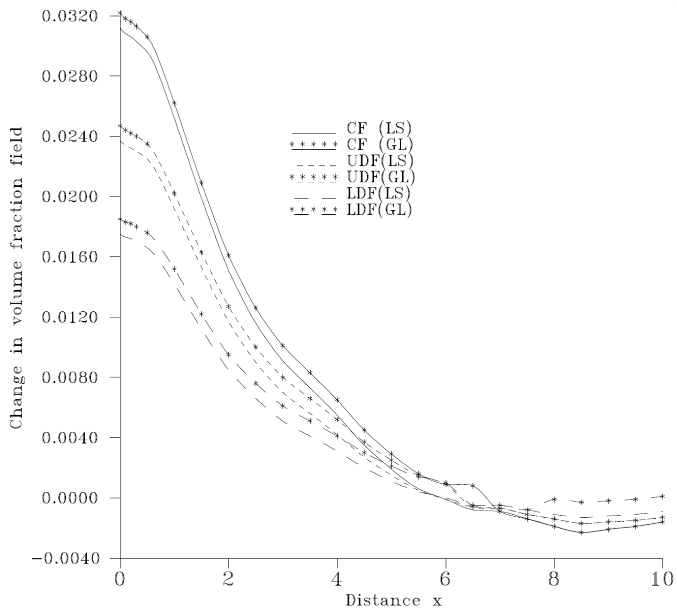

Figure 6 Variation of change in volume fraction field with distance x

Figure 6 shows the variation of change in volume fraction field ϕ with distance x. At the point of application of source the values of f for uniformly distributed source is larger than linearly distributed source and less than concentrated source for both the theories. The values of f decrease slowly in the range $0 \leq x \leq 10$ for both the theories and for the three sources.

8. Conclusions

1. The Laplace and Fourier transform technique is used to derive the components of stress, temperature distribution and change in volume fraction field.

2. As x diverses from the point of application of source the components of temperature distribution, stress and change in volume fraction field are observed to follow oscillatory pattern.

3. The variations of normal stress and temperature distribution for uniformly and linearly distributed source are same as those of concentrated source with a difference in their magnitudes.

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