

Application of the Energy Space in Chaotic Systems Research

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Received (18 March 2007)

Revised (25 April 2007)

Accepted (30 April 2007)

The new method of dynamic systems control is presented. Energy-vector space is introduced. Transformations of the traditional phase space to obtain the energy space. Applications of the new type system research are shown. Different types of systems dynamics are analysed with use of the energy space. Stability of the system is analysed with use of the Stefaski method of the largest Lyapunov exponent calculation [12]. New kind of maps are introduced and applied to examine system dynamics.

Keywords: energy flow, nonlinear dynamics

1. Introduction

In the article the new conception of the system dynamics research is proposed. The energy space is introduced and applied. The idea of this new space is based on the transformation of the traditionally applied phase space. After this transformation the norm $|\vec{x}|$ of the vector is the special function of the energy E , that is accumulated in the system. As the result the trajectory of the system, that accumulates constant amount of the energy lays on a surface of the multidimensional sphere, with the radius showing an amount of the accumulated energy. Vector $\vec{x}$ together with its derivative $\frac{d\vec{x}}{dt}$ determine the plane and direction of the temporary energy flow. Additionally the angle between these vectors gives the possibility of the energy E changes estimation. Application of the energy space allows also to control energy, that is accumulated in the oscillating system, or part of one, to estimate energy, that is dissipated during the motion. Moreover it possess all the possibilities of the system dynamics control, that are used in the traditional phase space. The phase space is nowadays one of the most important tools used in dynamical systems investigations. Many aspects of the systems dynamics can be concluded using this space, but it also allows for the intuitional, geometrical view on an existing dynamical phenomena [13]. The new conception of the space that is proposed in [9], [10], [11] allows for a geometrical view on energy changes in mechanical vibrating

systems. This space possesses all the advantages of the phase space but it also shows an amount of the energy accumulated in the system, an energy changes, flow, dissipation, synchronization, an energy attractors. Thus it increases our knowledge and intuition on energy changes in vibrating systems.

An energy flow modelling still arouses interests in the scientific world. Different methods are applied to solve problems connected with energy flow: Statistical Energy Analysis [1], [2], [3], Finite Element Method [4], [5]. But these methods do not allow for a special kind of a geometrical view on energy changes, which could develop our intuitional knowledge on energy flow phenomenon. This intuition is very important especially in modelling the systems, where we still can not measure energy flow, such as bioenergy, observed and applied in medicine diagnostics thanks to GDV method based on Kirlian effect [6], [7], [8], telepathy - the problem of an energy flow between two bioelectromagnetic systems, and so on.

In this article an application of the energy space in the mechanical vibrating system dynamics investigations is shown. Some aspects of the motion, energy flow, are considered. New types of the maps are introduced.

2. Energy space

Consider the vibrating system with the mathematical model given by four differential equations (1). These equations describe dynamics of the system consisted of two oscillators. The external harmonic force excites the oscillator μ . This oscillator is joined with the second oscillator μ_1 .

$$\begin{cases} \dot{x} = y \\ \dot{y} = (F \sin \eta \tau - cy - c_1 (y - y_1) - \sigma x^3 - \sigma_1 (x - x_1)) \frac{1}{\mu} \\ \dot{x}_1 = y_1 \\ \dot{y}_1 = (c_1 (y - y_1) + \sigma_1 (x - x_1)) \frac{1}{\mu_1} \end{cases} \quad (1)$$

Because of nonlinearity of the spring σ the transformation of the phase space, to obtain the energy space, in direction of x is nonlinear. In the considered case we have to show the transformation of the space, as function $f : R^4 \rightarrow R^4$, that transforms the phase space the following way:

$$f(x, y, x_1, y_1) = \begin{cases} \sqrt{\frac{\sigma}{2}} x^2 \text{sign}(x) = z_e \\ \sqrt{\frac{\mu}{2}} y = y_e \\ \sqrt{\frac{\sigma_1}{2}} (x_1 - x) = z_{1e} \\ \sqrt{\frac{\mu_1}{2}} y_1 = y_{1e} \end{cases} \quad (2)$$

To simplify the description of the transformation note, that in the energy space there are square roots of the potential or kinetic energies on each axis. Note also, that function f transforms energy into vector form.

For better understanding let us to analyse energy flow for the selected parameters. In the Fig.1 one can see the resonance diagram. Energy flow for the special point $\eta = 1$ was analysed. From the Fig. 1 one can see, that for $\eta = 1$ energy accumulated in the spring σ is close to zero. It means, that whole energy accumulated

in the oscillator μ system is close to zero, although this part of the system is forced by external force. It is the case, when the oscillator μ_1 works as the dynamical damper, that is for the excitation frequency equal to the system μ_1 free vibrations frequency.

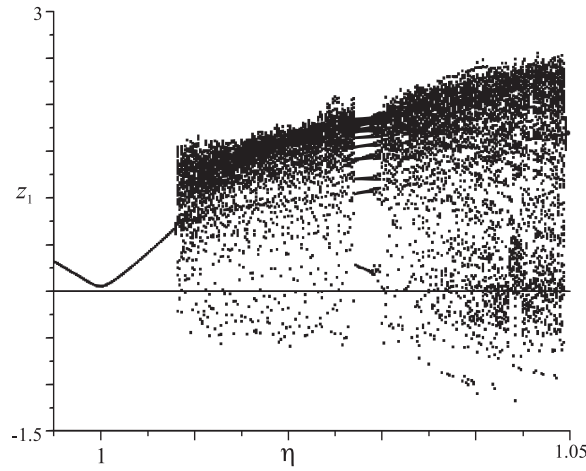


Figure 1 Resonance diagram of $z_1, \mu = 1\text{kg}, \mu_1 = 0.07\text{kg}, \sigma = 1\text{N/m}, \sigma_1 = 0.07\text{N/m}, c = 0.1\text{Ns/m}, c_1 = 0.0001\text{Ns/m}, F = 2.5\text{N}$

Energy flow for the considered case is shown in Fig. 2. In Fig. 2(a, b) one can see energy planes of the oscillators μ and μ_1 respectively.

The first one ($z_e : y_e$) – the energy plane of the oscillator μ system.

This plane is obtained after transformation of the plane ($x : y$). This transformation is nonlinear. Depending on actual x and parameters σ, μ the phase space is squeezed or stretched in directions of the basis vectors.

In Fig. 2a plane ($z_e : y_e$) can be seen. The norm of the vector projection on that plane shows the energy accumulated in the oscillator μ system. The value of z_e shows the potential energy of the spring σ , the value of y_e – the kinetic energy of the mass μ , and by means of these coordinates these energies can be calculated. One can see the changes from the potential energy into the kinetic one and vice versa. Note that total amount of the energy accumulated in the oscillator μ system is not constant. There exists an energy flow between the oscillators μ and μ_1 . This flow will be analysed further.

The second interesting plane is ($z_{1e} : y_{1e}$) – the energy plane of the oscillator μ_1 system.

There exist transformations of two kinds which were made on the phase plane ($x_1 : y_1$) to obtain this energy plane.

The first one: instead of the state variable x_1 we have the spring σ_1 deflection:
 $z_1 = x_1 - x$.

The second one is linear and one can find eigenvectors of the transformation as orthogonal basis vectors $[1,0]$ and $[0,1]$. Eigenvalues of the transformation are equal:

$$\lambda_1 = \sqrt{\frac{\sigma_1}{2}} \quad \lambda_2 = \sqrt{\frac{\mu_1}{2}} \quad \text{respectively.}$$

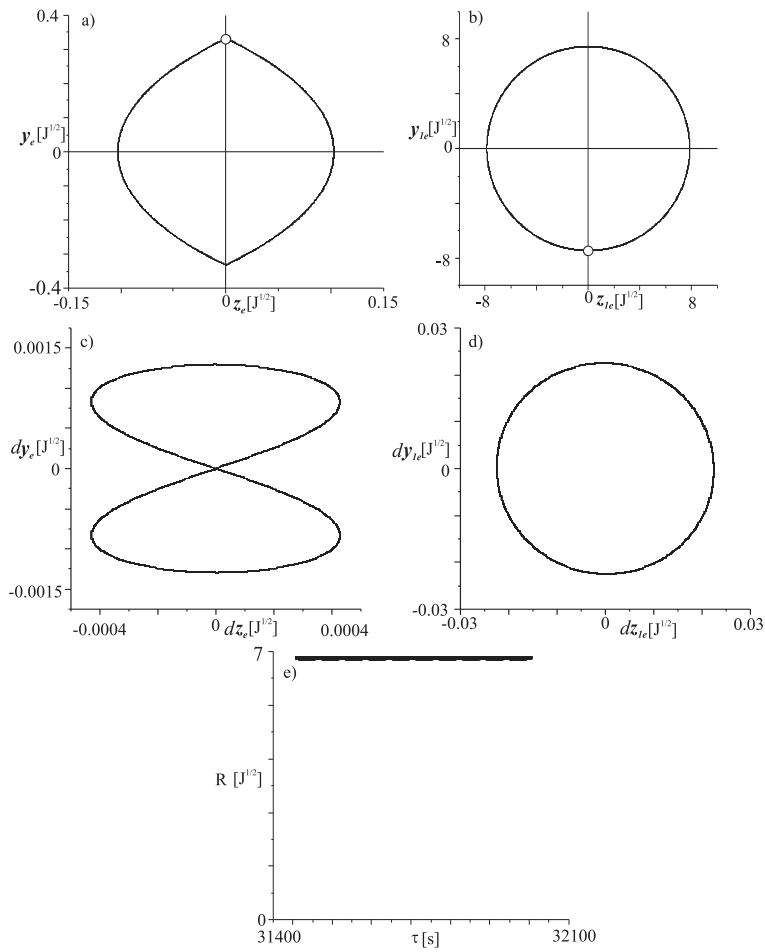


Figure 2 a) The energy plane of the oscillator μ system, b) The energy plane of the oscillator μ_1 system, c), d) Differences of the variables in $d\tau$ intervals, e) Radius of the attractor.
 $\eta = 1, \mu = 1kg, \mu_1 = 0.07kg, \sigma = 1N/m, \sigma_1 = 0.07N/m, c = 0.1Ns/m, c_1 = 0.001Ns/m,$
 $F = 2.5N$

Thus in time of this part of transformation, depending on σ_1 and μ_1 the phase space is just only squeezed or stretched in directions of the given eigenvectors.

In Fig. 2b plane $(z_{1e} : y_{1e})$ can be seen. The projection of the vector on that energy plane shows the energy accumulated in the oscillator μ_1 system. The value of z_{1e} shows the potential energy of the spring σ_1 and the value of y_{1e} – the kinetic energy of the mass μ_1 , and by means of these coordinates the energies can be calculated.

The position of the vector projections on the energy planes $(z_e : y_e)$ and $(z_{1e} : y_{1e})$ at the same moment of time is marked by a small circle.

Note, that difference between these oscillators is one time of magnitude order. Note also from the marked small circles, that system μ_1 oscillates in antiphase to the system μ . External force, that excites oscillator μ is in the same phase as this oscillator. Thus all the energy of the excitation just only flows through the μ system and is intercepted by μ_1 system. What is interesting, the energy of the μ_1 system is constant, what can be seen in Fig. 2(b, d). The energy accumulated in the system flows only between the masses μ and μ_1 systems and the excitation energy is dissipated in the dampers c and c_1 . The energy of the whole system is constant, what can be seen in Fig. 2e. The radius of the trajectory is constant, what means, that the trajectory lays on a surface of the four-dimensional sphere. The radius of this sphere is equal to the square root of the energy accumulated in the whole system.

3. Chaotic energy maps

Dependence of the system dynamics on the excitation force frequency is shown in the Fig. 3. The bifurcation diagrams of z_1 variable in grey, and the largest Lyapunov exponent λ_{max} in black can be seen in it. The value of largest Lyapunov exponent was estimated using Stefański method. It is based on the phenomenon of synchronization of identical systems [13]. Different types of dynamics and bifurcations leading to the chaotic state can be identified in Fig. 3. Chaotic energy flow is shown in Fig. 4(a, b). Intersection of the attractor using Poincare plane was applied. For comparison the same intersection of the attractor in the phase space was shown in Fig. 4(c, d). One can see the same type of dynamics in these charts. What can not be seen in the phase space is, that the oscillator μ accumulates much more energy than oscillator μ_1 . For the same system parameters new kind of maps is presented in Fig. 5. The system energy state on two energy levels can be seen in it. From the geometrical viewpoint it shows projections of the special attractor intersection. Attractor is cut using the sphere with the radius R matched up with the concerned energy level. Intersections by small Fig. 5(a, b) and big Fig. 5(c, d) spheres are presented. The scales of the charts are matched up with the intersection radius R . From these figures one can conclude, that the attractor is flat in directions of z_3 and z_4 . Especially the intersection by the big sphere shows that the attractor cuts the sphere in the “equator” plane, what is the reason of the circles, that can be seen in Fig. 5(a, d). Energy state of the system for an extreme values of the total energy accumulated in the system is shown in Fig. 6.

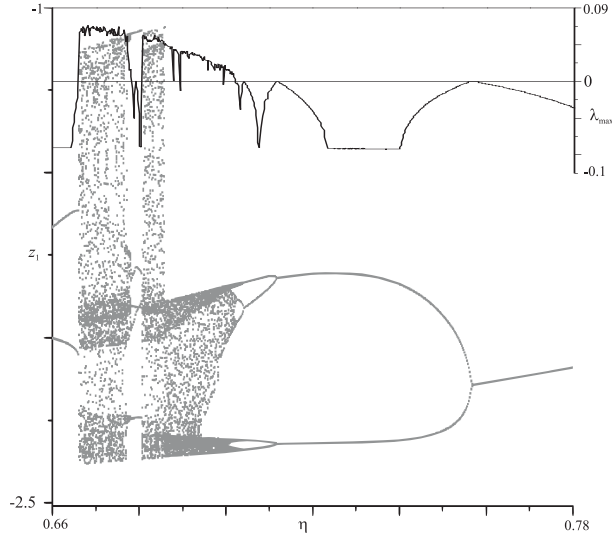


Figure 3 Bifurcation diagram of z_1 (in grey, left) The largest Lyapunov exponent λ_{max} (in black, right) $\mu = 1kg, \mu_1 = 0.07kg,$
 $\sigma = 1N/m, \sigma_1 = 0.07N/m, c = 0.1Ns/m, c_1 = 0.1Ns/m, F = 2.5N$

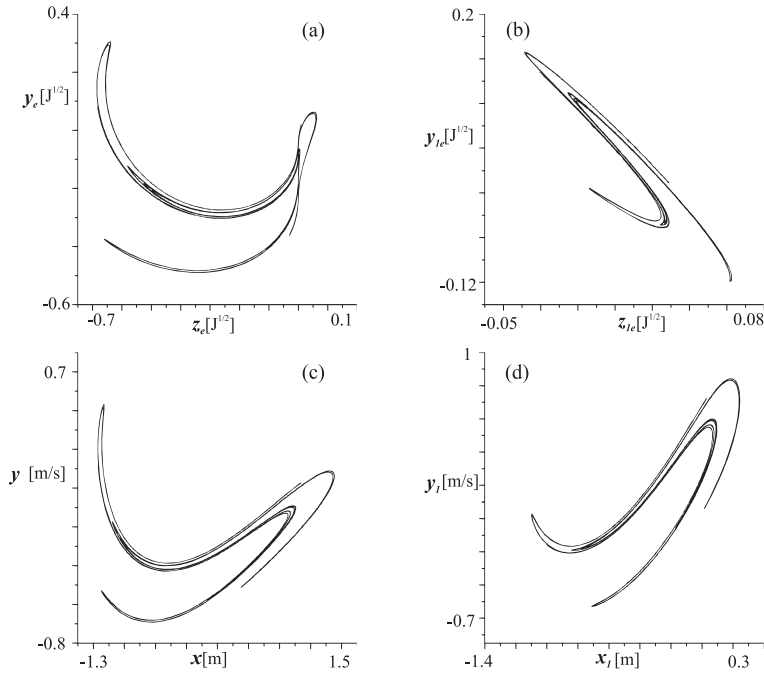


Figure 4 a) Energy plane of the oscillator μ system, b) Energy plane of the oscillator μ_1 system, c) Phase plane of the oscillator μ system, b) Phase plane of the oscillator.
 $\eta = 0.6025, \mu = 1kg, \mu_1 = 0.07kg, \sigma = 1N/m, \sigma_1 = 0.07N/m,$
 $c = 0.1Ns/m, c_1 = 0.1Ns/m, F = 2.5N$

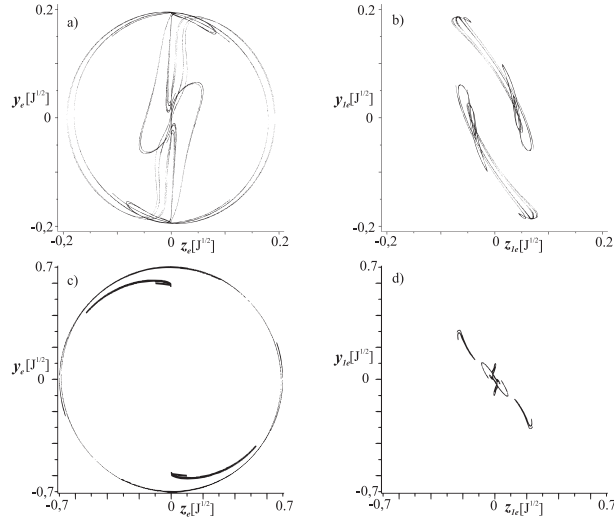


Figure 5 Sphere cut radius: a), b): $R = 0.2[J1/2]$, c), d): $R = 0.7[J1/2]$ a), c) Energy plane of the oscillator m system b), d) Energy plane of the oscillator m_1 system.
 $\eta = 0.6025, \mu = 1kg, \mu_1 = 0.07kg, \sigma = 1N/m, \sigma_1 = 0.07N/m,$
 $c = 0.1Ns/m, c_1 = 0.1Ns/m, F = 2.5N$

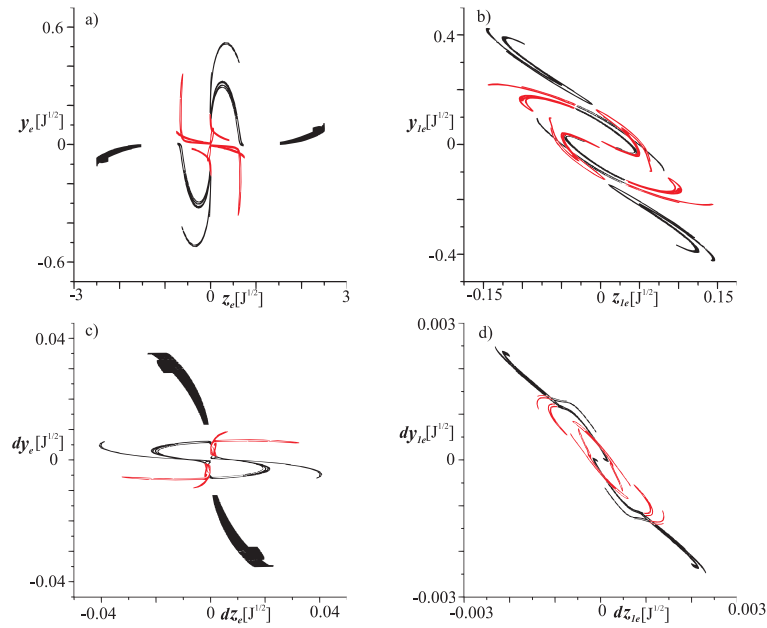


Figure 6 Energy max cut: a) Energy plane of the oscillator m system, b) Energy plane of the oscillator m_1 system, c), d) Differences of the variables in dt intervals.
 $\mu = 1kg, \mu_1 = 0.07kg, \sigma = 1N/m, \sigma_1 = 0.07N/m, c = 0.1Ns/m,$
 $c_1 = 0.1Ns/m, F = 2.5N$

3.1. Application of the energy - Vector analysis in the system with impacts

Consider the system shown in Fig. 7. For some ranges of the system parameters, it works as an impact damper of the motion of the oscillator μ [9], [10]. The system consists of three oscillators. The external harmonic force excites the oscillator μ . It is joined with the classical dynamical absorber μ_1 . This absorber is allowed to collide with the third oscillator μ_2 .

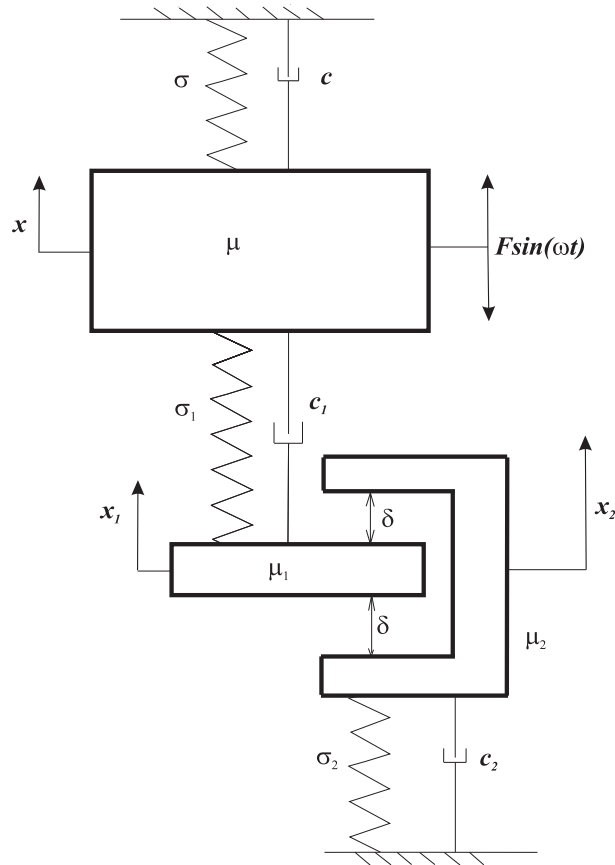


Figure 7 The physical model of the system

In the periods between the impacts the mathematical model of the system is given by six differential equations of the first order:

$$\begin{cases} \dot{x} = y \\ \dot{y} = (F \sin \eta \tau - cy - c_1(y - y_1) - \sigma x - \sigma_1(x - x_1)) \cdot \frac{1}{\mu} \\ \dot{x}_1 = y_1 \\ \dot{y}_1 = (-c_1(y_1 - y) - \sigma_1(x_1 - x)) \cdot \frac{1}{\mu_1} \\ \dot{x}_2 = y_2 \\ \dot{y}_2 = (-c_2 y_2 - \sigma_2 x_2) \cdot \frac{1}{\mu_2} \end{cases} \quad (3)$$

where:

- μ, μ_1, μ_2 – masses
- $\sigma, \sigma_1, \sigma_2$ – stiffness coefficients of the springs
- c, c_1, c_2 – damping coefficients
- F – amplitude of the external excitation force
- ω – frequency of the external excitation force

$$\eta = \frac{\omega}{\alpha}, \quad \tau = \alpha t, \quad \alpha = \sqrt{\frac{\sigma}{\mu}} \quad (4)$$

The impact between the dynamical and impact absorbers is put into the mathematical model over the restitution coefficient r .

The phase vector in the standard phase space R^6 is represented by six components:

$$x; \quad y; \quad x_1; \quad y_1; \quad x_2; \quad y_2 \quad (5)$$

Transform the phase space as follows:

- Instead of the displacement coordinates $\vec{x}$, $\vec{x}_1$, $\vec{x}_2$, take the deflections of the springs.

$$z = x; \quad z_1 = x_1 - x; \quad z_2 = x_2 \quad (6)$$

- Depending on the coefficients σ_i and μ_i squeeze and stretch the space in directions of z_i and y_i .

In these two steps a new energy space can be obtained.

In the first step, one obtains a new space V with the basis E :

$$E = (e_z, e_y, e_{1z}, e_{1y}, e_{2z}, e_{2y}) \quad (7)$$

where:

$$\begin{aligned} e_x &= [1, 0, 0, 0, 0, 0]^T, & e_y &= [0, 1, 0, 0, 0, 0]^T \\ e_{1x} &= [0, 0, 1, 0, 0, 0]^T, & e_{1y} &= [0, 0, 0, 1, 0, 0]^T \\ e_{2x} &= [0, 0, 0, 0, 1, 0]^T, & e_{2y} &= [0, 0, 0, 0, 0, 1]^T \end{aligned} \quad (8)$$

Change the basis vectors of the space V . Then, the new energy basis E_N of this space is:

$$E_N = (b_z, b_y, b_{1z}, b_{1y}, b_{2z}, b_{2y}) \quad (9)$$

where:

$$\begin{aligned} b_z &= [(\sqrt{\sigma})^{-1}, 0, 0, 0, 0, 0]^T, & b_y &= [0, (\sqrt{\mu})^{-1}, 0, 0, 0, 0]^T \\ b_{1z} &= [0, 0, (\sqrt{\sigma_1})^{-1}, 0, 0, 0]^T, & b_{1y} &= [0, 0, 0, (\sqrt{\mu_1})^{-1}, 0, 0]^T \\ b_{2z} &= [0, 0, 0, 0, (\sqrt{\sigma_2})^{-1}, 0]^T, & b_{2y} &= [0, 0, 0, 0, 0, (\sqrt{\mu_2})^{-1}]^T \end{aligned} \quad (10)$$

The transition matrix $A_{E_N \leftarrow E}$ from the basis E to the basis E_N of the energy space takes the form:

$$A_{E_N \leftarrow E} = \begin{bmatrix} \sqrt{\sigma} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{\mu} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\sigma_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\mu_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\sigma_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\mu_2} \end{bmatrix} \quad (11)$$

The coordinates of the vector $\vec{v}_e = [z_e, y_e, z_{1e}, y_{1e}, z_{2e}, y_{2e}]_{E_N}^T$ with respect to the energy basis E_N can be obtained from the vector $\vec{v}$ with respect to the basis E , using the transition matrix

$$v_e = A_{E_N \leftarrow E} v. \quad (12)$$

Then

$$v_e^- = [\sqrt{\sigma}z, \sqrt{\mu}y, \sqrt{\sigma_1}z_1, \sqrt{\mu_1}y_1, \sqrt{\sigma_2}z_2, \sqrt{\mu_2}y_2]_{E_N}^T \quad (13)$$

The norm of the vector $\vec{v}_e$ in the energy space with the energy product is as follows:

$$\begin{aligned} |v_e| &= \sqrt{\langle v_e, v_e \rangle} \\ &= \sqrt{\frac{1}{2} [\sqrt{\sigma}(z)^2 + (\sqrt{\mu}y)^2 + (\sqrt{\sigma_1}z_1)^2 + (\sqrt{\mu_1}y_1)^2 + (\sqrt{\sigma_2}z_2)^2 + (\sqrt{\mu_2}y_2)^2]} \\ &= 7\sqrt{\frac{\sigma z^2}{2} + \frac{\mu y^2}{2} + \frac{\sigma_1 z_1^2}{2} + \frac{\mu_1 y_1^2}{2} + \frac{\sigma_2 z_2^2}{2} + \frac{\mu_2 y_2^2}{2}} \\ &= \sqrt{E_p + E_k + E_{p1} + E_{k1} + E_{p2} + E_{k2}} \end{aligned} \quad (14)$$

where:

- E_p –the potential energy accumulated in the spring σ ,
- E_k –the kinetic energy of the mass μ ,
- E_{p1} –the potential energy accumulated in the spring σ_1 ,
- E_{k1} –the kinetic energy of the mass μ_1 ,
- E_{p2} –the potential energy accumulated in the spring σ_2 ,
- E_{k2} –the kinetic energy of the mass μ_2 .

The mathematical model of the system in the transformed space is given by following differential equations:

$$\begin{cases} \dot{z}_e = \sqrt{\frac{\sigma}{\mu}} y_e \\ \dot{y}_e = -\frac{c}{\mu} y_e - \frac{c_1}{\mu} y_e + \frac{c_1}{\sqrt{\mu\mu_1}} y_{1e} - \sqrt{\frac{\sigma}{\mu}} z_e + \sqrt{\frac{\sigma_1}{\mu}} z_{1e} + \frac{F}{\mu} \sin \eta \tau \\ \dot{z}_{1e} = \sqrt{\frac{\sigma_1}{\mu_1}} y_{1e} - \sqrt{\frac{\sigma}{\mu}} y_e \\ \dot{y}_{1e} = -\sqrt{\frac{\sigma_1}{\mu_1}} z_{1e} - \frac{c_1}{\mu_1} y_{1e} + \frac{c_1}{\sqrt{\mu\mu_1}} y_e \\ \dot{z}_{2e} = \sqrt{\frac{\sigma_2}{\mu_2}} y_{2e} \\ \dot{y}_{2e} = -\sqrt{\frac{\sigma_2}{\mu_2}} z_{2e} - \frac{c_2}{\mu_2} y_{2e} \end{cases} \quad (15)$$

where:

$$\begin{aligned} z_e &= \text{sign}(x) \sqrt{E_p} \\ y_e &= \text{sign}(y) \sqrt{E_k} \\ z_{1e} &= \text{sign}(x_1) \sqrt{E_{p1}} \\ y_{1e} &= \text{sign}(y_1) \sqrt{E_{k1}} \\ z_{2e} &= \text{sign}(x_2) \sqrt{E_{p2}} \\ y_{2e} &= \text{sign}(y_2) \sqrt{E_{k2}} \end{aligned} \quad (16)$$

3.1.1. Impact map

Let π be the plane determined by the basis vectors (Fig.8):

$$b_{1y} = [0, 0, 0, (\sqrt{\mu_1})^{-1}, 0, 0]^T \quad (17)$$

$$b_{2y} = [0, 0, 0, 0, (\sqrt{\mu_2})^{-1}]^T \quad (18)$$

The vectors y_{1e} and y_{2e} correspond to the kinetic energies of the masses μ_1 and μ_2 , respectively. The position of the vector is marked on this plane just before and after impact. Thus, this plane is a special kind of the impact map. In order to make this map clearer, note that it represents only the points for different types of impact. As the choice criterion,

$$y_{1e} < y_{2e}$$

has been applied. The consideration of the position and norm of the vector on this map allows one to conclude about the energy flow between the dynamical and impact absorbers and also about the energy dissipation during each collision. The energy dissipation is included into the mathematical model of the system over the restitution coefficient r .

Let $v_{e\pi}^{\rightarrow}$ be the projection of the $\vec{v}_e$ on the plane π just before the impact, and $v_{e\pi}^{\rightarrow'}$ after it.

The transformation of the vector $v_{e\pi}$ during each impact is given by the matrix

$$A_{EN}: \\ v_{e\pi}^{\rightarrow'} = A_{EN} v_{e\pi}^{\rightarrow}$$

where:

$$A_{E_N} = \begin{bmatrix} \frac{\mu-r}{\mu+1} & \frac{1+r}{\mu+1} \\ \frac{\mu+\mu r}{\mu+1} & \frac{1-\mu r}{\mu+1} \end{bmatrix} \quad (19)$$

$$\mu = \frac{\mu_1}{\mu_2}$$

r – the restitution coefficient.

In the case $\mu=1$ the eigenvalues of the A_{E_N} matrix are:

$$\lambda_1 = -r, \quad \lambda_2 = 1 \quad (20)$$

and the eigenvectors are:

$$\vec{w}_1 = [-1, 1]^T, \quad \vec{w}_2 = [1, 1]^T \quad (21)$$

respectively.

The directions of the eigenvectors are shown in Fig.8. Note that during impact the vector $\vec{v}_{e\pi}$ is transformed only in the direction given by the eigenvector $\vec{w}_1$.

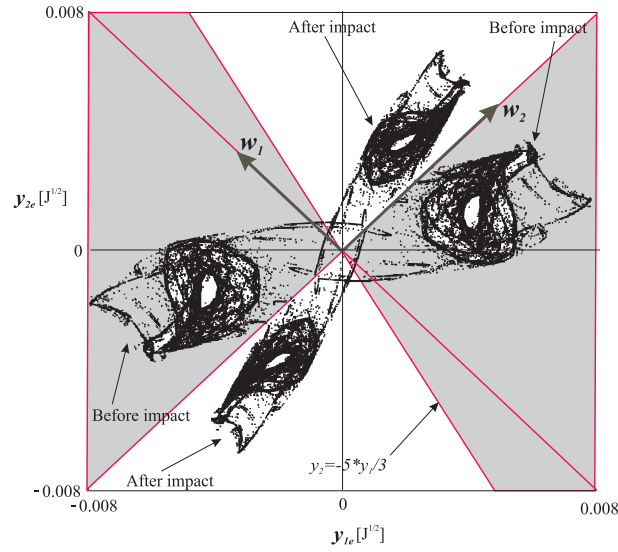


Figure 8 The impact map: $m = 1[kg]$, $m_1 = m_2 = 0.1[kg]$, $s = 1[N/m]$, $s_1 = s_2 = 0.1[N/m]$, $c = 0.04[Ns/m]$, $c_1 = 0.01[Ns/m]$, $c_2 = 0.02[Ns/m]$, $F = 0.002[N]$, $d = 0.0216[m]$, $r = 0.5$, $h = 1.165$

The energy dissipation in time of each collision can be found from the change of the norm of the energy space vector:

$$|\vec{v}_{e\pi}| - |\vec{v}'_{e\pi}| \quad (22)$$

The maximum dissipation of the energy takes place when the vector $\vec{v}_{e\pi}$ has the same direction as the eigenvector $\vec{w}_1$. Then

$$v'_{e\pi} = \lambda_1 \cdot v_{e\pi} \quad (23)$$

Taking in the consideration that

$$|\vec{v}_{e\pi}| = \sqrt{E_{K1} + E_{K2}} \quad (24)$$

where:

E_{K1} –the kinetic energy of the mass μ_1 before impact,
 E_{K2} –the kinetic energy of the mass μ_2 before impact,
 and

$$|v'_{e\pi}| = \sqrt{E'_{K1} + E'_{K2}} \quad (25)$$

where:

E'_{K1} –the kinetic energy of the mass μ_1 after impact,
 E'_{K2} –the kinetic energy of the mass μ_2 after impact,
 one can find easily that the energy relation after and before impact assumes the form:

$$\frac{E'_{K1} + E'_{K2}}{E_{K1} + E_{K2}} = \lambda_1^2 = r^2. \quad (26)$$

The closer the direction of the vector $\vec{v}_{e\pi}$ is to the second eigenvector $\vec{w}_2$, the less energy dissipation occurs. In the case when the directions of $\vec{v}_{e\pi}$ and $\vec{w}_2$ are almost the same, there is almost no energy dissipation during the collision. The velocities of the oscillators μ_1 and μ_2 are almost equal then, and in practice we do not know if impact occurs or not. It is so called grazing collision and it causes chaotic motion of the system. These special points can be seen in Fig.8 as common points of the before and after impact attractors.

The transformation matrix A_{EN} allows one also to divide the impact map into two kinds of fields: the first one for the case when the energy flows during impact from the dynamical to impact absorber and the second when the energy flows in the opposite direction.

Consider the matrix A_{EN} in the case $\mu_1 = \mu_2$:

$$A_{EN} = \begin{bmatrix} \frac{1-r}{2} & \frac{1+r}{2} \\ \frac{1+r}{2} & \frac{1-r}{2} \end{bmatrix} \quad (27)$$

Let:

$$v_{e\pi} = [v_1, v_2]^T \quad \text{and} \quad v'_{e\pi} = [v'_1, v'_2]^T \quad (28)$$

Then

$$v'_{e\pi} = A_{EN} \cdot v_{e\pi} \quad (29)$$

so

$$\begin{bmatrix} v'_1 \\ v'_2 \end{bmatrix} = \begin{bmatrix} \frac{1-r}{2} & \frac{1+r}{2} \\ \frac{1+r}{2} & \frac{1-r}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \quad (30)$$

The energy flow from the dynamical to impact absorber is given by the condition:

$$-v_1 \leq v'_1 \leq v_1 \quad (31)$$

Consider :

- $v'_1 = v_1$ The condition is satisfied when the directions of $v_{e\pi}$ and the eigenvector $\vec{w}_2$ are the same.
- $v'_1 = -v_1$

then

$$-v_1 = \frac{1-r}{2}v_1 + \frac{1+r}{2}v_2$$

and we obtain

$$v_2 = -\frac{3-r}{1+r} \cdot v_1 \quad (32)$$

For the case shown in Fig.4 $r = 0.5$ and then

$$v_2 = -\frac{5}{3} \cdot v_1 \quad (33)$$

As a result, the energy flow from the dynamical to the impact absorber occurs:

if $v_1 > 0$

$$-\frac{5}{3}v_1 \leq v_2 \leq v_1 \quad (34)$$

if $v_1 < 0$

$$v_1 \leq v_2 \leq -\frac{5}{3}v_1. \quad (35)$$

The field of energy flow in this direction is marked in Fig.8 by the grey colour. It can be seen that during impacts in the considered case energy flows in both directions, but definitely more often from the dynamical to the impact absorber.

4. Conclusions

The way of transformation of the phase space to obtain the energy space has been shown. It has been proved that this new kind of space allows for concluding about the energy state of a vibrating system. The norm of the vector in that space is equal to the square root of the total energy accumulated in the system. The projection of the vector space on energy subspaces show the amount of the energy that accumulates in some parts of the system. It has been shown that using this kind of spaces, all aspects of the kind of motion can be concluded about, like from the phase space and, moreover, the energy state, accumulation, flow and dissipation can be observed. Different types of the energy flow were shown. New kind of maps was introduced. It was shown, that the energy space allows for a new, geometrical view on energy changes in vibrating systems.

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