Mechanics and Mechanical Engineering
Vol. 11, No 1 (2007) 49–64

Efect of Reference Temperature on the Modulus of Elasticity in Case of 2-D Generalized Thermal Shock Problem for a Half-Space

Mohamed I. A. Othman
Faculty of Education, Department of Mathematics,
Salalah-211, P.O. Box 2801, Sultanate of Oman
e-mail:m_i_othman@yahoo.com

Received (10 October 2006)
Revised (15 March 2007)
Accepted (8 May 2007)

The model of the equations of two-dimensional coupled problem in thermo-elasticity for a thermally half-space solid whose surface is subjected to a thermal shock is established. The problem is in the context of the Green and Lindsay's generalized thermoelasticity theory with two relaxation times in an isotropic medium with the modulus of elasticity being dependent on the reference temperature. The normal mode analysis is used to obtain the exact expressions for the temperature, the displacement and thermal stress components. The resulting formulation is applied to two kinds of boundary conditions. Numerical results are illustrated graphically for each case considered. Comparison is carried out with the results predicted by the coupled theory and with the case where the modulus of elasticity is independent of temperature.

Keywords: Reference temperature, the modulus of elasticity, thermal shock, normal mode analysis.

1. Introduction

In the postwar years we have seen a rapid development of thermoelasticity stimulated by various engineering sciences Nowacki [1]. Most of the investigations were done under the assumption of the temperature-independent material properties, which limit the applicability of the solutions obtained to certain ranges of temperature. At high temperature the material characteristics such as the modulus of elasticity, the Poisson's ratio, the coefficient of thermal expansion and the thermal conductivity are no longer constants. In recent years due to progress in various fields in science and technology the necessity of taking into consideration the real behavior of the material characteristics became actual. In some investigations they were taken as functions of coordinates, Tanigawa [2] and Ootao et al. [3].

In this work we consider that the modulus of elasticity is the only temperature-dependent material parameter. The experimental data by Manson [4] show that the changes in Poisson's ratio and the coefficient of linear thermal expansion due to the high temperature can be neglected.

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observations. First, the equation of heat conduction of this theory does not contain any elastic terms, contrary to the fact that elastic changes produce heat effects. Second, the heat equation is of parabolic type, predicting infinite speeds of propagation for heat waves.

Biot [5] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories of diffusion type predict infinite speeds of propagation for heat waves contrary to physical observations. The theory of generalized thermoelasticity with two relaxation times was first introduced by Müller [6]. A more explicit version was then introduced by Green and Laws [7], Green and Lindsay [8] and independently by uhubi [9]. In this theory the temperature rates are considered among the constitutive variables. This theory also predicts finite speed of propagation as in Lord and Shulman's theory of generalized thermoelasticity with one relaxation time [10]. It differs from the latter in that Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Erbay and Şuhubi [11] studied wave propagation in finite cylinders. Ignaczak [12] studied a strong discontinuity wave and obtained a decomposition theorem for this theory [13]. This theory was extended by Dhaliwal and Sherief [14] to general anisotropic media in the presence of heat sources. Dhaliwal and Rokne have solved a thermal shock problem in [15]. Ezzat and Othman [16] constructed a model of two-dimensional equations of generalized magneto-thermoelasticity with two relaxation times in a perfectly conducting medium. Ezzat et al. [17] studied the effect of reference temperature on thermal stress distribution for the one-dimensional problems.

In the present work a comparison is made with the results predicted by the coupled theory and with the case where the modulus of elasticity is independent of temperature.

2. Formulation of the problem

We consider an isotropic elastic medium with temperature-dependent mechanical properties. The constitutive law for the theory of generalized thermoelasticity with two relaxation times is (Ezzat et al. [18])

$$\sigma_{ij} = \lambda e \delta_{i,j} + 2\mu\varepsilon_{ij} - \gamma[(T - T_o) + \nu_o \dot{T}]\delta_{ij} \tag{1}$$

The heat conduction equations

$$kT_{i,i} = \rho C_E(\dot{T} + \tau_o \ddot{T}) + \gamma T_o \dot{u}_{i,i} \tag{2}$$

The strain-displacement relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad e = u_{i,i} \tag{3}$$

We assume that

$$E = E_o f(T), \quad \lambda = E_o \lambda_o f(T), \quad \mu = E_o \mu_o f(T) \quad and \quad \gamma = E_o \gamma_o f(T) \tag{4}$$

where$f(T)$ is a given non-dimensional function of temperature. In the case where the modulus of elasticity is temperature independent, $f(T) \equiv 1$, and $E = E_o$.

The equations of motion are

$$\begin{aligned} \rho \ddot{u}_i &= E_o f[(\lambda_o + \mu_o)e_{,i} + \mu_o \nabla^2 u_i - \gamma_o (T + \nu_o \dot{T})_{,i}] \\ &\quad + E_o f_{,j}[\lambda_o e \delta_{ij} + 2\mu_o \varepsilon_{ij} - \gamma_o (T - T_o + \nu_o \dot{T})\delta_{ij}] \end{aligned} \tag{5}$$

Now we introduce the following non-dimensional variables

$$\begin{gathered} x^* = c_o \eta_o x, \quad y^* = c_o \eta_o y, \quad u_i^* = c_o \eta_o u_i, \quad t^* = c_o^2 \eta_o t \\ \tau_o^* = c_o^2 \eta_o \tau_o, \quad \nu_o^* = c_o^2 \eta_o \nu_o, \quad \theta^* = \frac{\gamma_o E_o}{\rho c_o^2}(T - T_o), \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\rho c_o^2} \end{gathered} \tag{6}$$

Omitting the asterisk for convenience, we have

$$\sigma_{ij} = [(2\beta - 1)e\delta_{ij} + (1 - \beta)(u_{i,j} + u_{j,i}) - (\theta + \nu_o \dot{\theta})\delta_{ij}]f(\theta) \tag{7}$$

$$\begin{aligned} \ddot{u}_i &= [\beta e_{,i} + (1 - \beta)\nabla^2 u_i - (\theta_{,i} + \nu_o \dot{\theta_{,i}}]f(\theta) + \\ &\quad + (2\beta - 1)e f_{,i} + (1 - \beta)(u_{i,j} + u_{j,i})f_{,j} - (\theta + \nu_o \dot{\theta})f_{,i} \end{aligned} \tag{8}$$

The heat conduction Eq.(2) by using Eq. (3) becomes:

$$\nabla^2 \theta = (\dot{\theta} + \tau_o \ddot{\theta}) + \varepsilon_1 f(\theta)\delta_o \dot{e} \tag{9}$$

We consider the special case in which

$$|T - T_o| << 1, \quad 0 \le \delta_o \le 1 \quad and \quad f(\theta) = (1 - \alpha^* T_o)$$

The equations of motion in two-dimension take the form

$$\alpha \ddot{u} = \nabla^2 u + \beta \frac{\partial^2 v}{\partial x \partial y} - \beta \frac{\partial^2 u}{\partial y^2} - (1 + \nu_o \frac{\partial}{\partial t})\frac{\partial \theta}{\partial x} \tag{10}$$

$$\alpha \ddot{v} = \nabla^2 v + \beta \frac{\partial^2 u}{\partial x \partial y} - \beta \frac{\partial^2 v}{\partial x^2} - (1 + \nu_o \frac{\partial}{\partial t})\frac{\partial \theta}{\partial y} \tag{11}$$

The equation of heat conduction becomes

$$\nabla^2 \theta = (\dot{\theta} + \tau_o \ddot{\theta}) + \varepsilon \dot{e} \tag{12}$$

$$\alpha \sigma_{xx} = \frac{\partial u}{\partial x} + (2\beta - 1)\frac{\partial v}{\partial y} - (1 + \nu_o \frac{\partial}{\partial t})\theta \tag{13}$$

$$\alpha \sigma_{yy} = \frac{\partial v}{\partial y} + (2\beta - 1)\frac{\partial u}{\partial x} - (1 + \nu_o \frac{\partial}{\partial t})\theta \tag{14}$$

$$\alpha \sigma_{xy} = (1 - \beta)(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \tag{15}$$

$$\alpha \sigma_{zz} = (2\beta - 1)e - (1 + \nu_o \frac{\partial}{\partial t})\theta \tag{16}$$

where

$$\alpha = \frac{1}{(1-\alpha^* T_o)}, \qquad \varepsilon = \varepsilon_1 \delta_o (1-\alpha^* T_o) \tag{17}$$

Differentiating Eq. (10) with respect to x and (11) with respect to y, then adding we arrive at

$$[\nabla^2 - \alpha\frac{\partial^2}{\partial t^2}]e = (1+\nu_o\frac{\partial}{\partial t})\nabla^2\theta \tag{18}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace's operator in two-dimensional.

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$[u, v, e, \theta, \sigma_{ij}](x, y, t) = [u^*(x), v^*(x), e^*(x), \theta^*(x), \sigma^*_{ij}(x)]exp(\omega t + iay) \tag{19}$$

where ω is the complex time constant and "a" is the wave number in the y-direction.

Using Eqs. (19), (12) and (18) take the form

$$[D^2 - a^2 - \omega(1+\tau_o\omega)]\theta^*(x) = \varepsilon\omega e^*(x) \tag{20}$$

$$[D^2 - a^2 - \alpha\omega^2]e^*(x) = (1+\nu_o\omega)(D^2-a^2)\theta^*(x) \tag{21}$$

where $D = \frac{\partial}{\partial x}$.

Eliminating $\theta^*(x)$ between Eqs. (20) and (21), we obtain the following fourth-order partial differential equation satisfied by $e^*(x)$

$$(D^4 - AD^2 + B)e^*(x) = 0 \tag{22}$$

where

$$A = 2a^2 + b_1 \tag{23}$$

$$B = a^4 + a^2 b_1 + b_2 \tag{24}$$

$$b_1 = [\alpha\omega^2 + \omega_2 + \varepsilon\omega_1] \tag{25}$$

$$b_2 = \alpha\omega^2\omega_2 \tag{26}$$

$$\omega_1 = \omega(1+\nu_o\omega), \ \omega_2 = \omega(1+\tau_o\omega) \tag{27}$$

In a similar manner we arrive at

$$(D^4 - AD^2 + B)\theta^*(x) = 0 \tag{28}$$

Eq. (22) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)e^*(x) = 0 \tag{29}$$

where, $k_i^2 (i = 1, 2)$ are the roots of the following characteristic equation

$$k^4 - Ak^2 + B = 0 \tag{30}$$

$$k_{1,2}^2 = \frac{1}{2}[A \pm \sqrt{A^2 - 4B}] \tag{31}$$

The solution of Eq. (29) has the form:

$$e^*(x) = \sum_{i=1}^{2} e_i^*(x) \tag{32}$$

where $e_i^*(x)$ is the solution of of the equation

$$(D^2 - k_i^2)e_i^*(x) = 0,\ i = 1, 2 \tag{33}$$

Substituting from Eq. (33) which is bounded as $x \to \infty$, is given by

$$e_i^*(x) = R_i(a, \omega)e^{-k_i x} \tag{34}$$

Thus, $e^*(x)$ has the form

$$e^*(x) = \sum_{i=1}^{2} R_i(a, \omega)e^{-k_i x}. \tag{35}$$

In a similar manner, we get

$$\theta^*(x) = \sum_{i=1}^{2} R_i'(a, \omega)e^{-k_i x} \tag{36}$$

where $R_i$andR_i' are some parameters depending on a and ω.

Substituting from Eqs. (35) and (36) into the Eq. (20) we obtain

$$R_i^{'}(a, \omega) = \frac{\varepsilon\omega}{[k_i^2 - a^2 - \omega_2]} R_i \tag{37}$$

Substituting from Eq. (37) into the Eq. (36) we obtain

$$\theta^*(x) = \sum_{i=1}^{2} \frac{\varepsilon\omega}{[k_i^2 - a^2 - \omega_2]} R_i e^{-k_i x} \tag{38}$$

In order to obtain the displacement u, in terms of Eq. (19) and using the following equation

$$e^* = Du^* + iav^* \tag{39}$$

from Eq. (10)

$$[(1-\beta)D^2 - (1-\beta)a^2 - \alpha\omega^2]u^*(x) = (1 + \nu_o\omega)D\theta^* - \beta De^* \tag{40}$$

$$(D^2 - m^2)u^*(x) = \frac{1}{(1-\beta)}[(1 + \nu_o\omega)D\theta^* - \beta De^*] \tag{41}$$

where

$$m^2 = a^2 + \frac{\alpha\omega^2}{(1-\beta)} \tag{42}$$

Substituting from Eqs. (35) and (37) into Eq. (41), we get

$$(D^2 - m^2)u^*(x) = \frac{1}{(1-\beta)} \sum_{i=1}^{2} k_i(\beta - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]})R_i e^{-k_i x} \tag{43}$$

The solution of Eq. (43), bounded as $x \to \infty$, is given by

$$u^*(x) = Ge^{-mx} + \frac{1}{(1-\beta)} \sum_{i=1}^{2} \frac{k_i}{(k_i^2 - m^2)}(\beta - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]})R_i e^{-k_i x} \tag{44}$$

From Eq. (35) we can obtain

$$v^* = \frac{-i}{a}[e^* - Du^*] \tag{45}$$

Substituting from Eqs. (35) and (44) into Eq. (45), we get

$$\begin{aligned} v^*(x) &= \frac{-i}{a}\{mGe^{-mx} + \sum_{i=1}^{2}[1 + \frac{k_i^2}{(1-\beta)(k_i^2 - m^2)}(\beta \\ &\quad - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]})]R_i e^{-k_i x}\} \end{aligned} \tag{46}$$

In terms of Eq. (19), substituting from Eqs. (38), (44) and (46) into Eqs. (13)-(16)

$$\begin{aligned} \sigma_{xx}^*(x) &= \frac{1}{\alpha}\{2m(\beta - 1)Ge^{-mx} - \sum_{i=1}^{2}[\frac{2\beta m^2}{(k_i^2 - m^2)} \\ &\quad + 1 - \frac{\varepsilon\omega_1(k_i^2 + m^2)}{(k_i^2 - m^2)[k_i^2 - a^2 - \omega_2]}]R_i e^{-k_i x}\} \end{aligned} \tag{47}$$

$$\begin{aligned} \sigma_{yy}^*(x) &= \frac{1}{\alpha}\{2m(1-\beta)Ge^{-mx} + \sum_{i=1}^{2}[1 + \frac{2\beta k_i^2}{(k_i^2 - m^2)} \\ &\quad - \frac{\varepsilon\omega_1(3k_i^2 - m^2)}{(k_i^2 - m^2)[k_i^2 - a^2 - \omega_2]}]R_i e^{-k_i x}\} \end{aligned} \tag{48}$$

$$\begin{aligned} \sigma_{xy}^*(x) &= \frac{i(1-\beta)}{\alpha a}\{(m^2 + a^2)Ge^{-mx} + \sum_{i=1}^{2} k_i[1 \\ &\quad + \frac{(k_i^2 + a^2)}{(1-\beta)(k_i^2 - m^2)}(\beta - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]})]R_i e^{-k_i x}\} \end{aligned} \tag{49}$$

$$\sigma_{zz}^* = \frac{1}{\alpha} \sum_{i=1}^{2}(2\beta - 1 - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]})R_i e^{-k_i x} \tag{50}$$

The normal mode analysis is, in fact, to look for the solution in the Fourier transformed domain. Assuming that all the relations are sufficiently smooth on the real line such that the normal mode analysis of these functions exist. In order to determine the parameters $R_i (i = 1, 2)$and G, we need to consider the boundary conditions at x = 0. We consider two kinds of boundary conditions respectively, and the details are described as the following

Case 1

1. Thermal boundary conditions that the surface of the half-space subected to a thermal shock
$$\theta(0, y, t) = f(y, t) \tag{51}$$
2. displacement boundary condition that the surface of the half-space is rigidly fixed
$$u(0, y, t) = v(0, y, t) = 0 \tag{52}$$

Substituting from the expressions of considered variables into the above boundary conditions we can obain the following equations satisfied by the parameters

$$\sum_{i=1}^{2} \frac{\varepsilon\, \omega}{[\, k_i^2 - a^2 - \omega_2\,]}\; R_i \;= f^*(a, \omega), \tag{53}$$

$$G + \frac{1}{(1-\beta)} \sum_{i=1}^{2} \frac{k_i}{(k_i^2 - m^2)} \left[\beta \;-\; \frac{\varepsilon\, \omega_1}{[\, k_i^2 - a^2 - \omega_2]} \right] R_i \;= 0\,, \tag{54}$$

$$m\, G \,+\, \sum_{i=1}^{2} \left[1 + \frac{k_i^2}{(1-\beta)(k_i^2 - m^2)} \left(\beta \;-\; \frac{\varepsilon\, \omega_1}{[\, k_i^2 - a^2 - \omega_2]} \right) \right] R_i = 0\,. \tag{55}$$

Solving Eqs. (53)-(55), we get the parameters $R_i,\ i = 1, 2$ and G with the following form respectively

$$G = \frac{1}{(\beta - 1)} \sum_{i=1}^{2} \frac{k_i}{(k_i^2 - m^2)} (\beta - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]}) R_i \tag{56}$$

$$R_1 = \frac{f^*(a, \omega) h_2}{s_1 h_2 - s_2 h_1} \tag{57}$$

$$R_2 = -\frac{f^*(a, \omega) h_1}{s_1 h_2 - s_2 h_1} \tag{58}$$

where

$$s_i = \frac{\varepsilon\omega}{[k_i^2 - a^2 - \omega_2]} \tag{59}$$

$$h_i = \frac{m}{(k_i + m)} + \frac{k_i}{(1-\beta)(k_i^2 - m^2)} (1 - \frac{\varepsilon\omega_1}{[k_i^2 - a^2 - \omega_2]}), \quad i = 1, 2 \tag{60}$$

Case 2

1. Thermal boundary conditions that the surface of the half-space subected to a thermal shock
$$\theta(0, y, t) = f(y, t). \tag{61}$$

2. Mechanical boundary condition that the surface of the half-space is traction free

$$\sigma_{xx}(0,y,t)=\sigma_{xy}(0,y,t)=0 \tag{62}$$

Similarly, we can obtain the following equations satisfied by the parameters

$$\sum_{i=1}^{2}\frac{\varepsilon\omega}{[k_i^2-a^2-\omega_2]}R_i=f^*(a,\omega) \tag{63}$$

$$2m(1-\beta)G-\sum_{i=1}^{2}(\frac{2\beta m^2}{(k_i^2-m^2)}+1-\frac{\varepsilon\omega_1(k_i^2+m^2)}{(k_i^2-m^2)[k_i^2-a^2-\omega_2]})R_i=0 \tag{64}$$

$$(m^2+a^2)G+\sum_{i=1}^{2}k_i(1+\frac{(k_i^2+a^2)}{(1-\beta)(k_i^2-m^2)}(\beta-\frac{\varepsilon\omega_1}{[k_i^2-a^2-\omega_2]}))R_i=0 \tag{65}$$

From Eqs. (63)-(65), we get

$$G=\frac{1}{2m(1-\beta)}\sum_{i=1}^{2}(\frac{2\beta m^2}{(k_i^2-m^2)}+1-\frac{\varepsilon\omega_1(k_i^2+m^2)}{(k_i^2-m^2)[k_i^2-a^2-\omega_2]})R_i \tag{66}$$

$$R_1=\frac{f^*(a,\omega)g_2}{s_1g_2-s_2g_1} \tag{67}$$

$$R_2=-\frac{f^*(a,\omega)g_1}{s_1g_2-s_2g_1} \tag{68}$$

where

$$\begin{aligned} g_i \quad &= \quad \frac{m^2+a^2}{2m(\beta-1)}(\frac{2\beta m^2}{(k_i^2-m^2)}+1-\frac{\varepsilon\omega_1}{(k_i^2-m^2)[k_i^2-a^2-\omega 2]}) \\ &\quad +k_i[1+\frac{(k_i^2+a^2)}{(1-\beta)(k_i^2-m^2)}(\beta-\frac{\varepsilon\omega_1}{[k_i^2-a^2-\omega_2]})] \end{aligned} \tag{69}$$

4. Numerical results

The copper material was chosen for the purpose of numerical evaluations. In the caculation process, the material constants necessary to be known can be found by Sherief and Helmy [19].

Since we have $\omega=\omega_o+i\,\zeta$, where i is imaginary unit, $e^{\omega\,t}=e^{\omega_o t}(\,cos\,\zeta\,t+i\,sin\,\zeta\,t\,)$ and for small values of time, we can take $\omega=\omega_o$ (real). The other constants of the problem are taken as: $\varepsilon=0.003$; $\delta_o=0.0199$; $\omega_o=2$; $a=5$; $\nu_o=0.05$; and $\tau_o=0.02$. The computations were carried out for a value of time $t=0.2$. The real part of $\theta(x,y,t)$, $u(x,y,t)$, $\sigma_{xx}(x,y,t)$ and $\sigma_{yy}(x,y,t)$ are caculated in accordance with two different values of α, for $\alpha=1$ when $\alpha^*=0$ and for $\alpha=1.5$ when $\alpha^*=0.0012$ [1/K], (corresponding respectively, to the cases of independence and dependence on temperature, $T_o=273$ [K]. Both values were studied for the two different cases.

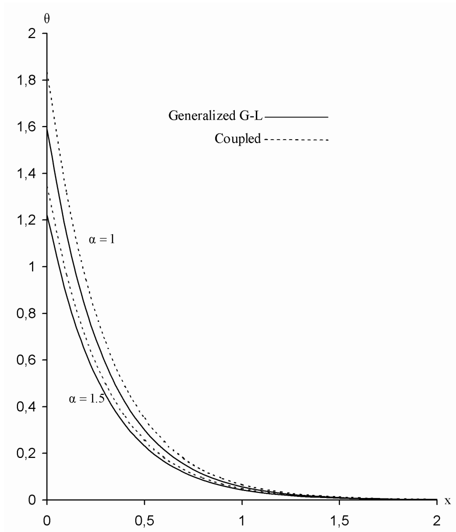

Figure 1 Temperature distribution θ for case 1

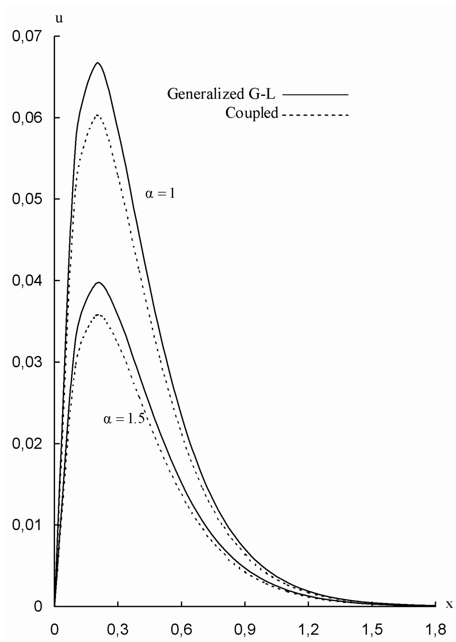

Figure 2 Horizontal displacement distribution u *for case 1*

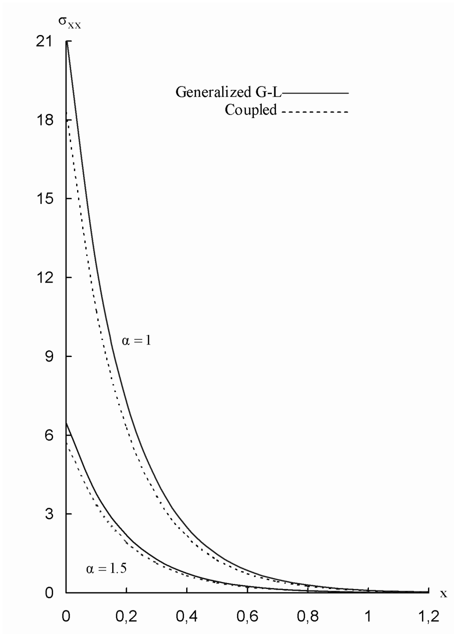

Figure 3 The distribution of stress component σ_{xx} for case 1

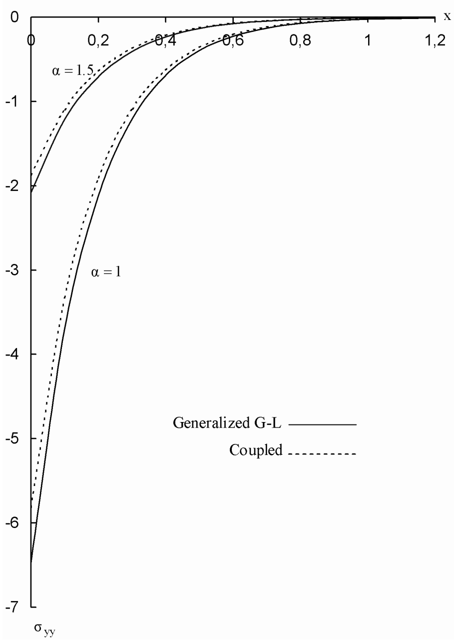

Figure 4 The distribution of stress component σ_{yy} for case 1

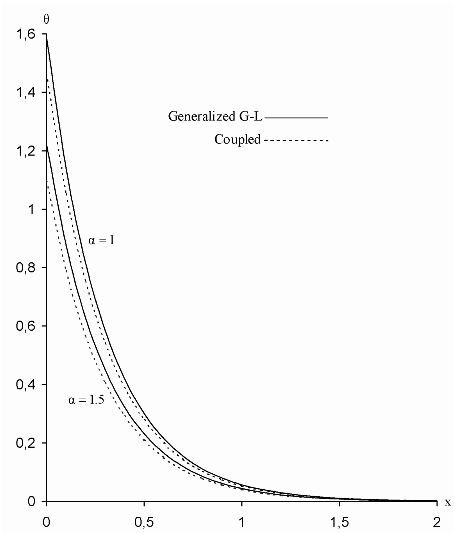

Figure 5 Temperature distribution θ for case 2

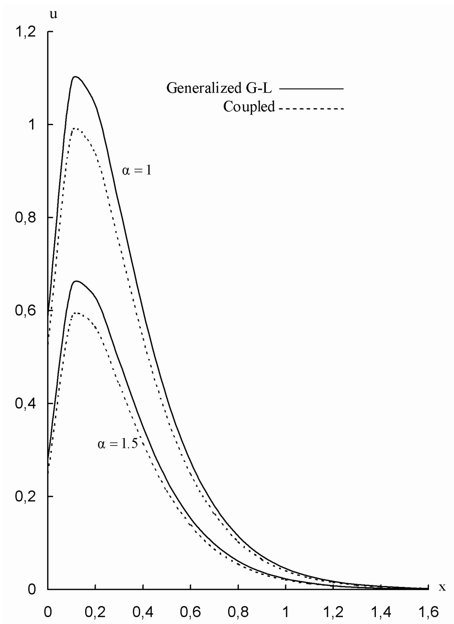

Figure 6 Horizontal displacement distribution u for case 2

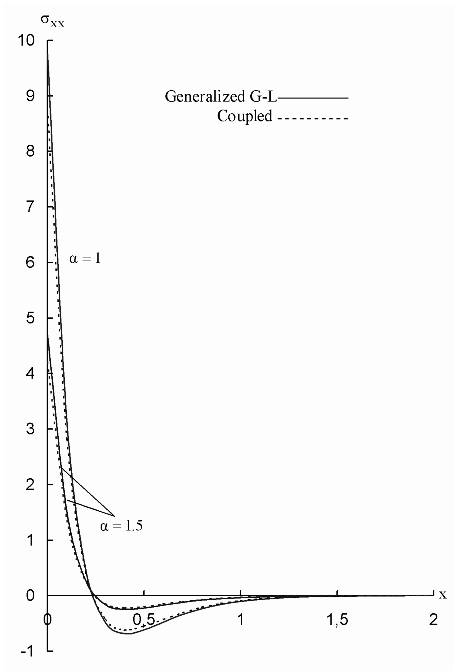

Figure 7 The distribution of stress component σ_{xx} for case 2

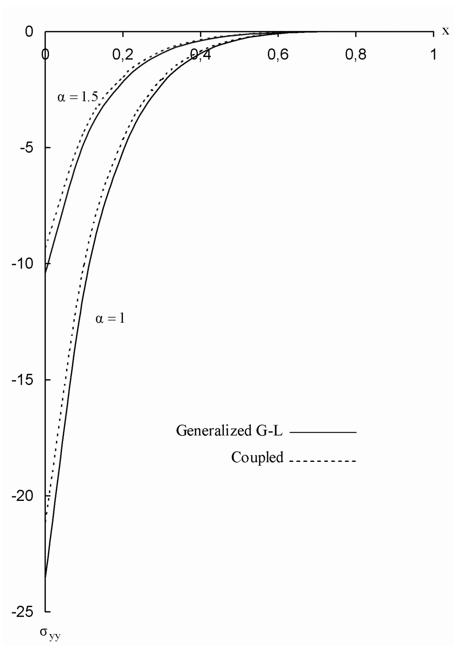

Figure 8 The distribution of stress component σ_{yy} for case 2

Due to the symmertries of geometrical shape and thermal boundary condition, the displacement component $v(x, y, t)$ and the stress component $\sigma_{xy}(x, y, t)$ are zero when $y = 0$. The results are shown in Figs. 1-8, respectively. The graph shows the four curves predicted by different theories of thermoelasticity . In these figures, the solid lines represent the solution in the generalized Green and Lindsay's theory, and the dotted lines represent the solution derived using the coupled equation of heat conduction ($\tau_o = 0\,,\ \nu_o = 0$). It was found that near the surface of the solid, where the boundary conditions dominate, the generalized and the coupled theories give very close results. We also notice that the results for the considered variables when the relaxation times is included in the equation of motion and the heat equation are distinctly different from those when the relaxation times are not mentioned in the equation of motion and the heat equation, because the thermal waves in the Fourier theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non-Fourier case. This demonstrates clearly the difference between the coupled and the generalized theories of thermoelasticity. Also, it was found that the dependent of the modulus of elasticity on the reference temperature effects to decrease the magnitude of the considred variables.

5. Concluding remarks

Due to the complicated nature of the governing equations for generalized thermoelasticity, with two relaxation times, few attempts have been made to solve problems in this field, Nowacki [20]; these attempts utilized approximate method valid for only a specific range of some parameters. In this work, the method of normal mode analysis is introduced for the solution of two different cases in generalized thermoelasticity in which the temperature, displacement and stress are coupled. This method gives exact solutions without any assumed restrictions on temperature, displacement and stress distributions. The normal mode analysis is applied to a wide range of problems in different branches (Othman, [21], Ezzat et al., [17], [18]). It can be applied to boundary-layer problems, which are described by the linearized Navier-Stokes equations in electrohydrodynamic (Othman, [22] and Othman and Ezzat, [23]).

References

[1] **Nowacki,W.** Thermoelasticity, *Addison-Wesley Pub. Com. Inc. London*, 1962.

[2] **Tanigawa, Y.**: Some Basic Thermoelastic Problems for Non- homogeneous Structural Materials, *Appl. Mech. Rev.*, 1995, **48**, 115.

[3] **Ootao, Y., Akai, T., Tanigawa, Y.**: Three-dimensional Transient Thermal Stress Analysis of a Non-homogeneous Hollow Circular Cylinder due to Moving Heat Source in the Axial Direction, *J. Thermal Stresses*, 1995, **18**, 89.

[4] **Manson, S. S.**: Behaviour of Material under Conditions of Thermal Stress. *NACA Report*, 1954, **1170**, 317-350.

[5] **Biot, M.**: Thermoelasticity and Irreversible Thermodynamics, *J. Appl.Phys.*, 1956, **27**, 240-253.

[6] **Müller, I.**: The Coldness, A Universal Function in Thermo-elastic Solids, *Arch. Rat. Mech. Anal.*, 1971, **41**, 319-332.

[7] **Green,A. E., Laws, N.** : On the Entropy Production Inequality, *Arch. Rat. Mech. Anal.*, 1972, **45**, 47-53.

[8] **Green, A. E., Lindsay, K. A.**: Thermoelasticity *J. Elast.*, 1972, **2**, 1-7.

[9] **Şuhubi, E. S.**: Themoelastic Solids. *In Continuum Physics II, Chapter 2, ed. A. C. Eringen. Academic, Press, New York*, 1975.

[10] **Lord, H., Shulman, Y.**: A Generalized Dynamical Theory of Thermo-elasticity. *J. Mech. Phys. Solid*, 1967, **15**, 299-309.

[11] **Erbay, S., Şuhubi, E. S.**: Longitudinal Wave Propagation in Generalized Elastic Cylinder, *J. Therm. Stresses*, 1986, **9**, 279-295.

[12] **Ignaczak, J.**: A Strong Discontinuity Wave in Thermoelasticity with Relaxation Times, *J. Therm. Stresses*, 1985, **8**, 25-40.

[13] **Ignaczak, J.**: Decomposition Theorem for Thermoelasticity with Finite Wave Speeds, *J. Therm. Stresses*, 1978, **1**, 41-52.

[14] **Dhaliwal, R., Sherief, H. H.**: Generalized Thermoelasticity for an- Isotropic Media, *Quart. Appl. Math.*, 1980, **33**, 1-8.

[15] **Dhaliwal, R., Rokne**: Thermal Shock Problem in Generalized Thermoelastic,*J. Therm. Stresses*, 1989, **12**, 259.

[16] **Ezzat, M., Othman M. I. A.**: Electromagneto-thermoelasticity Plane Waves with Two Relaxation Times in a Medium of Perfect Conductivity, *Int. J. Engng Sci.*, 2000, **38**, 107-120.

[17] **Ezzat, M., Othman, M. I. A., El-Karamany, A. S.**: The Dependence of the Modulus of Elasticity on the Reference Temperature in Generalized Thermo- elasticity, *J. Therm. Stresses*, 2001, **24**, 1159-1176.

[18] **Ezzat, M., Othman, M. I. A., El-Karamany,A. S.**: Electromagneto-thermoelasticity Plane Waves with Thermal Relaxation in a Medium of Perfect Conductivity, *J. Therm. Stresses*, 2001, **24**,411-432.

[19] **Sherief, H. H., Helmy, K. A.**: A Two-Dimensional Problem for a Half-Space in Magneto-thermoelasticity with Thermal Relaxation, *Int. J. Engng Sci.*, 2002, **40**, 578-604 .

[20] **Nowacki,W.**: Dynamic Problems of Thermoelasticity, *Noordhoof Int., The Netherlands.* 1975.

[21] **Othman, M. I. A.**: Lord-Shulman Theory Under the Dependence of the Modulus of Elasticity on the Reference Temperature in Two-Dimensional Generalized Thermoelasticity, *J. Thermal Stresses*, 2002 **25**, 1027-1045.

[22] **Othman, M. I. A.**: Electrohydrodynamic Stability in a Horizontal Viscoelastic Fluid Layer in the Presence of a Vertical Temperature Gradient, *Int. J. Engng. Sci.*, 2001**39**, 1217-1232 .

[23] **Othman M. I. A., M. Ezzat**: Electromagneto-hydrodynamic Instability in Horizontal Viscoelastic Fluid Layer with One Relaxation Time, *Acta Mech.*,2001, 150, **1-2**, 1-9 .

Nomenclature

λ, μ	Lame's constants
ρ	density
C_E	specific heat at constant strain
$E(T)$	temperature dependent modulus of elasticity
α^*	empirical material constant [1/K]
E_o	constant modulus of elasticity at $\alpha^* = 0$
ν	Poisson's ratio
t	time
T	absolute temperature
α_T	coefficient of linear thermal expansion
δ_o	non-dimensional constant
c_o^2	$\frac{(\lambda_o+2\mu_o)E_o}{\rho}$
T_o	$\frac{\delta_o \rho c_o^2}{\gamma_o E_o} = \frac{\delta_o}{\alpha_T}(\frac{1-\nu}{1+\nu})$, reference temperature
σ_{ij}	components of stress tensor
ε_{ij}	components of strain tensor
$e = \varepsilon_{ii}$	dilatation
u_i	components of displacement vector
k	thermal conductivity
ν_o, τ_o	two relaxation times
ε_1	$\frac{\gamma_o E_o}{\rho C_E}$
$f(T)$	is a given nondimensional function of temperature
ε	$\varepsilon_1 \delta_o f(T)$
μ_o	$\frac{1}{2(1+\nu)}$
λ_o	$\frac{\nu}{(1+\nu)(1-2\nu)}$
γ_o	$\frac{\alpha_T}{1-2\nu}$
β	$\frac{E_o(\lambda_o+\mu_o)}{\rho c_o^2} = \frac{1}{2(1-\nu)}$
η_o	$\frac{\rho C_E}{k}$