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Mixed Convection on a Vertical Flat Plate with Variable Magnetic Field

M. F. Dimian and M. Kh. Hadhoda
Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt

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The steady laminar incompressible boundary layer mixed convection flow of an electrically conducting fluid on a vertical flat plate in the presence of an applied magnetic field has been studied. The effect of the induced magnetic field has been considered in the analysis. The resulting partial differential equations are transformed into a system of ordinary differential equations which have been solved numerically using shooting method. Two cases are considered here for the buoyancy force: (i) when it acts in the same direction as the forced flow ($T_w > T_\infty$), (ii) when it acts in the opposite direction to the forced flow ($T_w < T_\infty$). The velocity profiles, temperature profiles, the skin friction on the plate and the rate of heat transfer coefficient (Nusselt number) are computed and discussed for different values of the magnetic force number β, the thermal buoyancy force λ_1, reciprocal of the magnetic Prandtl number α and viscous dissipation parameter (Eckert number) Ec for the two cases.

Keywords: convection flow, conducting fluid, magnetic field

1. Introduction

The study of mixed convection flow finds applications in several industrial and technical processes such as nuclear reactors cooled during emergency shutdown, solar central receivers exposed to winds, electronic devices cooled by fans, heat exchangers placed in a low-velocity environment, etc.

The mixed convection flows become important when the buoyancy forces due to the temperature difference between the wall and the free stream becomes large. The mixed convection around heated vertical surfaces has been studied by Ramachandran et al. [1], Mahmood and Merkin [2] and Merkin and Mahmood [3]. They [1-3] have obtained similarity solutions of the governing equations. These studies deal with steady flows. The analogous unsteady case was recently studied by Surma Devi, et al. [4]. It has been observed that in a nuclear reactor, magnetic field affects considerably the flow and heat transfer. The steady forced convection flow over a flat plate with a magnetic field has been studied by Glauert [5] and Na [6].

Hossain and Ahmed [7] have studied a combined effect of forced and free convection with uniform heat flux in the presence of a strong magnetic field. Hossain [8] also studied the effect of viscous and Joule heating on the flow of an electrically conducting and viscous incompressible fluid past a semi infinite plate of which temperature varies linearly with the distance from the leading edge and in the presence of uniform transverse magnetic field. Ibrahim [9] also studied the effect of the magnetic field on boundary layer equations of a non-Newtonian power law fluid when the induced magnetic field is small. The study of unsteady laminar incompressible mixed convection flow of an electrically conducting fluid at the stagnation point of a two dimensional body and an axisymmetric body in the presence of an applied magnetic field has been studied by Kumari, et al. [10], they explained the effect of the induced magnetic field. The unsteady laminar boundary layer flow of an electrically conducting fluid past a semi infinite flat plate with an aligned magnetic field when at time t > 0, the plate is impulsively moved with a constant velocity which is in the same or opposite direction to that of free stream velocity has been studied by Takhar, et al. [11]. They solved the non linear partial differential equations numerically using finite-difference method.

The present paper is continuing of the last works. Taking into account the effect of viscosity and Joule heating in the heat equation also we take the effect of buoyancy force in the two cases. We will use the similarity solution and shooting method to solve the non linear partial differential equations. There are discussions of the results.

2. Formulation of the problem

Consider a vertical flat plate aligned parallel to a uniform free stream with velocity U and temperature T_∞. The plate is maintained at a constant temperature . Let x-axis and y-axis are in the direction of the plate upward and the normal to it, respectively and let the gravitational force acts in the direction opposite to the x direction as in Fig. 1. The buoyancy force then acts in the same direction as the forced flow when ($T_w > T_\infty$) in the opposite direction to the forced flow when ($T_w < T_\infty$).

We consider the steady laminar incompressible viscous electrically conducting fluid flow with constant properties. A magnetic field H_o is imposed parallel to the surface (i.e. along the x-axis) outside the boundary layer. The effects of the induced magnetic field, viscous dissipation and Joule heating have been included in the analysis. However, the Hall effect is neglected. It is assumed that there is no applied voltage which implies the absence of the electric field (i.e E = 0). The electrical currents flowing in the fluid give rise to an induced magnetic field which would exist if the fluid were an electrical insulator. Here it is assumed that the normal component of the induced magnetic field vanishes at the plate and the parallel component approaches its given value H_o far from the plate [5, 10, 23]. The plate is assumed to have constant temperature T_w. Under these assumptions, the approximation boundary layer equations governing the steady mixed convection flow under Boussinesq's approximations can be expressed as [10, 11, 12, 13].

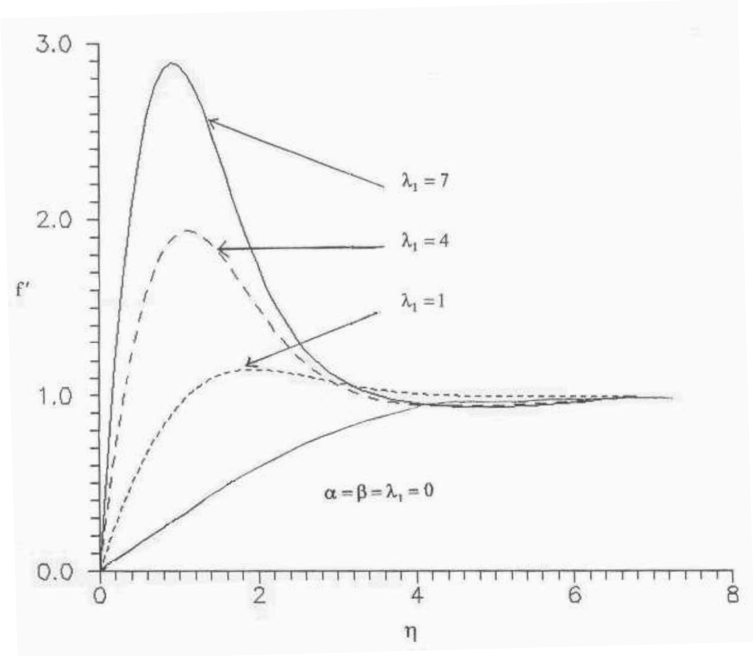

Figure 1 Variation of the velocity profile f' with λ_1 for the first case ($T_w > T_\infty$) when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0.2, \lambda_1 = 1, 4, 7$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \pm g\beta_T\left(T - T_\infty\right) + \frac{\mu_m}{\rho}\left(H_1\frac{\partial H_1}{\partial x} + H_2\frac{\partial H_1}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = K\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{C_p\rho\sigma}\left(\frac{\partial H_1}{\partial y}\right)^2 \tag{4}$$

$$\frac{\partial H_1}{\partial y} = \frac{1}{\alpha_1}\left(vH_1 - uH_2\right) \tag{5}$$

in which u and v are the components of the velocity of the fluid in x and y directions respectively, and are the induced magnetic field components in x and y direction respectively, ν is the kinematic viscosity, plus or minus sign in equation (3) corresponds to a positively or negatively buoyant force, g is the acceleration due to the gravity, β_T is the coefficient of thermal expansion, T is the temperature of the fluid, ρ is the density of the fluid, is the magnetic permeability, K is the thermal diffusivity, is the specific heat at constant pressure, is the electrical conductivity and is the magnetic diffusivity or magnetic viscosity [19] $\alpha_1 = (\sigma\ \mu_m)^{-1}$. The boundary conditions associated with equations (1)-(5) are

$$u = v = 0 \quad , \quad T = T_w \quad , \quad H_2 = 0 \qquad at \qquad y = 0 \tag{6}$$

$$u = U \quad , \quad T = T_\infty \quad , \quad H_1 = H_0 \qquad as \qquad y \to y_\infty \tag{7}$$

The continuity equations (1) and (2) can be satisfied by using a stream functions such that

$$u = \frac{\partial \psi_1}{\partial y}. \quad v = -\frac{\partial \psi_1}{\partial x}, \quad H_1 = \frac{\partial \psi_2}{\partial y}, \quad H_2 = -\frac{\partial \psi_2}{\partial x} \tag{8}$$

To transform equations (3-5) into a set of ordinary differential equations, we use the transformations: [10]

$$\begin{aligned} \eta &= \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y, \quad \psi_1(\eta) = (U\nu x)^{\frac{1}{2}} f(\eta) \\ \psi_2(\eta) &= H_0 \left(\frac{\nu x}{U}\right)^{\frac{1}{2}} \phi(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \end{aligned} \tag{9}$$

where is dimensionless distance from the plate, f() and are dimensionless stream function of velocity and magnetic field respectively and is the dimensionless temperature. By using equations (8) and (9), equations (3)-(7) transform to :

$$f''' + \frac{1}{2} f f'' - \frac{1}{2}\beta\phi\phi'' \pm \lambda_1 \theta = 0 \tag{10}$$

$$\theta'' + \frac{1}{2} \Pr f\,\theta' \pm \Pr Ec\left(f''^2 + \alpha\beta\phi''^2\right) = 0 \tag{11}$$

$$\phi'' + \frac{1}{2\alpha}(f\phi' - f'\phi) = 0 \tag{12}$$

with boundary conditions :
at

$$a)\quad \eta = 0, \quad b)\ f(0) = f'(0) = 0, \quad c)\ \theta(0) = 1, \quad d)\ \phi(0) = 0 \tag{13}$$

as

$$\eta \to \eta_\infty, \quad a)\ f'(\eta_\infty) = 1, \quad b)\ \theta(\eta_\infty) = 0, \quad c)\ \phi'(\eta_\infty) = 1 \tag{14}$$

where $\beta = \frac{\mu_m H_0^2}{\rho U^2}$ is the magnetic force number which is the square of the ratio of the Alfven wave velocity [20] $U_A = \left(\frac{\mu_m H^2}{\rho}\right)^{1/2} = \frac{B}{(\mu_m \rho)^{1/2}}$, $B = \mu_m H$ to free stream velocity; is the ratio of Grashof number $Gr = \frac{g\,\beta_T (T_w - T_\infty)\,x^3}{4\,\nu^2}$ to the square of Reynolds number $Re(x) = \frac{x\,U}{\nu}$ and it is represented as the thermal buoyancy force. We notice that λ_1 is a dimensionless quantity but it is a function of x. In the similarity solution we can give λ_1 any value and in this case $Gr(x)$ and $Re(x)$ are called local Grashof number and local Reynolds number [1, 24, 25]. $\Pr = \frac{\nu}{K}$ is Prandtl number; is Eckert number (viscous dissipation parameter) and $\alpha = \frac{\alpha_1}{\nu} = \frac{(\sigma\mu_m)^{-1}}{\nu} = (\sigma\mu_m\nu)^{-1}$ is the reciprocal of the magnetic Prandtl number

Pr_m, which is the ratio of the kinematic viscosity to the magnetic diffusivity. Thus one can define the magnetic Prandtl number as $\mathrm{Pr}_m = \frac{1}{\alpha} = \frac{R_m}{Re}$, where R_m is magnetic Reynolds number [21, (Chapter 10)] ($R_m = \frac{U\,x}{\alpha_1}$) and Re is Reynolds number. Plus or minus ($\pm$) in equations (10) and (11) are according to the direction of the buoyancy force with the direction of the forced flow or in opposite to it respectevily. The prime denotes derivative with respect to η.

3. Numerical solution:

The nonlinear ordinary differential equations (10) , (11) and (12) with the boundary conditions (13) and (14) have been solved by the fourth-order Runge Kutta integration scheme along with the Nachtshem- Swigert shooting technique [14] with error of order 10^{-4}. The procedure is to estimate the unknown values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$.

In order to verify the accuracy of our present method, we have compared our results with those of Pop et al. [15] in their special case for the viscosity is constant i.e. μ= constant and if we have $\beta = \lambda_1 = Ec = 0$, our equations and their equations will reduce to

$$f''' + \frac{1}{2} f\, f'' = 0 \tag{15}$$

$$\theta'' + \frac{1}{2}\ \mathrm{Pr}\ f\, \theta' = 0 \tag{16}$$

If the boundary conditions are : at $\eta = 0$: f(0) = 0, $f'(0) = 0$ and $\theta(0) = 1$; as $\eta \to \infty$: $f'(\infty) = 1$, $\theta(\infty) = 0$. Our results were $f''(0) = -0.4445887$ and $\theta'(0) = -0.3542833$, but in Pop, et al. [15] $f''(0) = -0.4445517$ and $\theta'(0) = -0.3507366$. Therefore our results are in very good agreement with [15]. In the special case, when $\beta = \lambda_1 = 0$, equation (10) together with the boundary conditions (13a, b) and equation (14a) reduces to the well known Blasius equation ($f''' + \frac{1}{2} f\, f' = 0$). Our result for this special case was $f''(0) = 0.3320654$, which agrees very well with Schlichting [16].

4. Results and discussion

4.1. *Thermal buoyancy force λ_1 in the same direction of the forced flow $T_w > T_\infty$*

In the computation performed, we can find important results. From figure (1), it can be seen that:

1. In the interval ($0 < \eta \cong 3.2$), the dimensionless longitudinal velocity f' increases as the thermal buoyancy force$\lambda_1$increases.

2. In the middle of this interval, the velocity f' has values more than one (the outer flow velocity). This overshoot is due to the effect of the buoyancy force (λ_1) on the velocity.

3. After this interval i.e. far from the plate the effect of the buoyancy force λ_1 is very weak.

From figure (2), it is noticed that Eckert number (Ec) also increases this overshoot of the velocity, so the velocity f' increases as Eckert number Ec (viscous dissipation parameter) increases. Figure (3) illustrates that there is an overshoot in the temperature near the plate for values $\lambda_1 \geq 4$. That is near the plate, the temperature θ increases as the buoyancy force λ_1 increases. This result agrees with Kumari, et. al. [10] and Ibrahim and Terbeche [22] for their case of non-Newtonian fluid (when n is kept constant). Also from this figure, we notice that at Eckert number $Ec = 0$ there is no overshoot in the temperature but when $Ec = 0.2$ the overshoot appears which prove that the cause of this overshoot is due to the existence of Eckert number i.e. the last two terms in equation (4) which are the viscosity term and Joule heating. Important result is also noticed from this figure that the temperature θ increases as the buoyancy force λ_1 increases for $Ec > 0$ but the temperature θ decreases as the buoyancy force λ_1 increases for $Ec = 0$(there is no viscous dissipation). From figure (4), it is observed that near the plate (for $\eta < 1$) the value of the velocity f' approximately does not depend on the value of the magnetic field parameter . But after this interval (for $\eta > 1$), the dimensionless velocity f' increases as the magnetic field parameter increases. Figure (5) represents the effect of the reciprocal of the magnetic Prandtl number (α) on the velocity. We found that as α increases the velocity f' decreases. That is the magnetic Prandtl number has direct effect on the velocity f' . Numerical calculations are carried out for the effects of the magnetic field parameter $\beta$and the reciprocal of the magnetic Prandtl number (α) on the temperature θ, it is found that they have very small effects.

4.2. *Thermal buoyancy force in the opposite direction of the forced flow* $T_w < T_\infty$

$$f''' + \frac{1}{2}\, f\, f'' - \frac{1}{2}\,\beta\,\phi\,\phi'' - \lambda_1\,\theta = 0 \tag{17}$$

$$\theta'' + \frac{1}{2}\,\mathrm{Pr}\; f\,\theta' - \mathrm{Pr}\; Ec\,(f''^2 + \alpha\,\beta\,\phi''^2) = 0 \tag{18}$$

$$\phi'' + \frac{1}{2\alpha}(f\phi' - f'\phi) = 0 \tag{19}$$

with the boundary conditions (13) and (14). The transformation (in this case)

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_w < T_\infty \tag{20}$$

illustrates that the dimensionless temperature θ decreases as the temperature T increases. Numerical calculations are carried out for this case as before in the case ($T_w < T_\infty$). From figure (6) it can be seen that:

1. The dimensionless longitudinal velocity f' near the plate has negative value in the interval $0 \leq \eta \leq 1$ due to the effect of the opposite direction of the buoyancy force parameter. Also this negative value according to the effect of the normal componant of the induced magnetic field H_2

$$H_2 = \frac{H_0}{2}\sqrt{\frac{\nu}{Ux}}\;(\eta\phi'(\eta) - \phi(\eta))$$

In this interval, H_2 takes negative value.

2. In the interval $0 \leq \eta \leq 3.2$, the velocity f' increases as the buoyancy parameter increases.

3. Far from the plate $(3.2 \leq \eta \leq \eta_\infty)$ the dominant force is due to the value of the induced magnetic field component H_1. Consequently an inverse effect for on f' is noticed.

4. It is observed also that in the middle zone of the velocity boundary layer $(2 \leq \eta \leq \eta_\infty)$, the velocity f' increases and has value more than one (the outer flow velocity).

This overshoot is due to the effect of the buoyancy parameter and Eckert number Ec. Figure (7) represents the effect of the parameter on the temperature θ. It is noticed that the temperature θdecreases as increases in the interval $0 \leq \eta \leq 2$ but an inverse behavior after this interval is noticed. Figures (8), (9) and (10) illustrat the effects of the magnetic field parameter , the reciprocal of the magnetic Prandtl number α and Eckert number Ec on the velocity respectively. They have the same behavior on the velocity as the buoyancy parameter . Figure (8) illustrates that the magnitude of the velocity f' increases very small as the magnetic field parameter increases in the interval $0 < \eta \cong 3$, but an inverse behavior after this interval is noticed. Figure (9) illustrates that the dimensionless velocity f' increases as the reciprocal of the magnetic Prandtl numberα increases. Also from figure (10), it is noticed that the velocity f' increases as Eckert number Ec increases in the interval $0 \leq \eta \leq 2.4$, but after this interval f' decreases as Eckert number Ec increases. From figure (11) it is noticed that increasing of Ec causes decreasing of the temperature θ until $\eta \cong 2$ (this result is different from the result in the case $(T_w < T_\infty)$). For $\eta > 2$ the temperature θ increases as Ec increases. The effects of the magnetic field parameter and the reciprocal of the magnetic Prandtl number α on the temperature θ are calculated numerically and found that their effects are very small.

The physical quantities of interest in this problem are the skin friction coefficient $f''(0)$, the dimensionless coefficient of heat transfer $-\theta'(0)$ i.e. the Nusselt number and the displacement thickness (the boundary layer thickness) δ_1 which are defined respectively as ([15] and [22]):

$$C_f = \frac{2\tau_w}{\rho U^2}, \quad \tau_w = \mu_w(\partial u/\partial y)_{y=0} \tag{21}$$

$$N_u = \frac{xq_w}{k(T_w - T_\infty)}, \quad q_w = -k(\partial T/\partial y)_{y=0} \tag{22}$$

$$\delta_1 = \int_0^{y_\infty} (1 - \frac{u}{U})\, dy = \int_0^{\eta_\infty} (1 - f')\, d\eta \tag{23}$$

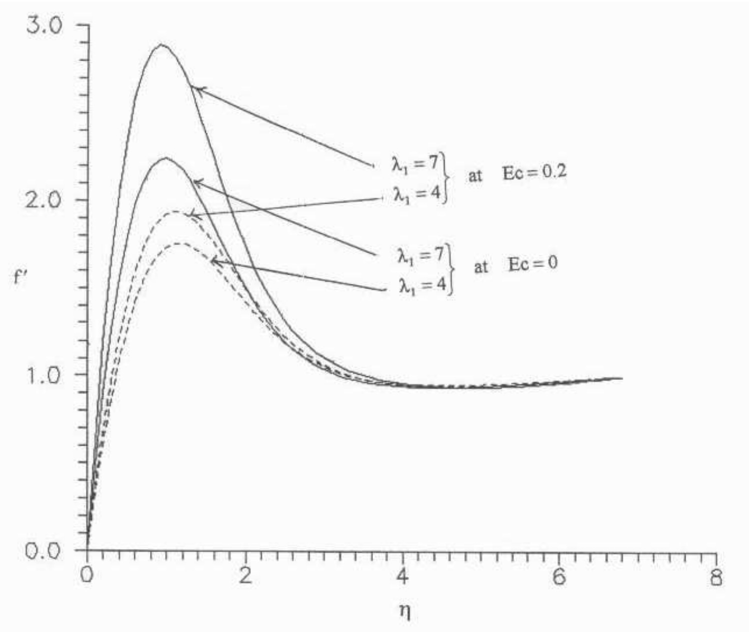

Figure 2 Variation of the velocity profile f' with λ_1 for the first case $(T_w > T_\infty)$ when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0, 0.2, \lambda_1 = 4, 7$

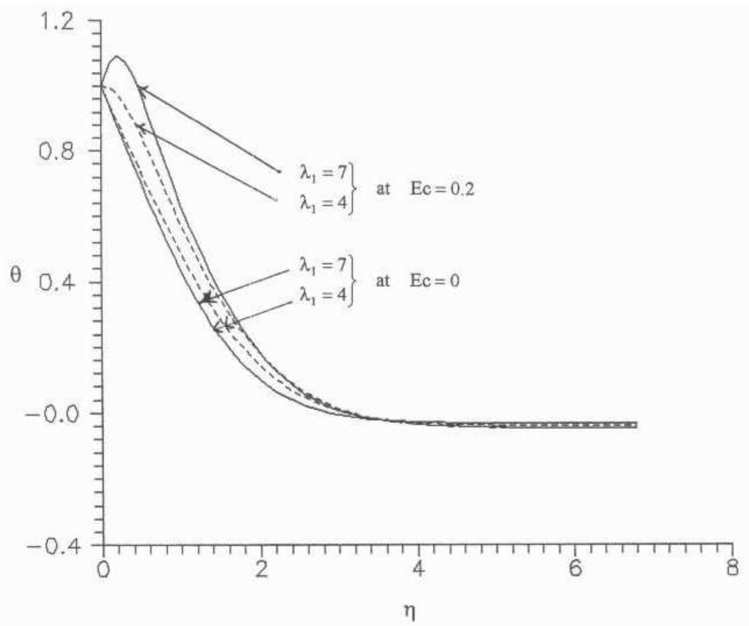

Figure 3 Variation of the temperature profile θ with λ_1 for the first case $(T_w > T_\infty)$ when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0, 0.2, \lambda_1 = 4, 7$

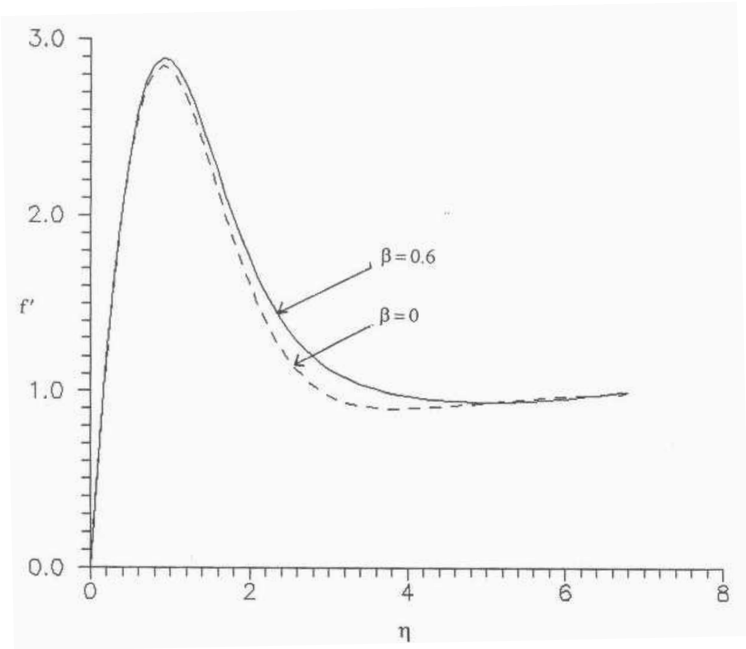

Figure 4 Variation of the velocity profile f' with β for the first case ($T_w > T_\infty$) when $Pr = 0.7, \beta = 0, 0.6, \alpha = 6, Ec = 0.2, \lambda_1 = 0.5$

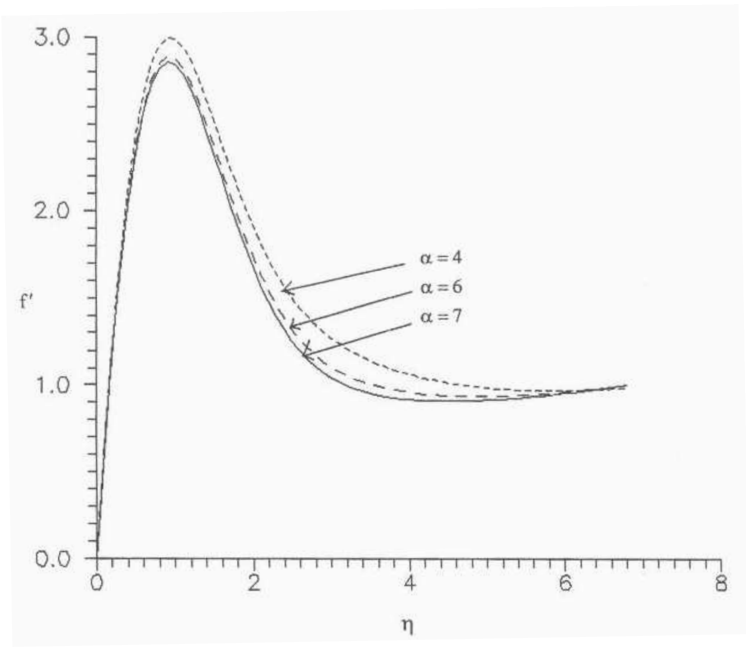

Figure 5 Variation of the velocity profile f' with α for the first case ($T_w > T_\infty$) when $Pr = 0.7, \beta = 0.5, \alpha = 4, 6, 7, Ec = 0.2, \lambda_1 = 7$

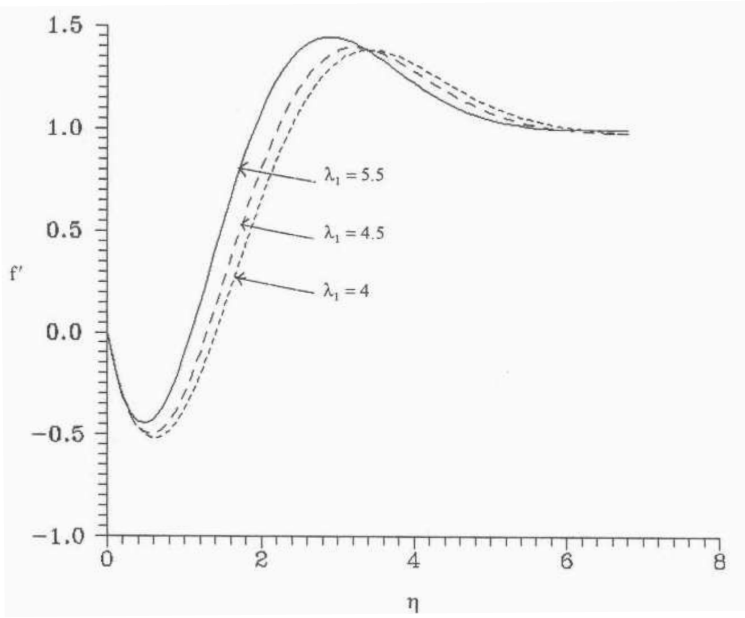

Figure 6 Variation of the velocity profiles f' with λ_1 for the second case ($T_w < T_\infty$) when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0.7, \lambda_1 = 4, 4.5, 5.5$

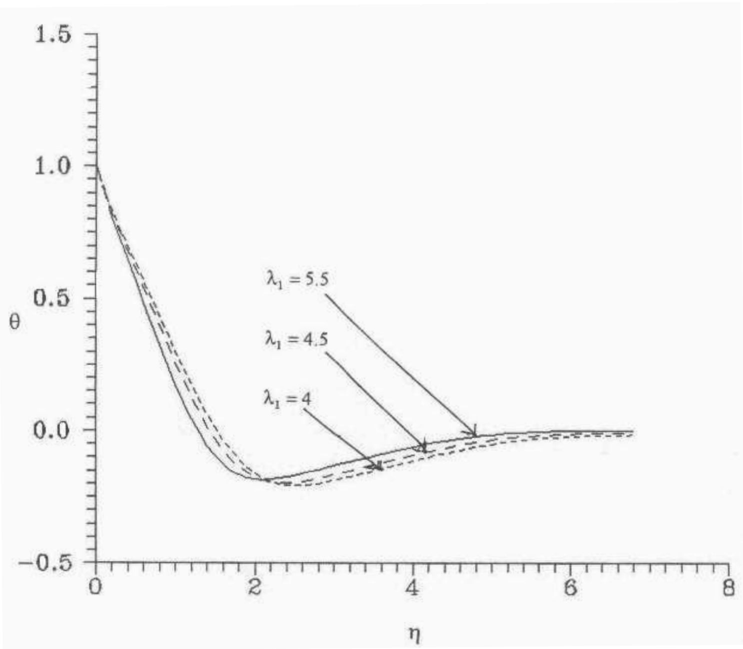

Figure 7 Variation of the temperature profile f' with λ_1 for the second case ($T_w < T_\infty$) when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0.7, \lambda_1 = 4, 4.5, 5.5$

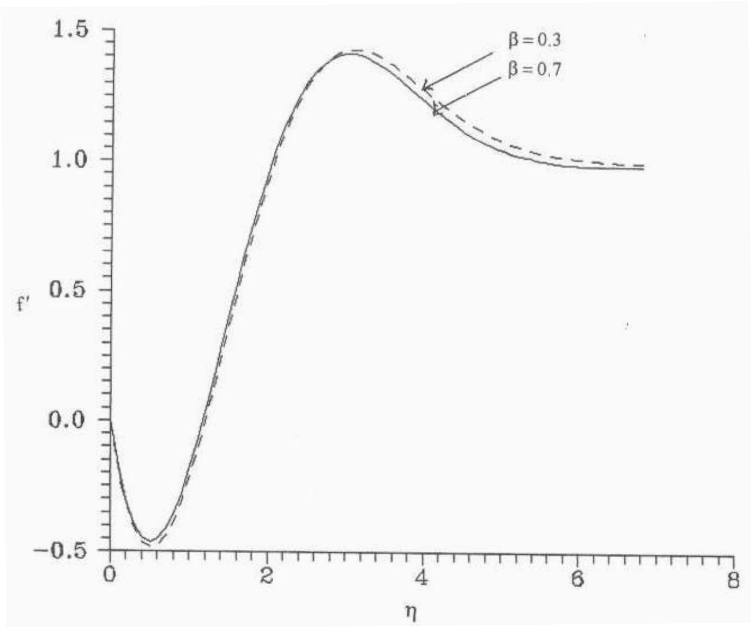

Figure 8 Variation of the velocity profiles f' with β for the second case $(T_w < T_\infty)$ when $Pr = 0.7, \beta = 0.3, 0.7, \alpha = 6, Ec = 0.7, \lambda_1 = 5$

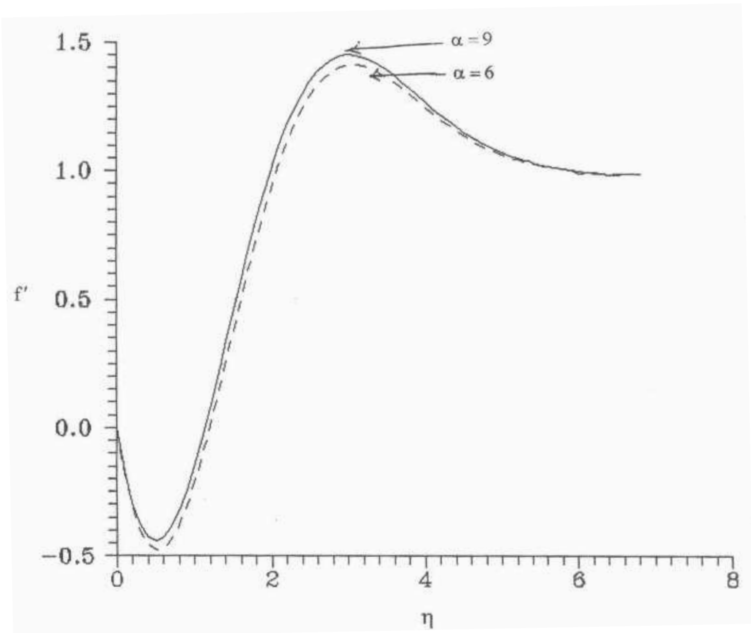

Figure 9 Variation of the velocity profiles f' with α for the second case $(T_w < T_\infty)$ when $Pr = 0.7, \beta = 0.5, \alpha = 5, 9, Ec = 0.7, \lambda_1 = 5$

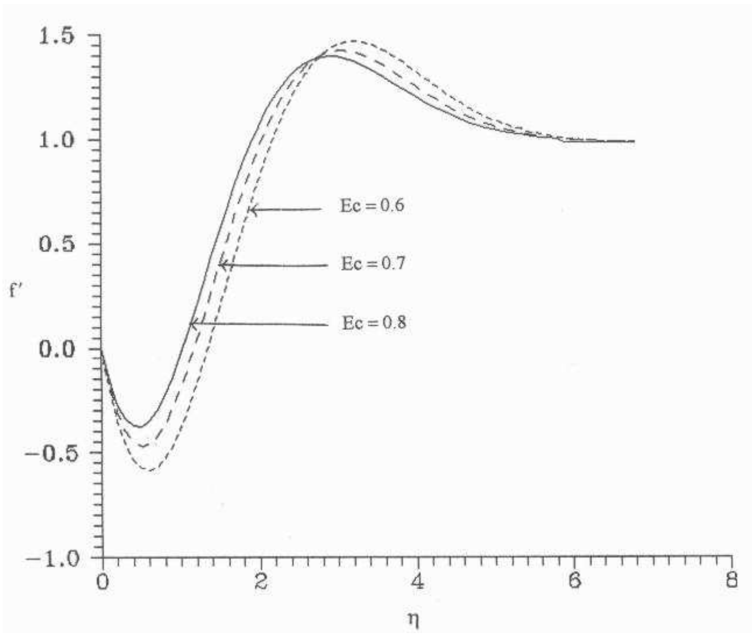

Figure 10 Variation of the velocity profiles f' with Ec for the second case ($T_w < T_\infty$) when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0.6, 0.7, 0.8, \lambda_1 = 5$

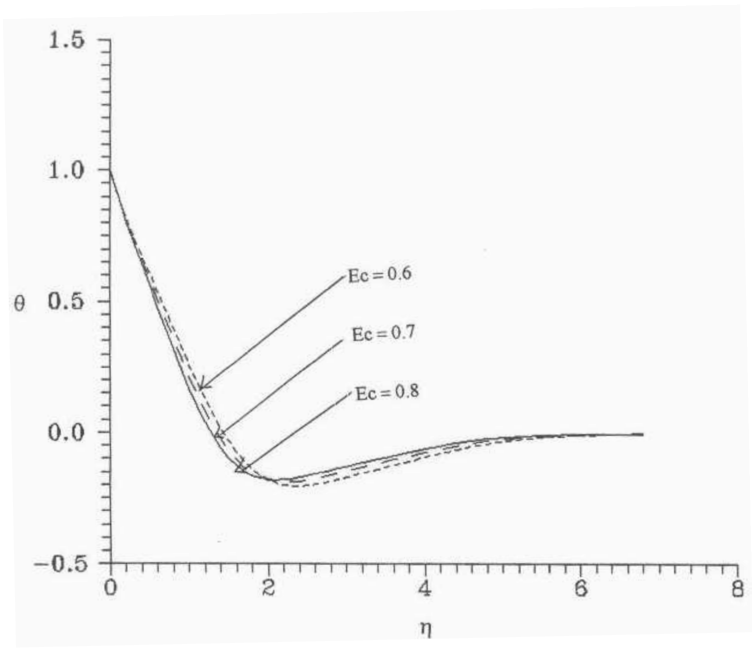

Figure 11 Variation of the temperature profiles f' with Ec for the second case ($T_w < T_\infty$) when $Pr = 0.7, \beta = 0.5, \alpha = 6, Ec = 0.60.7, 0.8, \lambda_1 = 5$

Using (8) and (9), quantities (26) and (27) can be expressed as

$$C_f \;=\; 2(Re)^{\frac{-1}{2}} f''(0), \quad Nu = -(Re)^{\frac{1}{2}} \theta'(0) \tag{24}$$

From table (1); $f''(0)$, $\theta'(0)$ and $\phi'(0)$ are listed for Prandtl number $Pr = 0.7$ and several sets of values of β, λ_1 , $\alpha$and Ec. It is seen that for increasing the magnetic force parameter β, all of the skin friction coefficient $f''(0)$; $f''(0) = \left[(Re)^{1/2}\, C_f/2\right]$, heat transfer coefficient in term of Nusselt number $\theta'(0)$; $\theta'(0) = \left[-(Re)^{-1/2} Nu\right]$ and the induced magnetic field $\phi'(0)$ increase. These results agree with Kumari, et al. [10]. Also the effects of the buoyancy force parameter λ_1 and Eckert number Ec on $f''(0)$, $\theta'(0)$ and $\phi'(0)$are the same as the magnetic force parameter β i.e. $f''(0)$, $\theta'(0)$ and $\phi'(0)$ increase as the buoyancy parameter λ_1 increases also as Eckert number Ec increases. But an inverse behavior of the reciprocal of the magnetic Prandtl number α on them. That is $f''(0)$, $\theta'(0)$ and $\phi'(0)$ decrease as α increases.

Table 1 Values of the skin friction $f''(0)$, the local Nusselt number $\theta'(0)$ and the induced magnetic field $\phi'(0)$ with $Pr = 0.7$ for the case $T_w > T_\infty$

β	λ_1	α	Ec	$f''(0)$	$\theta'(0)$	$\phi'(0)$
0	7	6	0.2	6.59578	0.97839	1.58533
0.2	7	6	0.2	6.62086	0.98768	1.60411
0.4	7	6	0.2	6.63612	0.99088	1.61463
0.6	7	6	0.2	6.64502	0.99335	1.62865
0.5	1	6	0.2	1.41021	-0.28571	0.89167
0.5	3	6	0.2	3.03018	-0.10649	1.12282
0.5	5	6	0.2	4.68836	0.28617	1.35001
0.5	7	6	0.2	6.64233	0.99166	1.61973
0.5	7	4	0.2	6.80785	1.07912	1.79468
0.5	7	5	0.2	6.71417	1.02904	1.70013
0.5	7	6	0.2	6.64423	0.99282	1.62162
0.5	7	7	0.2	6.59297	0.96721	1.56469
0.5	7	6	0	5.19609	-0.59447	1.35373
0.5	7	6	0.1	5.72216	-0.02262	1.45275
0.5	7	6	0.2	6.64302	0.99208	1.62033

From table (2) which represents the second case for cooling plate ($T_w < T_\infty$). we can notice that the skin friction $f''(0)$, the local Nusselt number$\theta'(0)$ and the induced magnetic field $\phi'(0)$ increase as the magnetic field parameter β and the reciprocal of the magnetic Prandtl number increase as in the first case. But an inverse behavior for the buoyancy parameter λ_1. It is noticed that only $\phi'(0)$ increases but

$f''(0)$ and $\theta'(0)$ decrease as the buoyancy parameter λ_1 increases. Finally $f''(0)$ and $\phi'(0)$ increase as Eckert number Ec (viscous dissipation parameter) increases but $\theta'(0)$ decreases as Eckert number (viscous dissipation parameter) increases.

Table 2 Values of the skin friction $f''(0)$, the local Nusselt number $\theta'(0)$ and the induced magnetic field $\phi'(0)$with $\Pr = 0.7$ for cooling plate ($T_w < T_\infty$)

β	λ_1	α	Ec	$f''(0)$	$\theta'(0)$	$\phi'(0)$
0.3	5	6	0.7	-2.02626	-1.04905	0.61019
0.4	5	6	0.7	-2.01089	-1.04676	0.61641
0.5	5	6	0.7	-1.99778	-1.04465	0.62099
0.6	5	6	0.7	-1.98709	-1.04368	0.62830
0.5	4	6	0.7	-1.87022	-0.94976	0.56822
0.5	4.5	6	0.7	-1.94942	-1.00083	0.59416
0.5	5	6	0.7	-1.99778	-1.04465	0.62099
0.5	5.5	6	0.7	-2.04826	-1.08894	0.64819
0.5	5	5	0.7	-2.01649	-1.04831	0.58736
0.5	5	7	0.7	-1.99606	-1.04463	0.65274
0.5	5	8	0.7	-1.96974	-1.04033	0.68022
0.5	5	9	0.7	-1.94704	-1.03688	0.70425
0.5	5	6	0.6	-2.21526	-1.03046	0.58339
0.5	5	6	0.65	-2.21526	-1.03816	0.60497
0.5	5	6	0.7	-1.98554	-1.04384	0.62423
0.5	5	6	0.8	-1.80303	-1.06216	0.70425

5. Conclutions

The effect of buoyancy-induced streamwise pressure gradients on laminar forced convective flow and heat transfer over a vertical flat plate are studied analytically by the local similarity method of solution in tow cases for the buoyancy force:

1. when it acts in the same direction as the forced flow ($T_w > T_\infty$),
2. when it acts in the opposite direction to the forced flow ($T_w < T_\infty$).

We found that:

1. In the first case ($T_w > T_\infty$) i.e. for heating plate, the longitudinal dimensionless longitudinal velocity $f'$and the temperature θ of the fluid are increasing as the buoyancy parameter λ_1 and the the magnetic field parameter β increase. But they (f' and θ) decrease as the reciprocal of the magnetic Prandtl number increases. In the second case ($T_w < T_\infty$) i.e. for cooling the plate, the longitudinal dimensionless longitudinal velocity f' and the temperature θ of the fluid decrease as the buoyancy parameter λ_1 increases (near the plate).
2. In the two cases, near the plate ($\eta < 1$), the effect of the magnetic field is very weak on the velocity f'. That is f' does not depend on the magnetic field near the plate, but far from the plate ($\eta > 1$), f' increases as the magnetic field parameter β increases.

3. For heating plate ($T_w > T_\infty$); the skin friction coefficient $f''(0)$, heat transfer coefficient $\theta'(0)$ and the induced magnetic field $\phi'(0)$ increase as the magnetic force parameter β and the buoyancy parameter λ_1 increase. But for cooling plate ($T_w < T_\infty$), it is noticed that only the induced magnetic field $\phi'(0)$ increases as the buoyancy parameter λ_1 increases but the skin friction coefficient $f''(0)$ and heat transfer coefficient $\theta'(0)$ decrease as buoyancy parameter λ_1 increases.

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